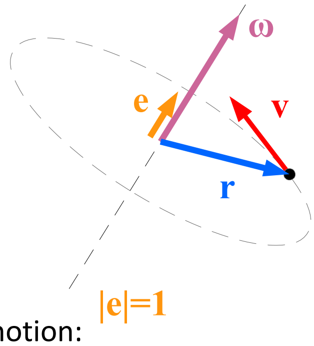
THE MOST IMPORTANT FORMULAE

VECTOR DESCRIPTION OF MOTION

- **Position vector** $\mathbf{r}(t)=[x(t)\ ;\ y(t)\ ;\ z(t)]$
- **Velocity vector** $\mathbf{v}(t)=\dot{\mathbf{r}}(t)=[\dot{x}(t)\ ;\ \dot{y}(t)\ ;\ \dot{z}(t)]$
- **Acceleration vector** $\mathbf{a}(t)=\ddot{\mathbf{r}}(t)=[\ddot{x}(t)\ ;\ \ddot{y}(t)\ ;\ \ddot{z}(t)]$

  - **Tangent acceleration vector** $\mathbf{a}_s = \dfrac{\mathbf{a}\circ\mathbf{v}}{|\mathbf{v}|^2}\cdot\mathbf{v}$
  - **Normal acceleration vector** $\mathbf{a}_n = \mathbf{a}-\mathbf{a}_s$

Rotational motion



- **Angular distance** $\alpha(t)$
- **Angular velocity** $\omega(t) = \dot{\alpha}(t)$ $\quad \boldsymbol{\omega}=\omega\,\mathbf{e}$
- **Angular acceleration** $\varepsilon(t) = \ddot{\alpha}(t)$ $\quad \boldsymbol{\varepsilon}=\varepsilon\,\mathbf{e}$

Other measures of motion expressed by measures of rotational motion:

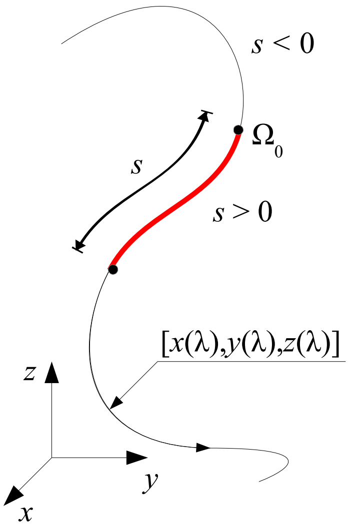
- **velocity vector** $\mathbf{v} = \boldsymbol{\omega}\times\mathbf{r}$
- **tangent acceleration vector** $\mathbf{a}_s = \boldsymbol{\varepsilon}\times\mathbf{r}$
- **normal acceleration vector** $\mathbf{a}_n = \boldsymbol{\omega}\times\mathbf{v}$

Rotation in plane XY around fixed axis $\mathbf{r}(t) = [R\cos(\alpha(t))\ ;\ R\sin(\alpha(t))\ ;\ 0]$

NATURAL DESCRIPTION OF MOTION

- **Parametric equation of trajectory** –system of equations for coordinates of a point on trajectory, dependent on chosen parameter.

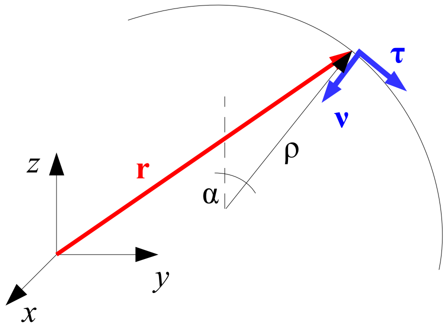
$$\begin{cases} x = x(\lambda) \\ y = y(\lambda) \\ z = z(\lambda) \end{cases}$$



- **Initial point** Ω_0
- **Orientation** – an agreement on on what side of the initial point the measure of distance is considered positive or negative.
- **Equation of motion –** a time dependent information on the distance *s* covered within time *t*: $s = s(t)$. Measure of distance (also „measure of length") means that it may be positive or negative,depending on chosen orientation of trajectory. A specific case is when the trajectory is parametrized with co calle *natural parameter* $\lambda = s$, the absolute value of which is equal the length of the section of trajectory starting from the initial point.

MEASURES OF MOTION IN NATURAL DESRIPTION

- **Tangent versor** $\boldsymbol{\tau} = \dfrac{\mathrm{d}\mathbf{r}}{\mathrm{d}s}$
- **Normal versor** $\boldsymbol{\nu} = \dfrac{\mathrm{d}\boldsymbol{\tau}}{\mathrm{d}\alpha}$
- **Velocity** $\mathbf{v} = \dot{s}\,\boldsymbol{\tau}$
- **Acceleration** $\mathbf{a} = \ddot{s}\,\boldsymbol{\tau} + \dfrac{\dot{s}^2}{\rho}\boldsymbol{\nu}$
  - **Tangent acceleration** $\mathbf{a}_s = \ddot{s}\,\boldsymbol{\tau}$
  - **Normal acceleration** $\mathbf{a}_n = \dfrac{\dot{s}^2}{\rho}\boldsymbol{\nu}$



VECTOR DESCRIPTION → NATURAL DESCRIPTION

1. Position vector $\mathbf{r}(t)$ gives us the equations of trajectory which may be parametrized with time $\lambda = t$

$$\mathbf{r}(t) \quad \Rightarrow \quad \mathbf{r}(\lambda = t): \begin{cases} x = x(\lambda) \\ y = y(\lambda) \\ z = z(\lambda) \end{cases}$$

2. Initial point Ω_0 is the one corresponding with $t=0$.
3. Equation of motion is obtained via integration along trajectory starting from $t=0$:

$$s = \int ds = \int_{\lambda_0=0}^{\lambda=t} \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}\lambda}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}\lambda}\right)^2 + \left(\frac{\mathrm{d}z}{\mathrm{d}\lambda}\right)^2}\,\mathrm{d}\lambda \quad \Rightarrow \quad s(t)$$

4. Orientation is chosen in such a way that the measure of a length of an arc of trajectory was positive at that side towards which the motion is performed.

NATURAL DESCRIPTION → VECTOR DESCRIPTION

1. Having the trajectory parametrized with λ , we may find the measure of length of section of the trajectory. It is found as a curvilinear integral of a unitary scalar field along trajectory. Lower bound of integration is the value of parameter corresponding with initial point.

$$s(\lambda) = \int_{\Omega_0}^{P} ds = \int_{\lambda_0}^{\lambda} \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}\lambda}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}\lambda}\right)^2 + \left(\frac{\mathrm{d}z}{\mathrm{d}\lambda}\right)^2}\,\mathrm{d}\lambda$$

2. This measure must be equal the equation of motion.

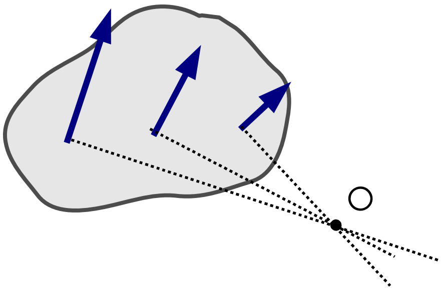
$$s(\lambda) = \int_{\lambda_0}^{\lambda} \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}\lambda}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}\lambda}\right)^2 + \left(\frac{\mathrm{d}z}{\mathrm{d}\lambda}\right)^2}\,\mathrm{d}\lambda = \pm s(t)$$

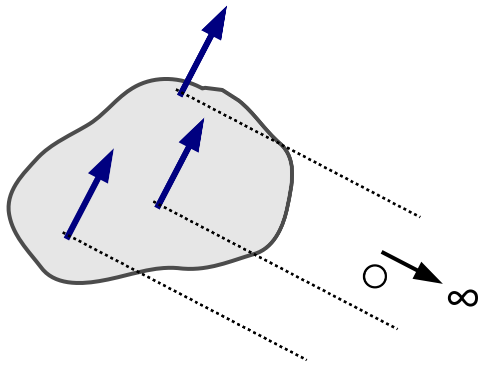
The sign is according to the chosen orientation – since $s(\lambda)$ will always increase with increase of λ (the integrand is positive), so if the point moves from the initial point towards positive orientation (function $s(t) > 0$), then in the above equation we take „+", otherwise „-".

3. As a result we obtain dependency $\lambda(t)$, which may be substituted in the equations of trajectory giving us a vector desription.

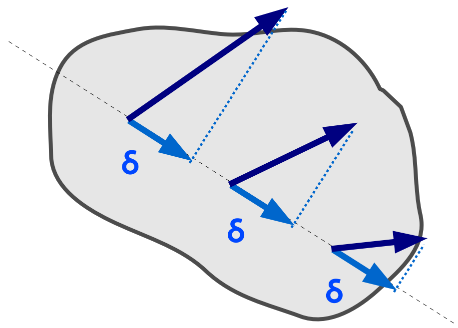
THEOREMS ON DISTRIBUTION OF VELOCITY IN RIGID BODY

1. **Any planar motion of a rigid body in any time *t* may be considered a rotation about an instant center of rotation.**
   - In general, instant center of rotation moves itself
   - In particular the instant center of rotation may be located in infinity but in certain direction – the rotation becomes then a parallel translation in a perpendicular direction.

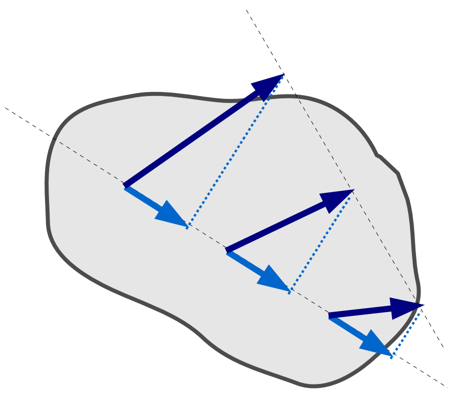




2. **For points lying on a single straight line projections of their velocity vectors on that line are the same.**



3. **For points lying on a single straight line the heads of their velocity vectors line on a single straight line (in general a different one).**

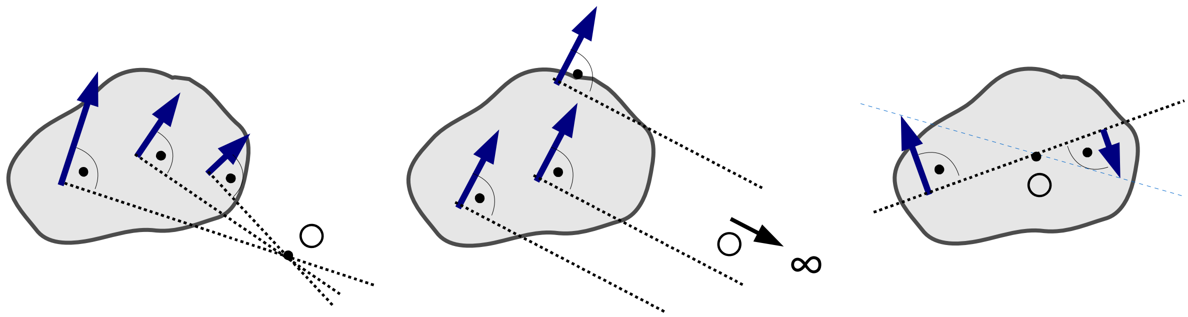


CONCLUSION

1. Velocity vector is always perpendicular to a line connecting the given point with the instant center of rotation.

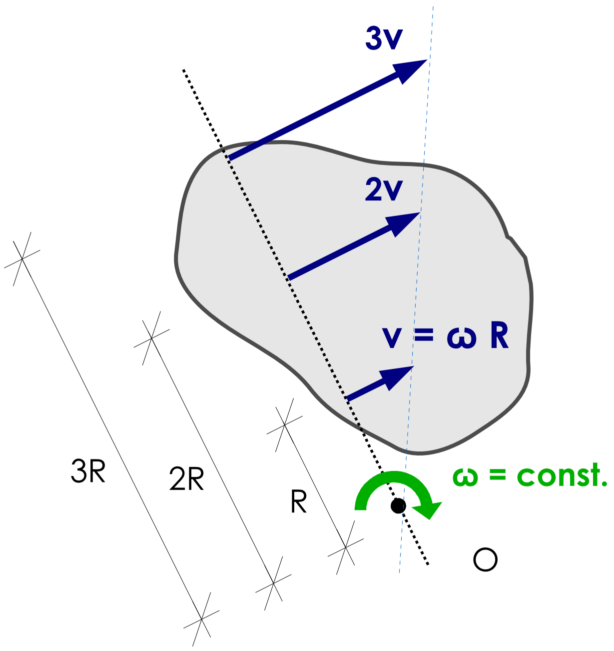
2. If we know directions of velocity vectors in two points not lying on a line, which is perpendicular to those directions, then the instant center of rotation lies at intersections of lines, which are perpendicular to the velocity vectors (in particular the intersection may be in infinity for parallel lines)

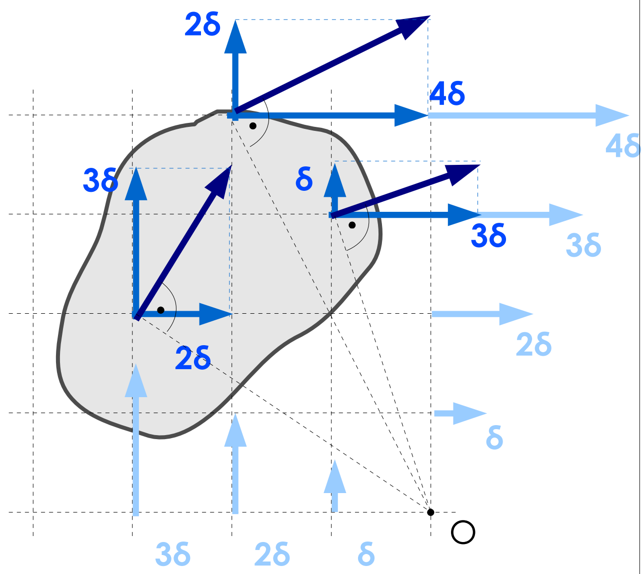
3. If we know directions of velocity vectors in two points lying on a line, which is perpendicular to those directions, then the instant center of rotation lies at intersections of this line line with a line connecting the heads of velocity vectors.



4. Velocity vectors of points lying on a single straight line connecting them with instant center of rotation have their magnitudes proportional to the distance from this center. Proportionality coefficient is the angular velocity.

5. Vertical components of velocity vector may be translated vertically.
   Horizontal components of velocity vector may be translated horizontally.





EXERCISE 1

Motion of a point is given in vector description:

$$\mathbf{r}(t) = \begin{cases} x(t) = 4t^2+1 \\ y(t) = -2t \\ z(t) = 3t^2-2 \end{cases}$$

Find the velocity vector, acceleration vector and its tangent and normal components..

ROZWIĄZANIE:

Velocity vector:

$$\mathbf{v}(t) = \frac{\mathrm{d}\,\mathbf{r}}{\mathrm{d}\,t} = \begin{cases} \dfrac{\mathrm{d}\,x}{\mathrm{d}\,t} = 8t \\ \dfrac{\mathrm{d}\,y}{\mathrm{d}\,t} = -2 \\ \dfrac{\mathrm{d}\,z}{\mathrm{d}\,t} = 6t \end{cases}$$

Acceleration vector:

$$\mathbf{a}(t) = \frac{\mathrm{d}\,\mathbf{v}}{\mathrm{d}\,t} = \begin{cases} \dfrac{\mathrm{d}^2\,x}{\mathrm{d}\,t^2} = 8 \\ \dfrac{\mathrm{d}^2\,y}{\mathrm{d}\,t^2} = 0 \\ \dfrac{\mathrm{d}^2\,z}{\mathrm{d}\,t^2} = 6 \end{cases}$$

Tangent acceleration vector is obtained via projection of acceleration vector on direction of velocity vector:

$$\mathbf{a}_s = \frac{\mathbf{a} \circ \mathbf{v}}{|\mathbf{v}|^2} \cdot \mathbf{v} = \frac{8\cdot(8t)+0\cdot(-2)+6\cdot(6t)}{\sqrt{(8t)^2+(-2)^2+(6t)^2}}\,[8t\ ;\ -2\ ;\ 6t] = \frac{100t}{100t^2+4}\,[8t\ ;\ -2\ ;\ 6t] =$$
$$= \left[\frac{200t^2}{25t^2+1}\ ;\ -\frac{50t}{25t^2+1}\ ;\ \frac{150t^2}{25t^2+1}\right]$$

Normal acceleration vector:

$$\mathbf{a}_n = \mathbf{a} - \mathbf{a}_s = \left[8-\frac{200t^2}{25t^2+1}\ ;\ \frac{50t}{25t^2+1}\ ;\ 6-\frac{150t^2}{25t^2+1}\right] = \left[\frac{8}{25t^2+1}\ ;\ \frac{50t}{25t^2+1}\ ;\ \frac{6}{25t^2+1}\right]$$

EXERCISE 2

A material point moves along a circle of radius $R = 2\ \mathrm{m}$. Distance covered in time *t* is describet by function $s(t) = 2t^2$.

Find:

- Velocity vector and acceleration vector
- Angular velocity and angular acceleration
- Angular velocity vector and angular acceleration vector. With the use of those quantities find the velocity vector as well as tangent and normal acceleration vectors.

ROZWIĄZANIE:

Let's determine position vector. Let's chose a Cartesian coordinate system, the origin of which lies in the center of circle and the initial point lies on x axis. Position of any point of this circle is described with equations:

$$\mathbf{r} = \begin{cases} x = R\cos\alpha \\ y = R\sin\alpha \end{cases}$$

Distance covered by a point moving along a circular trajectory is the length of an arch $s(t) = \ell$, which is in the following relation with angular distance (expressed in **radians**!):

$$s(t) = R\cdot\alpha(t) \quad \Rightarrow \quad \alpha(t) = \frac{s(t)}{R} = t^2$$

Position vector: $$\mathbf{r}(t) = \begin{cases} x(t) = R\cos t^2 \\ y(t) = R\sin t^2 \end{cases}$$

Velocity vector: $$\mathbf{v}(t) = \dot{\mathbf{r}}(t) = \begin{cases} \dot{x}(t) = -2tR\sin t^2 \\ \dot{y}(t) = 2tR\cos t^2 \end{cases}$$

Acceleration vector: $$\mathbf{a}(t) = \ddot{\mathbf{r}}(t) = \begin{cases} \ddot{x}(t) = -2R\sin t^2 - 4t^2R\cos t^2 \\ \ddot{y}(t) = 2R\cos t^2 - 4t^2R\sin t^2 \end{cases}$$

Having the angular distance expressed as a function of time $\alpha(t)$ we may determine:

- angular velocity: $\omega = \dot{\alpha} = 2t$
- angular acceleration: $\varepsilon = \ddot{\alpha} = 2$

Angular velocity vector and angular acceleration vector are perpendicular to the plane of motion:

- angular velocity vector: $\boldsymbol{\omega} = [0\ ;\ 0\ ;\ \omega] = [0\ ;\ 0\ ;\ 2t]$
- angular acceleration vector: $\boldsymbol{\varepsilon} = [0\ ;\ 0\ ;\ \varepsilon] = [0\ ;\ 0\ ;\ 2]$

Velocity and acceleration vectors are found from the following relations:

$$\mathbf{v} = \boldsymbol{\omega}\times\mathbf{r} \quad , \quad \mathbf{a}_s = \boldsymbol{\varepsilon}\times\mathbf{r} \quad , \quad \mathbf{a}_n = \boldsymbol{\omega}\times\mathbf{v} \quad , \quad \mathbf{a} = \mathbf{a}_s + \mathbf{a}_n$$

Velocity vector:

$$\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r} = [0\ ;\ 0\ ;\ 2t] \times [R\cos t^2\ ;\ R\sin t^2\ ;\ 0] = [-2t\,R\sin t^2\ ;\ 2t\,R\cos t^2\ ;\ 0]$$

Tangent acceleration vector:

$$\mathbf{a}_s = \boldsymbol{\varepsilon} \times \mathbf{r} = [0\ ;\ 0\ ;\ 2] \times [R\cos t^2\ ;\ R\sin t^2\ ;\ 0] = [-2R\sin t^2\ ;\ 2R\cos t^2\ ;\ 0]$$

Normal acceleration vector:

$$\mathbf{a}_n = \boldsymbol{\omega} \times \mathbf{v} = [0\ ;\ 0\ ;\ 2t] \times [-2t\,R\sin t^2\ ;\ 2t\,R\cos t^2\ ;\ 0] = [-4t^2 R\cos t^2\ ;\ -4t^2 R\sin t^2\ ;\ 0]$$

Acceleration vector:

$$\mathbf{a} = \mathbf{a}_s + \mathbf{a}_n = [-2R\sin t^2 - 4t^2 R\cos t^2\ ;\ 2R\cos t^2 - 4t^2 R\sin t^2\ ;\ 0]$$

EXERCISE 3

A material point moves along a trajectory given by parametric equations:

$$K: \begin{cases} x(\lambda) = 2\lambda \\ y(\lambda) = 4-2\lambda \\ z(\lambda) = \lambda+3 \end{cases}$$

The initial point is $\Omega_0=(4\ ;\ 0\ ;\ 5)$. The trajectory is oriented in such a way that the measure of covered distance $s>0$ for $z<5$. Equation of motion is $s = t^2$. Find the position vector as a function of time (vector description)

SOLUTION:

Our task is to transform a natural description into a vector one.

- Initial point Ω_0 corresponds with $\lambda=2$.
- Increment of distance covered as a function of parameter λ is equal:

$$s(\lambda) = \int_{\lambda_0}^{\lambda} \sqrt{\left(\frac{\mathrm{d}\,x}{\mathrm{d}\,\lambda}\right)^2+\left(\frac{\mathrm{d}\,y}{\mathrm{d}\,\lambda}\right)^2+\left(\frac{\mathrm{d}\,z}{\mathrm{d}\,\lambda}\right)^2}\mathrm{d}\,\lambda = \int_{2}^{\lambda} \sqrt{(2)^2+(-2)^2+(1)^2}\mathrm{d}\,\lambda = 3\int_{2}^{\lambda} \mathrm{d}\,\lambda = 3(\lambda-2)$$

- The result is compared with the equation of motion:

$$\pm s(\lambda) = s(t) \qquad \Rightarrow \qquad \pm 3(\lambda-2) = \pm t^2$$

- The sign is chosen by checking if the sign in increment of distance covered is the same in the left hand side and right hand side of the above relation. According to the orientation of trajectory we should have $s>0 \quad \Leftrightarrow \quad z<5$. Substituting $s(\lambda)$ and the last of parametric equations of trajectory, we obtain:

$$\begin{gathered} \pm 3(\lambda-2)>0 \quad \Leftrightarrow \quad \lambda+3<5 \\ \pm[3\lambda-6]>0 \quad \Leftrightarrow \quad \lambda<2 \\ \pm[\lambda]>2 \quad \Leftrightarrow \quad \lambda<2 \end{gathered}$$

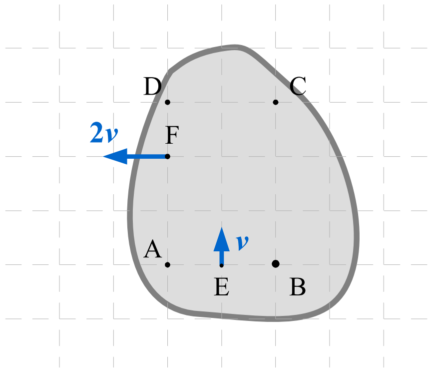
  Incerement of distance in expression $s(\lambda)=3(\lambda-2)$ is opposite then the one resulting from orientation, so we chose $-s(\lambda)=s(t)$.

$$-3(\lambda-2) = t^2 \quad \Rightarrow \quad \lambda = 2-\frac{1}{3}t^2$$

- Substituting it in the equations of trajectory, we obtain the position vector as a function of time:

$$\mathbf{r}(t) = \begin{cases} x(t) = 4-\dfrac{2}{3}t^2 \\ y(t) = \dfrac{2}{3}t^2 \\ z(t) = 5-\dfrac{1}{3}t^2 \end{cases}$$

EXERCISE 4



Knowing the velocity vectors in two point of a plne rigid body, find velocity vectors in point A, B, C, D. Grid lines are spaced with a single unitary distance.

SOLUTION:

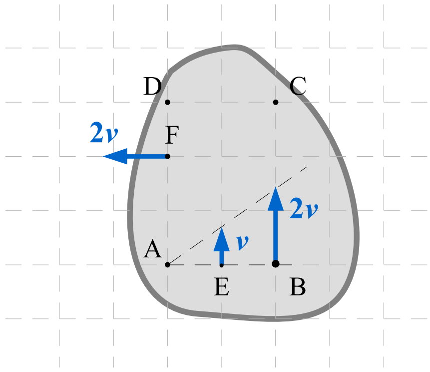
POINT A:

•Projection of $\mathbf{v}_E$ on line AE is zero,so **horizontal component of $\mathbf{v}_A$ is zero**.

•Projection of $\mathbf{v}_F$ on line FA is zero, so **vertical component of $\mathbf{v}_A$ is zero**.

•Since both horizontal and vertical component of $\mathbf{v}_A$ is zero, then $\mathbf{v}_A$ must be a zero vector and point A does not move at all.

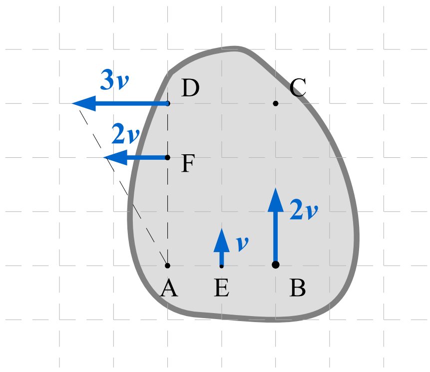
POINT B:



•Projection of $\mathbf{v}_E$ on line AB is zero **,** so **horizontal component of $\mathbf{v}_B$ is zero** – it is **a vertical vector**.

•Points A, E, B lies on a single line, so heads of their velocity vectors must also lie on a line. Similarity of triangles gives us $|\mathbf{v}_B| = 2v$.

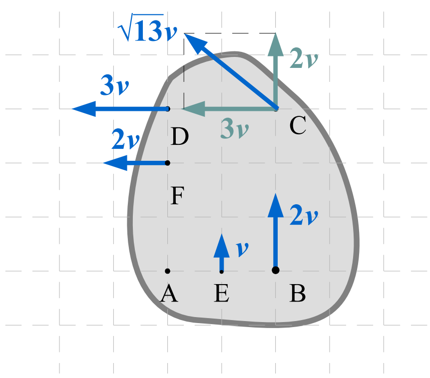
POINT D:



•Projection of $\mathbf{v}_F$ on line FD is zero, so **vertical component of $\mathbf{v}_D$ is zero** – it is a **horizontal vector**.

•Points A, F, D lies on a single line, so heads of their velocity vectors must also lie on a line. Similarity of triangles gives us $|\mathbf{v}_D| = 3v$.

POINT C:



•Projection of $\mathbf{v}_D$ on horizontal line DC is equal $3v$, so **horizontal component of $\mathbf{v}_C$ is equal** $3v$.

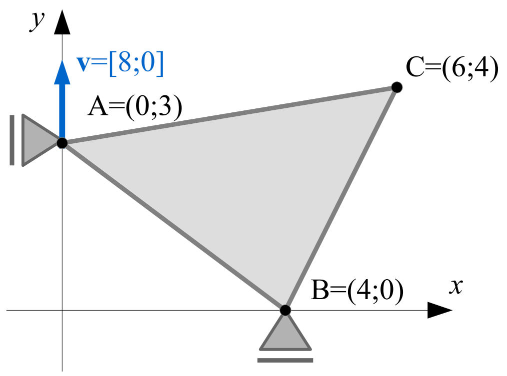
•Projection of $\mathbf{v}_B$ on vertical line BC is equal $2v$, so **vertical component of $\mathbf{v}_C$ is equal** $2v$

• $|\mathbf{v}_C| = \sqrt{(3v)^2+(2v)^2} = \sqrt{13}\,v$

EXERCISE 5

Find velocity vectors in points B, C, D, knowing that velocity in A is equal:

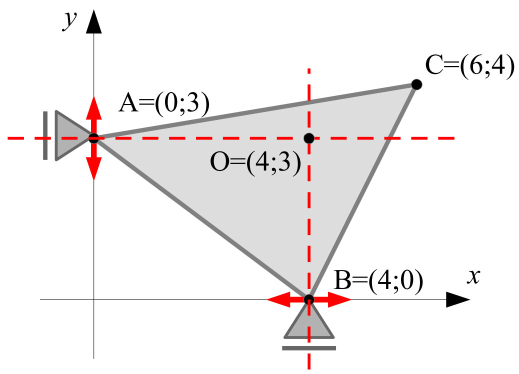
$$\mathbf{v}_A = [0{,}8]\ \mathrm{m/s}.$$



SOLUTION:

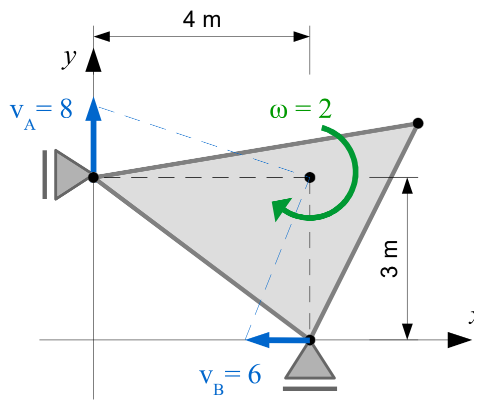
We know that planar motion of a rigid body may be interpreted as a rotation about instant center of rotation. If we find the position of this center and determine the angular velocity, we will be able to find velocity of any point of the body.

Instant center of rotation is always placed at intersection of lines which are perpendicular to the permissible directions of velocities. Support in point A allows for vertical motion only. Support in point B allows for horizontal motion only. It allows us to find the location of instant center of rotation.

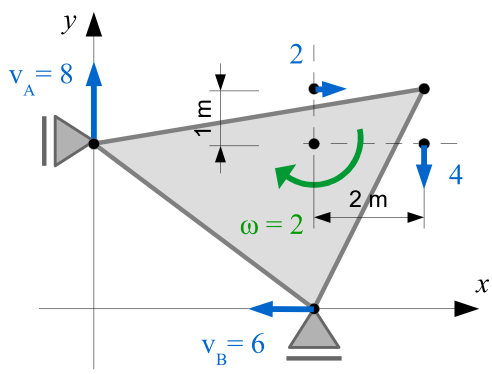


Linear and angular velocities hold the following relation: $v = \omega R$, where *R* is the distance from the center of rotation. Magnitude of velocity vector in A is equal $v_A = |\mathbf{v}| = 8\ \mathrm{m/s}$. Distance from A to O is equal $4\,\mathrm{m}$, hence $\omega = 2\ \mathrm{rad/s}$. Basing on it, we can find velocity in B which is surely horizontal and oriented in such a way that it is consistent with clockwise rotation of the body:

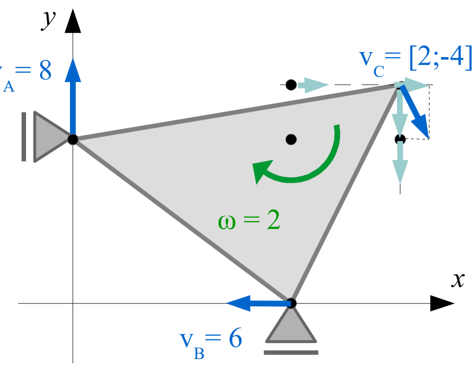
$$v_B = \omega R_B = 2\,\mathrm{rad/s}\cdot 3\ \mathrm{m} = 6\,\mathrm{m/s} \quad .$$



In order to find components of velocity vector in C we will first determine velocities in two fictitious points which do not belong to the body and which will have a horizontal and vertical velocity respectively, equal to the (respectively) horizontal and vertical component of velocity vector in C.

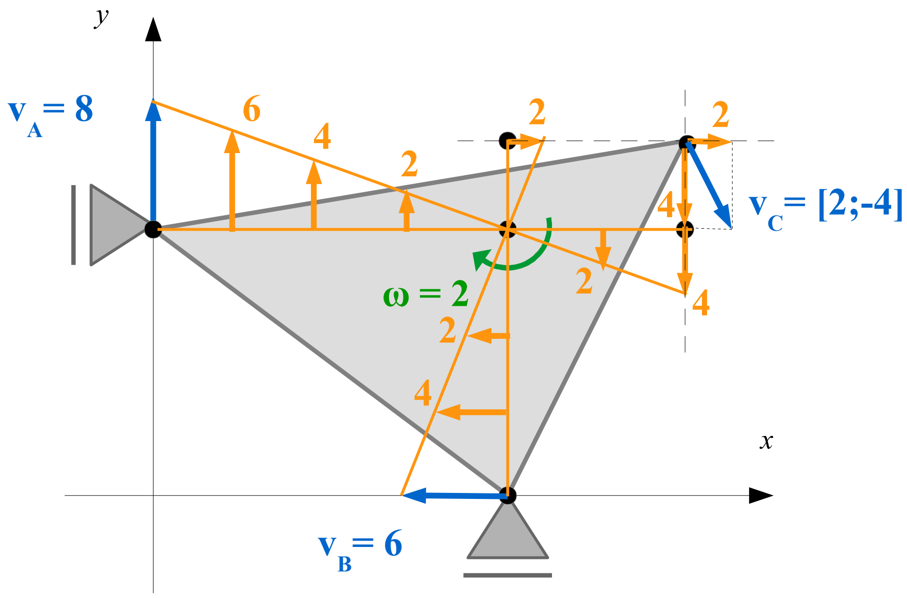


Components of velocity vector in C are found by translating appropriate projections along axes which are parallel to them.



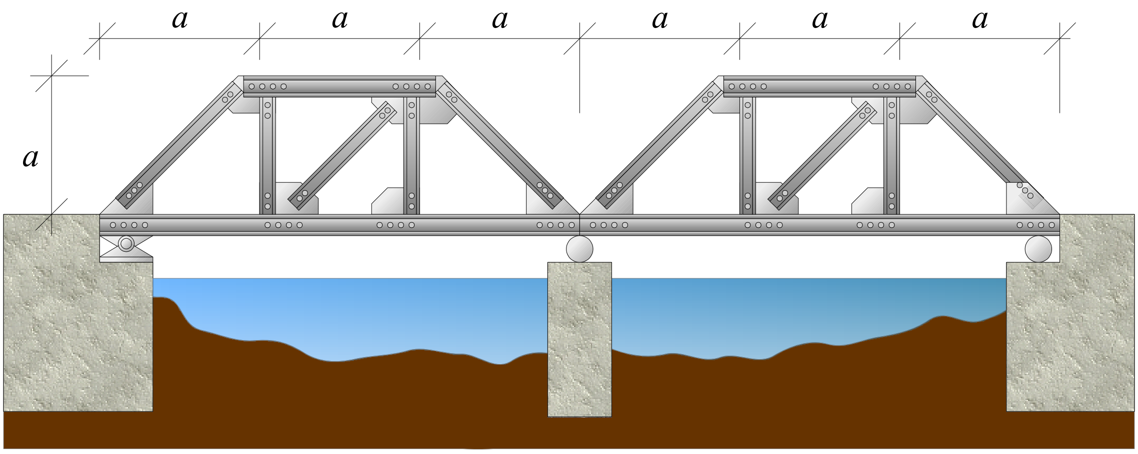
Finally:

$$\mathbf{v}_A = [0\ ;\ 8]\ \mathrm{m/s}$$
$$\mathbf{v}_B = [-6\ ;\ 0]\ \mathrm{m/s}$$
$$\mathbf{v}_C = [2\ ;\ -4]\ \mathrm{m/s}$$



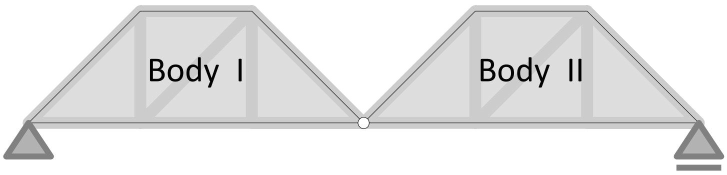
EXERCISE 6

There is a system of two truss bodies connected with a joint (hinge) over a middle support. Find the velocities of nodes of trusses after removing the middle support.

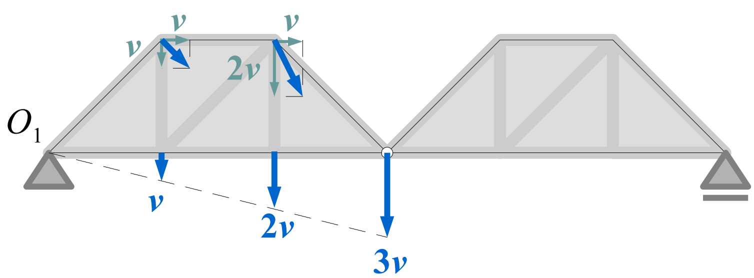


SOLUTION:

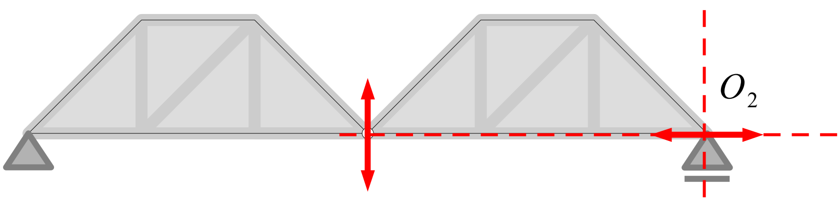
Static diagram after removing the support..



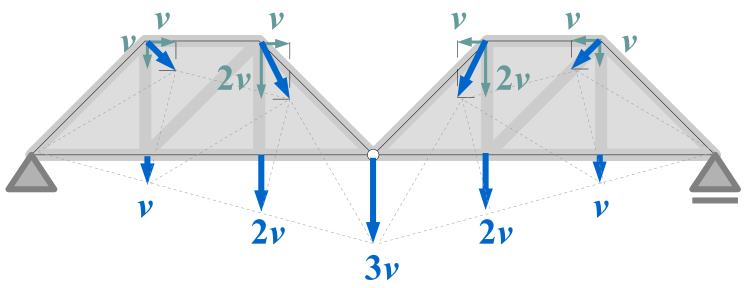
Let's determine the center of rotation of body I. Since it is pinned in a single point, this point must be the center of rotation O_1 . Let's assume the angular velocity $\omega = v/a$.



A common point of two bodies may move only vertically. Supported point of the second body may move only horizontally. We draw lines which are perpendicular to the permissible direction of velocities in those two point of body II. At intersection of those lines an instant center of rotation O_2 for that body lies.

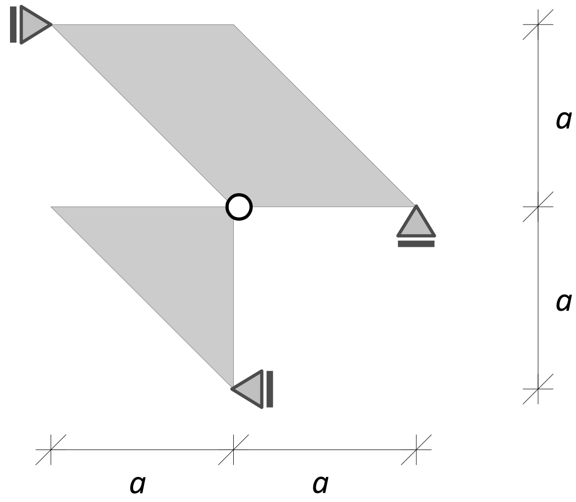


We determine velocities of nodes of truss respective to angular velocity determined according to the known position of center of rotation and known linear velocity in joint.



EXERCISE 7

Determine the distribution of velocity in the mechanical system shown below:



OUTLINE OF SOLUTION

(*a system of rigid bodies connected with a joint with no known velocities*)

1. For each body we have to determine its instant center of rotation and angular velocity – knowing them we are able to determine velocity vectors in any point of each body.
2. We start our analysis with a body to which the greatest number of constraints (supports) are applied. Usually it is the one which is supporte with
   - two roller supports or
   - single pinned support
3. Determine the location of instant center of rotation.
4. Assuming a chosen angular velocity, determine velocities in all points of the body, in particular in the joint connecting it with another body.
5. Knowing the velocity in joint and supports applied to the second body (or velocities in other point of the second body) find the instant center of rotation and angular velocity for that body.
6. Repeat steps 4 and 5 until all required velocities are found.

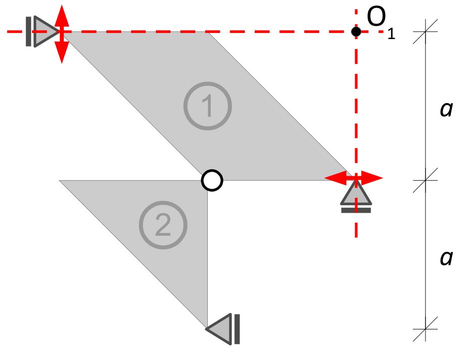
REMARKS:

- If at any step of our solution we obtain any incosistency (e.g. due to theorems on distribution of velocities a velocity is required along a direction which is not allowed due to support, instant center of rotation has non-zero velocity etc.) it means that the assumption on the motion of the first body is incorrect. We assume then that this first body holds still and the joint becomes an instant center of rotation for the connected body (it is immovable and allows rotation). Such a change of initial assumption may happen more than once.

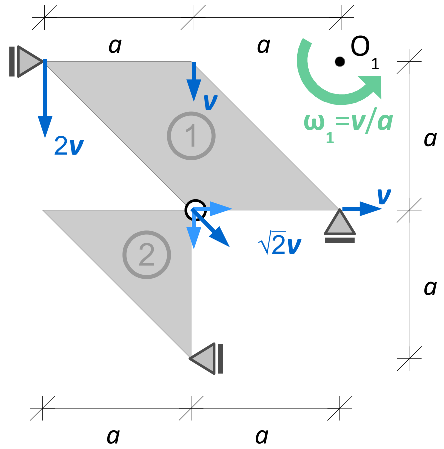
- If the instant center of rotation is an improper point in infinity (lines perpendicular to the permissible directions of velocities are parallel), then the body performs a parallel translation (no rotation) and velocity vectors in all points of that body are the same.

SOLUTION:

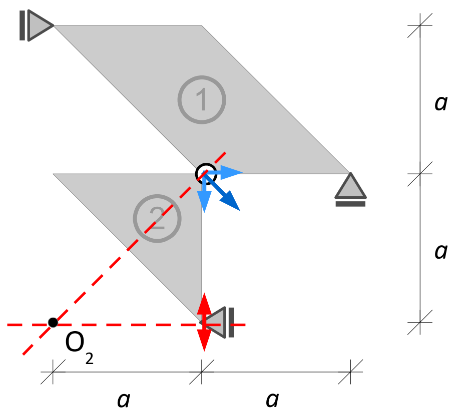
Let's start with body 1 (notation as in the picture) since there are two roller supports. We determine the permissible directions of velocities in supported points. At intersection of lines perpendicular to that permissible directions there is an instant center of rotation.



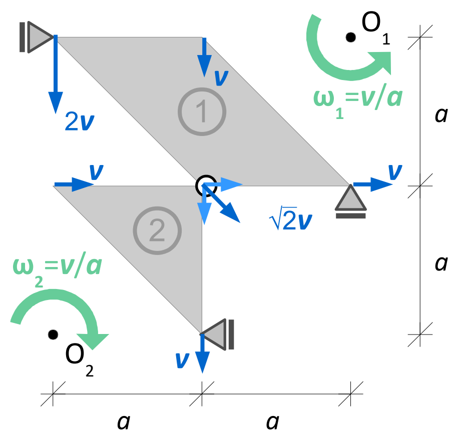
Let's assume angular velocity $\omega_1 = v/a$ - orientation may be chosen arbitrary. In any point at distance a from the center of rotation the velocity is equal v. Direction of velocity is always perpendicular to the line connecting the considered point with the center of rotation. Orientation yields from orientation of angular velocity.



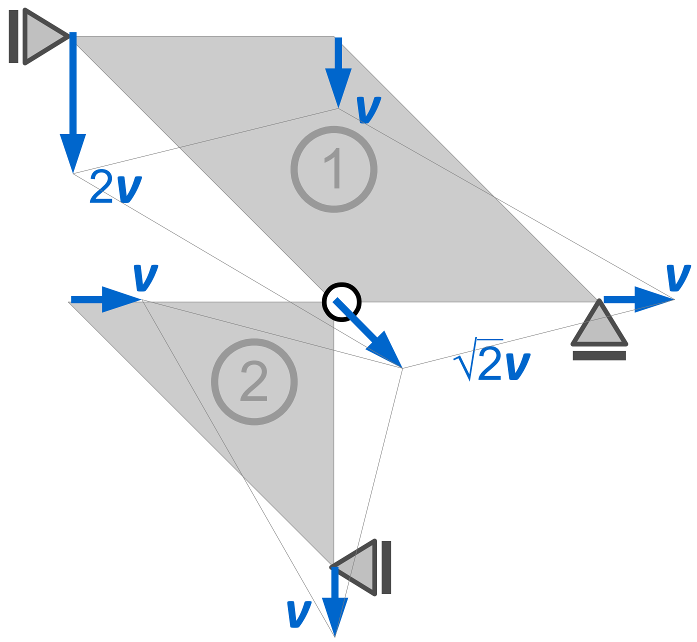
Knowing the direction of velocity in joint and permissible direction of velocity on support in the 2nd body, we draw lines perpendicular to those directions and find the instant center of rotation for body 2 at their intersection.



Angular velocity for the 2nd body is found with the use of relation $\omega = v/r$ knowing the velocity in joint. It is at distance $r = a\sqrt{2}$, hence the angular velocity $\omega_2 = v/a$. Knowing the angular velocity we find the velocities in all other points of the body.

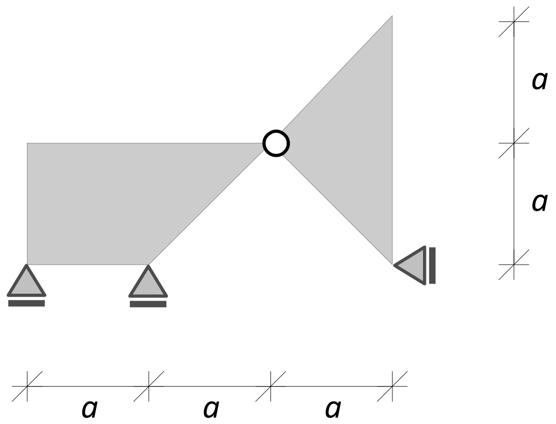


Approximate deformation:



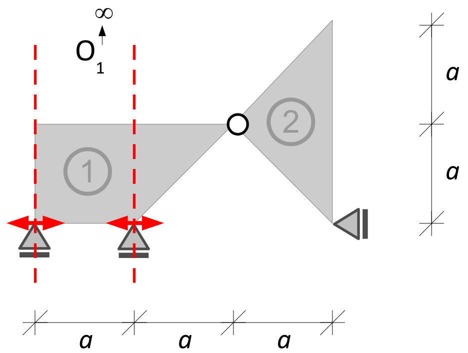
EXERCISE 8

Determine the distribution of velocity in the mechanical system shown below:

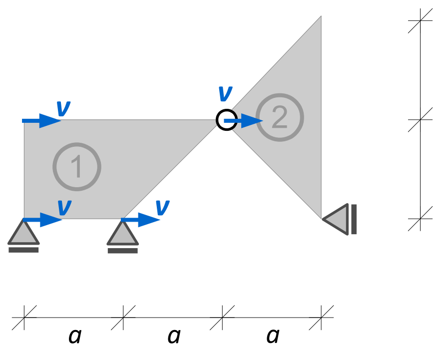


SOLUTION:

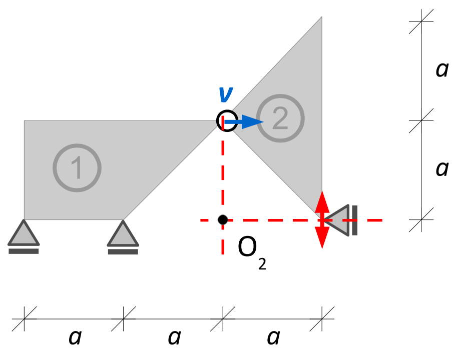
Let's start with body 1 (notation as in the picture) since there are two roller supports. We determine the permissible directions of velocities in supported points. At intersection of lines perpendicular to that permissible directions there is an instant center of rotation. It emerges to be an improper point (in infinity).



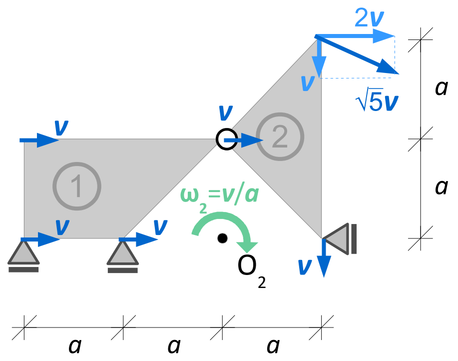
Since the instant center of rotation is in infinity, the body performs parallel translation – velocity vectors in all points are all the same. Supports allow only for horizontal displacements. Orientation and magnitude of velocity may be chosen arbitrary. Let's assume v towards right. In particular this is also the velocity of joint.



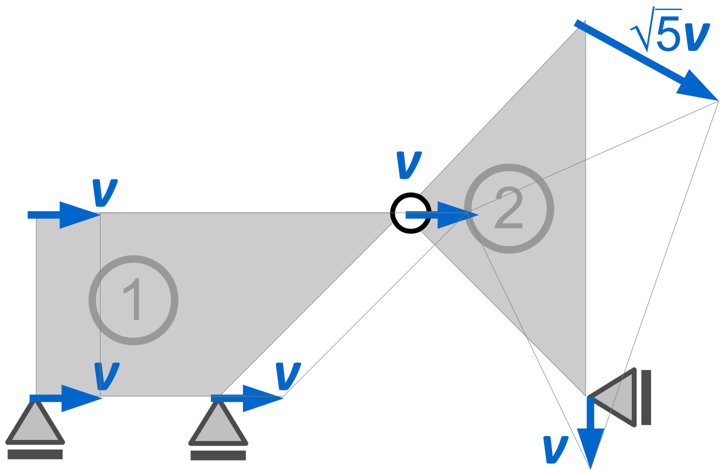
Knowing the direction of velocity in joint and permissible direction of velocity on support in the 2[nd] body, we draw lines perpendicular to those directions and find the instant center of rotation for body 2 at their intersection.



Angular velocity for the 2nd body is found with the use of relation $\omega = v/r$ knowing the velocity in joint. It is at distance $r = a$, hence the angular velocity $\omega_2 = v/a$. Knowing the angular velocity we find the velocities in all other points of the body.

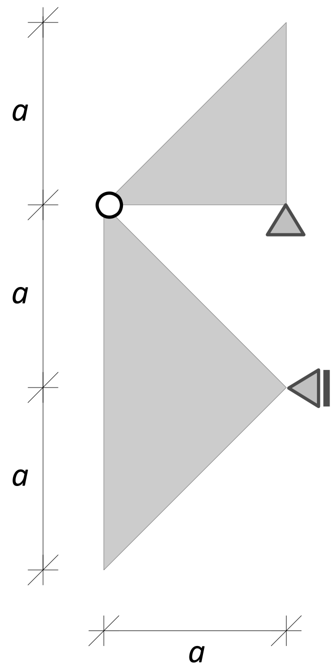


Approximate deformation:

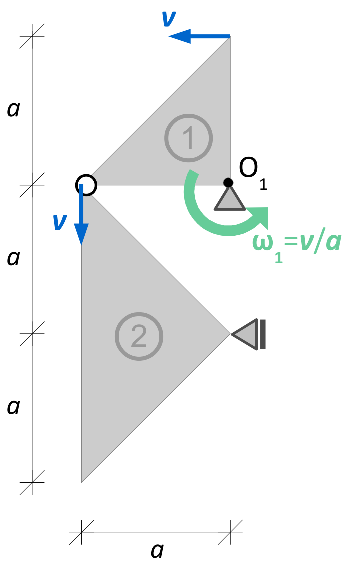


EXERCISE 9

Determine the distribution of velocity in the mechanical system shown below:

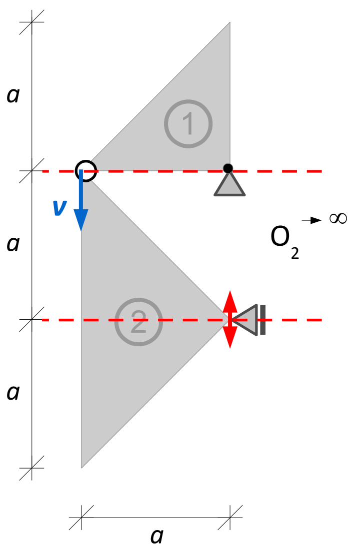


SOLUTION:

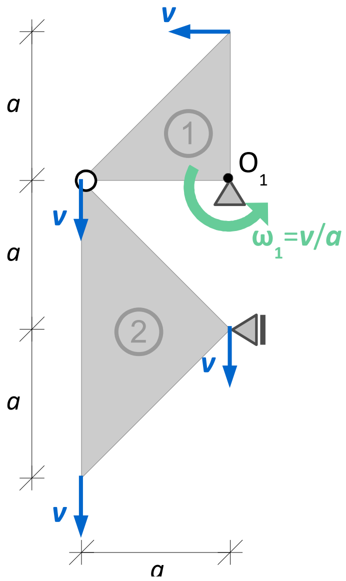


Let's start with body 1 (notation as in the picture) since there isa pinned support. Instant center of rotation is located at the support for body 1. Let's assume angular velocity $\omega_1 = v/a$ -- orientation may be chosen arbitrary. In any point at distance a from the center of rotation the velocity is equal v. Direction of velocity is always perpendicular to the line connecting the considered point with the center of rotation. Orientation yields from orientation of angular velocity.

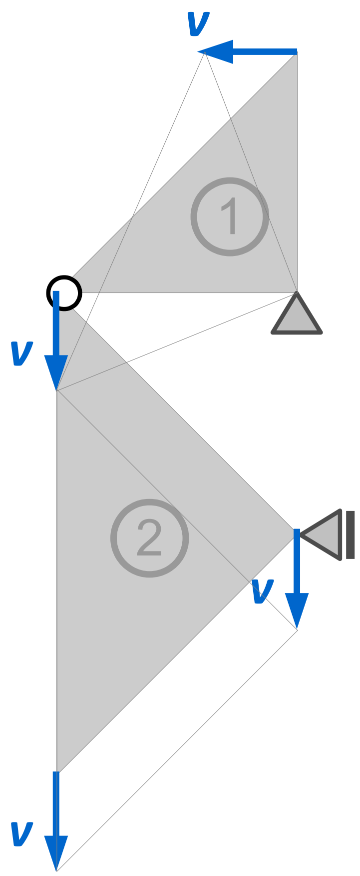
Knowing the direction of velocity in joint and permissible direction of velocity on support in the 2nd body, we draw lines perpendicular to those directions and find the instant center of rotation for body 2 at their intersection. It emerges to be an improper point (in infinity).



Since the instant center of rotation is in infinity, the body performs parallel translation – velocity vectors in all points are all the same. Since the joint moves with v downwards it will also be the velocity of all other points of that body.

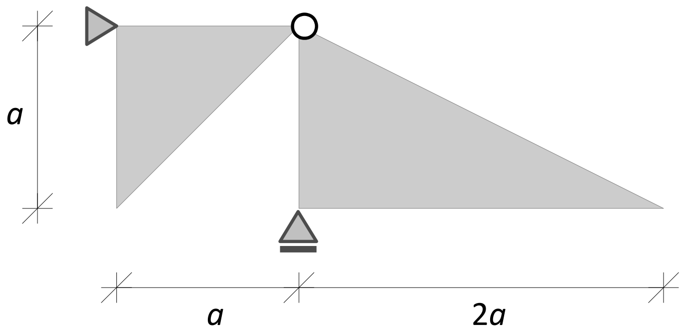


Approximate deformation:

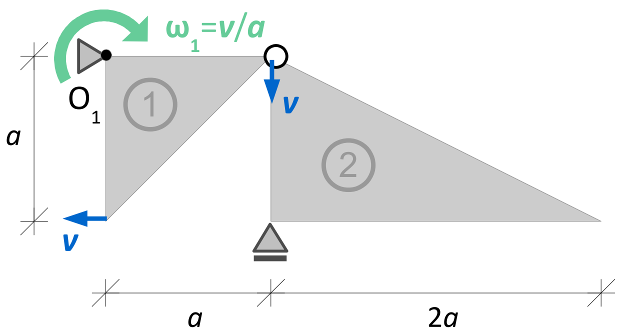


EXERCISE 10

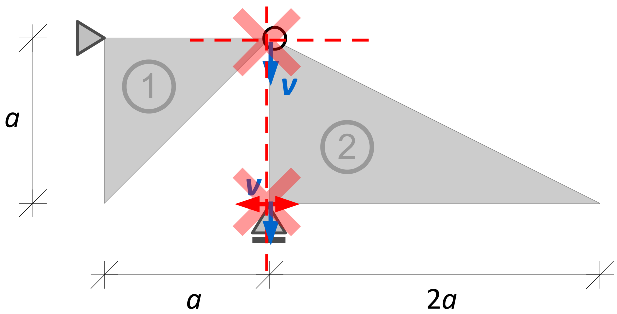
Determine the distribution of velocity in the mechanical system shown below:



SOLUTION:

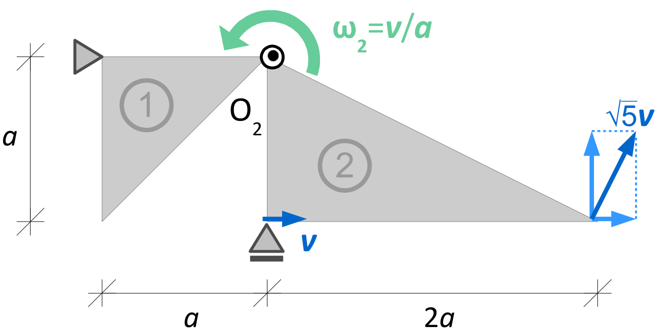


Let's start with body 1 (notation as in the picture) since there is a pinned support. Instant center of rotation is located at the support for body 1. Let's assume angular velocity $\omega_1 = v/a$ -- orientation may be chosen arbitrary. In any point at distance a from the center of rotation the velocity is equal v. Direction of velocity is always perpendicular to the line connecting the considered point with the center of rotation. Orientation yields from orientation of angular velocity.



Knowing the direction of velocity in joint and knowing the permissible direction of velocity at support in body 2 we draw lines perpendicular to that directions and we find the instant center of rotation at intersection of those lines. It emerges that the intersection is in the joint, in which the velocity is non-zero – it is an inconsistency, since **instant center of rotation must be a point in which the velocity is zero**. What's more, if we wanted to translate the vertical component of velocity in joint along a vertical line down to the support, we would have ti have a vertical **component of velocity at support which do not allow for such a motion**. Both those facts indicate that our initial assumption on rotation of body 1 around the pinned support was incorrect. As it is the only way in which body 1 may move, we conclude that it must remain still.

Since body 1 is immovable, then also the joint does not move. As it is immovable, allows for rotation and belongs to body 2 it must be the instant center of rotation for body 2. Let's assume angular velocity $\omega_2 = v/a$ -- orientation may be chosen arbitrary. In any point at distance a from the center of rotation the velocity is equal v . Direction of velocity is always perpendicular to the line connecting the considered point with the center of rotation. Orientation yields from orientation of angular velocity.



Approximate deformation: