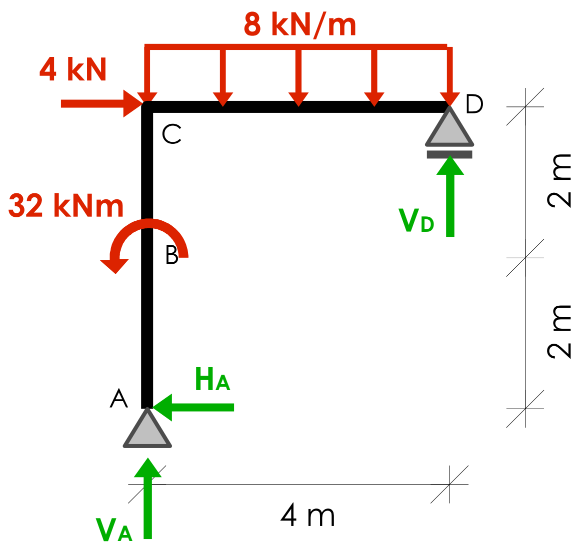
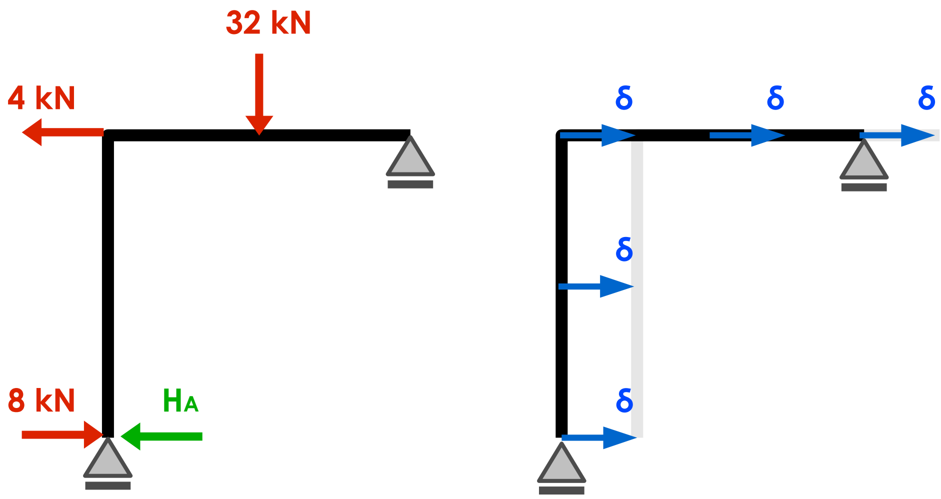
EXERCISE 1

Wyznacz każdą z reakcji podporowych za pomocą ZPW.



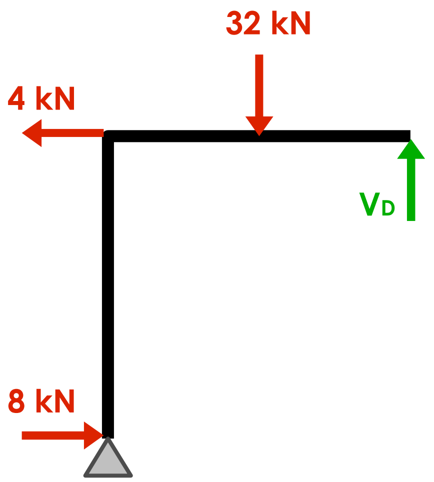
- Distributed load must be replaced with a resultant point force in proper place.
- Point moment *M* must be replaced with a couple of forces *P* in distance *d, where M* = *Pd* - forces of this couple may be applied to any point of a body, to which the moment is applied. It is convenient to apply them in places where another load is applied in order to simplify calculations.

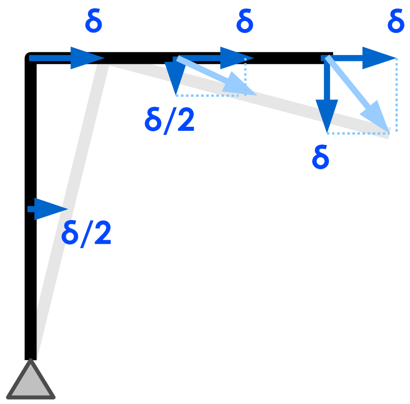
Reaction H_A



$$\delta L = 8 \cdot \delta - H_A \cdot \delta - 4 \cdot \delta = 0 \quad \Rightarrow \quad H_A = 4$$

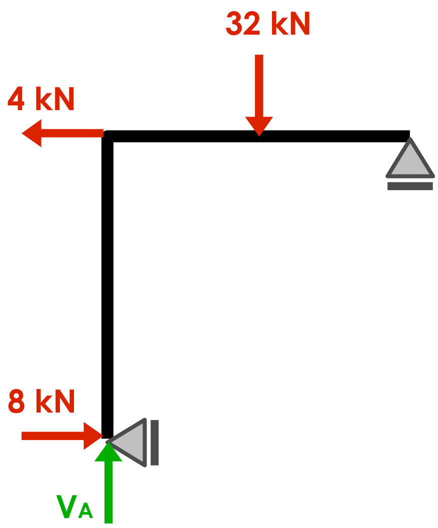
Reaction V_D

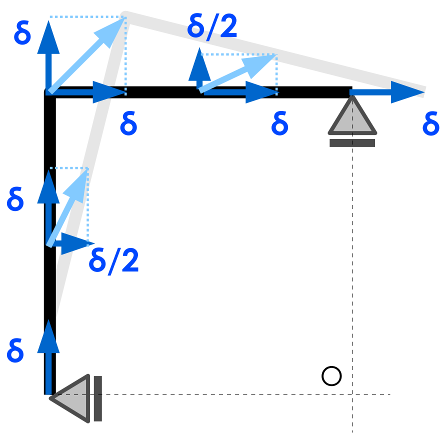




$$\delta L = -4 \cdot \delta + 32 \cdot \frac{\delta}{2} - V_D \cdot \delta = 0 \quad \Rightarrow \quad V_D = 12$$

Reaction V_A

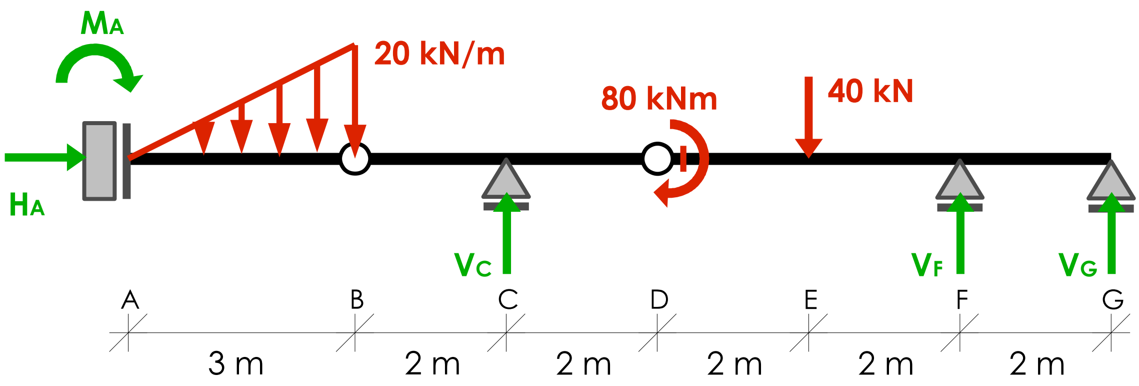




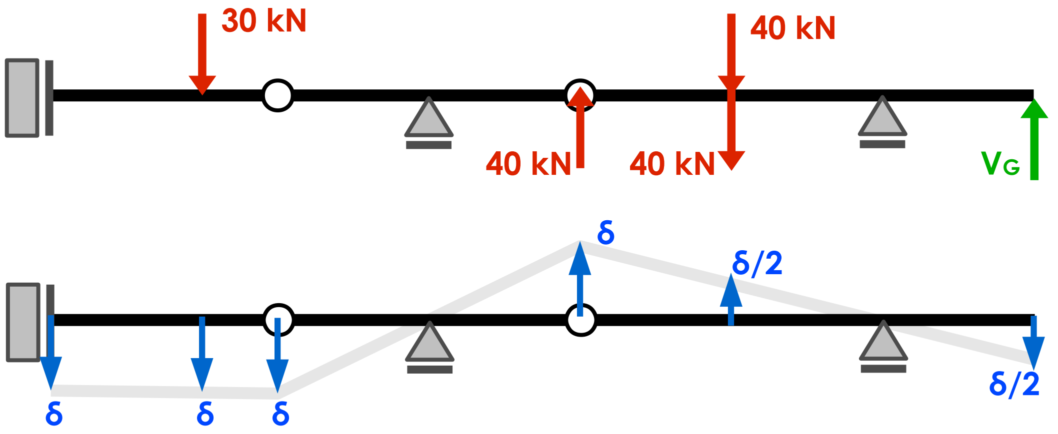
$$\delta L = V_A \cdot \delta - 4 \cdot \delta - 32 \cdot \frac{\delta}{2} = 0 \quad \Rightarrow \quad V_A = 20$$

EXERCISE 2

Determine the vertical reaction with the use of PVW.

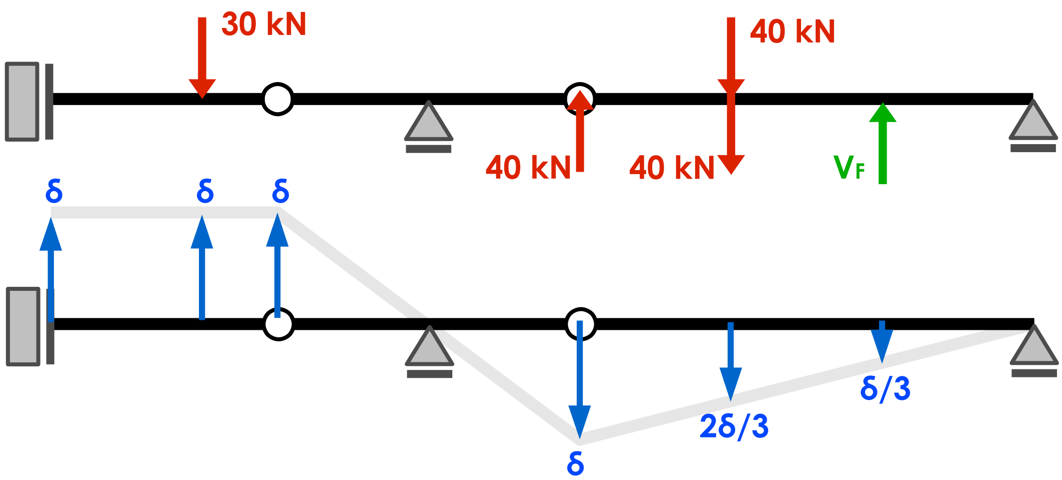


Reaction V_G:



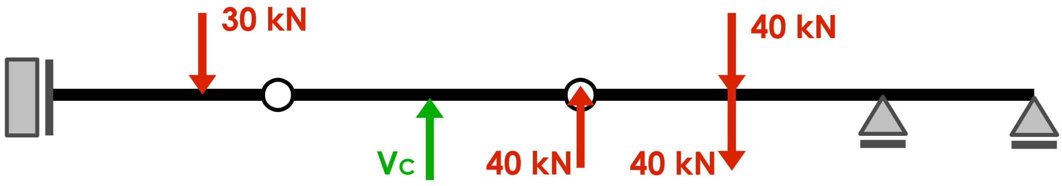
$$\delta L = 30\cdot\delta + 40\cdot\delta - (40+40)\cdot\frac{\delta}{2} - V_G\cdot\frac{\delta}{2} = 0 \quad \Rightarrow \quad V_G = 60$$

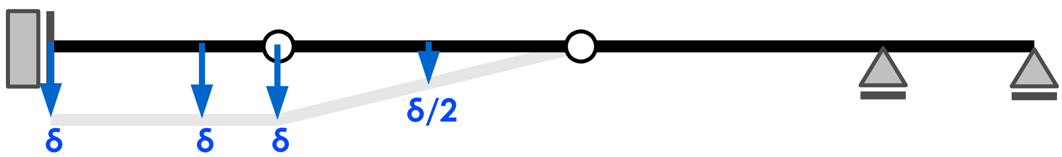
Reaction V_F:



$$\delta L = -30\cdot\delta - 40\cdot\delta + (40+40)\cdot\frac{2\delta}{3} - V_G\cdot\frac{\delta}{3} = 0 \quad \Rightarrow \quad V_F = -50$$

Reaction V_C:

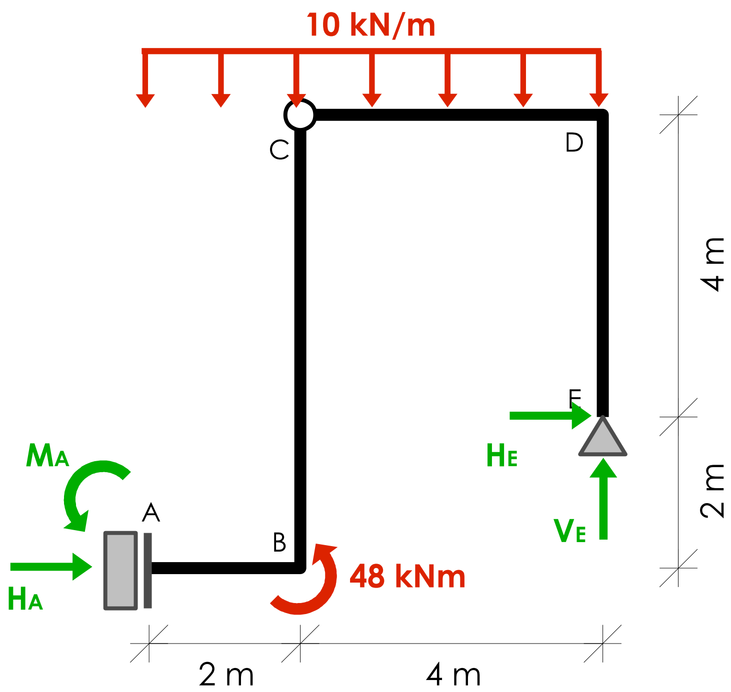




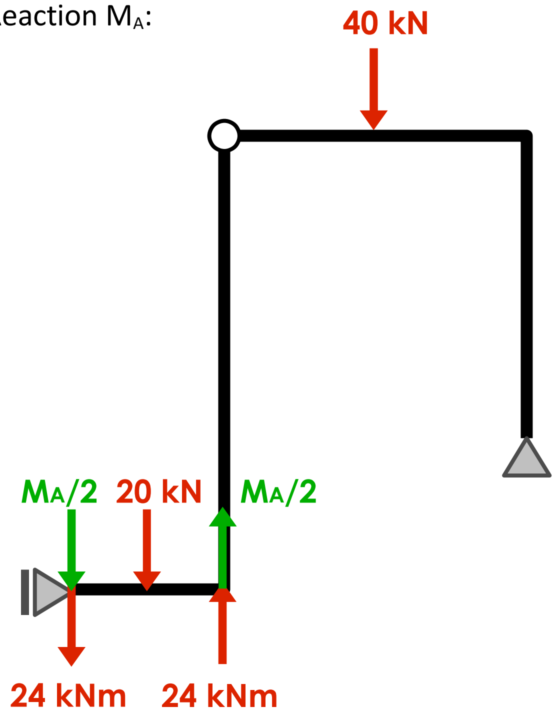
$$\delta L = 30 \cdot \delta - V_C \cdot \frac{\delta}{2} = 0 \quad \Rightarrow \quad V_C = 60$$

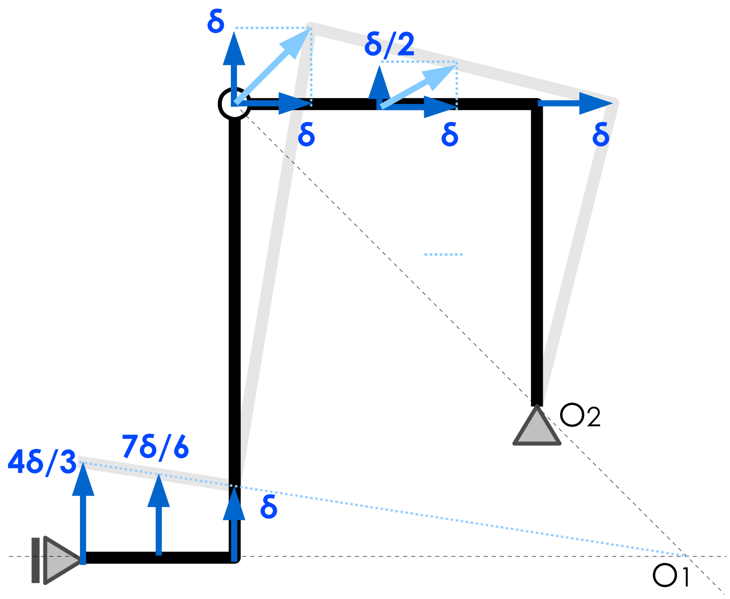
EXERCISE 3

Determine reactions at support A with the use of PVW.



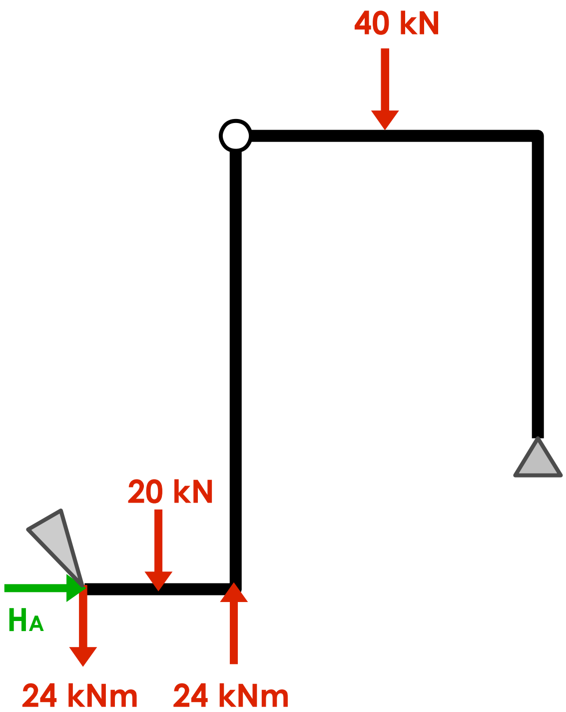
Reaction M_A:

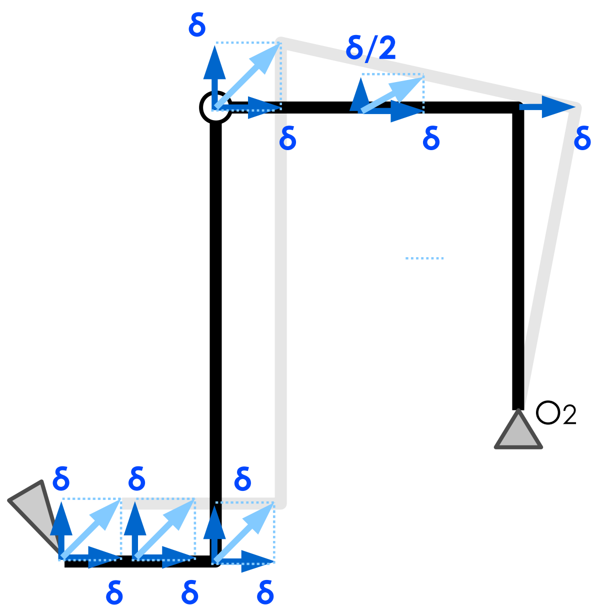




$$\delta L = -\frac{M_A}{2}\cdot\frac{4\delta}{3} - 24\cdot\frac{4\delta}{3} - 20\cdot\frac{7\delta}{6} + \frac{M_A}{2}\cdot\delta + 24\cdot\delta - 40\cdot\frac{\delta}{2} = 0 \quad \Rightarrow \quad M_A = -308$$

Reaction H_A:

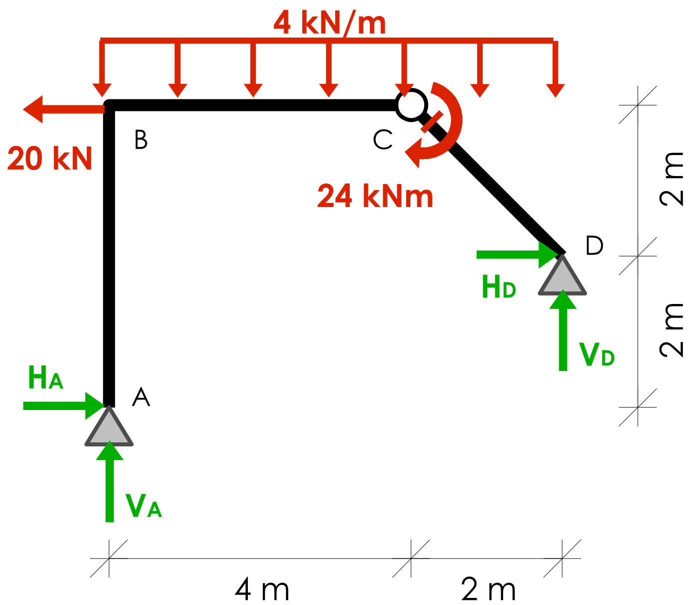




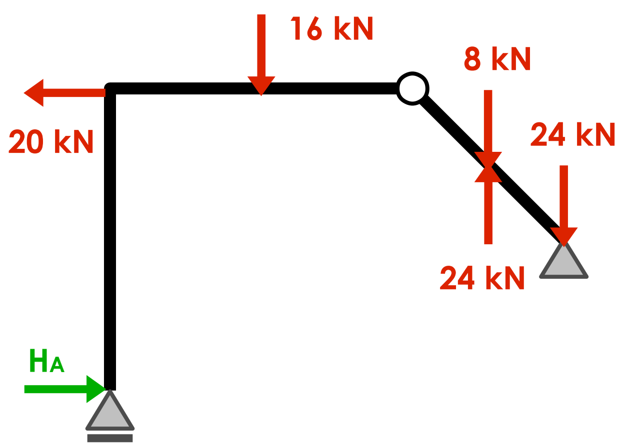
$$\delta L = H_A\cdot\delta - 24\cdot\delta - 20\cdot\delta + 24\cdot\delta - 40\cdot\frac{\delta}{2} = 0 \quad \Rightarrow \quad H_A = 40$$

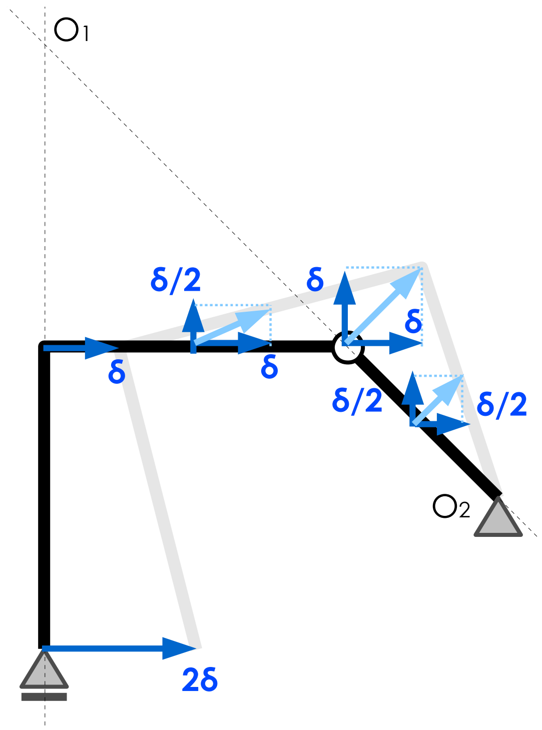
EXERCISE 4

Determine reactions H_A and V_D with the use of PVW.



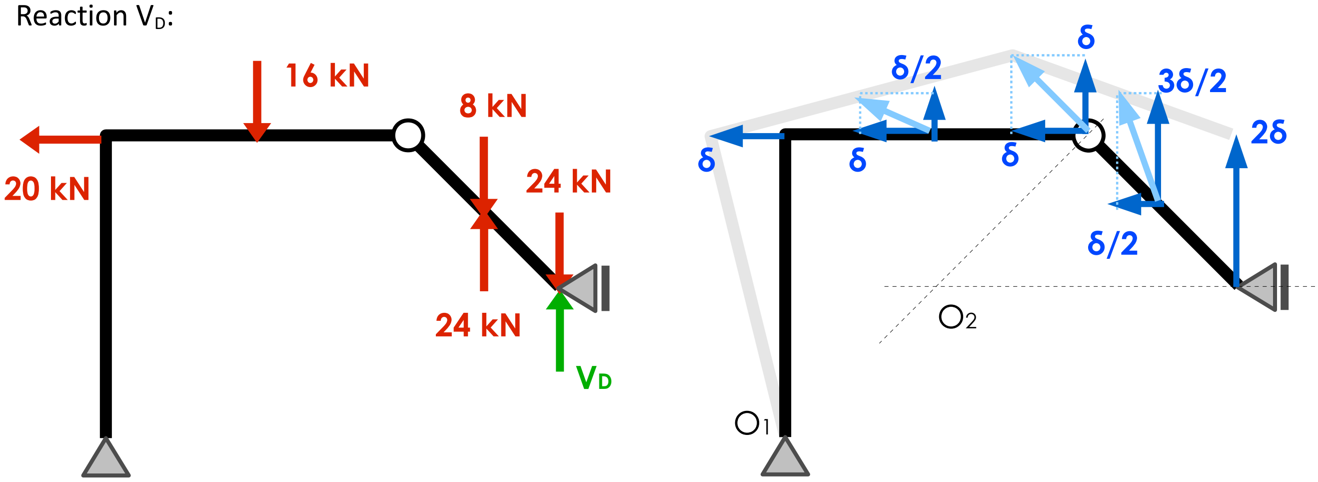
Reaction H_A:





$$\delta L = H_A \cdot 2\delta - 20 \cdot \delta - 16 \cdot \frac{\delta}{2} - 8 \cdot \frac{\delta}{2} + 24 \cdot \frac{\delta}{2} = 0 \quad \Rightarrow \quad H_A = 10$$

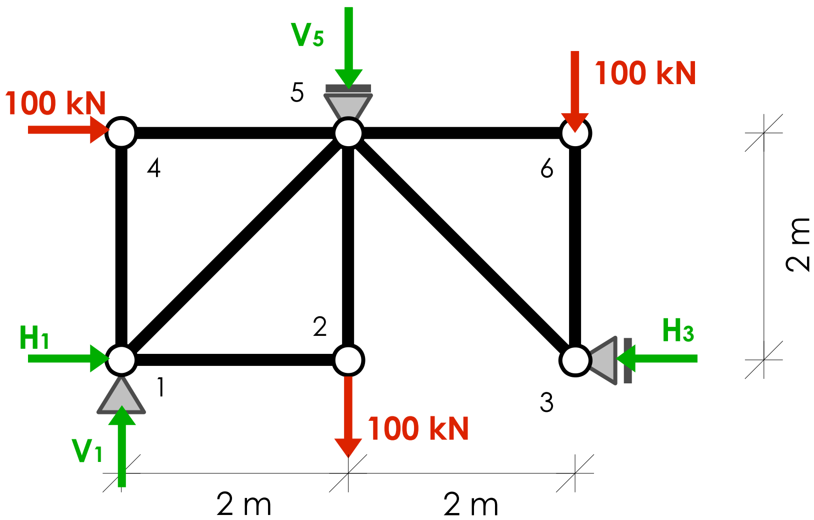
Reaction V_D:



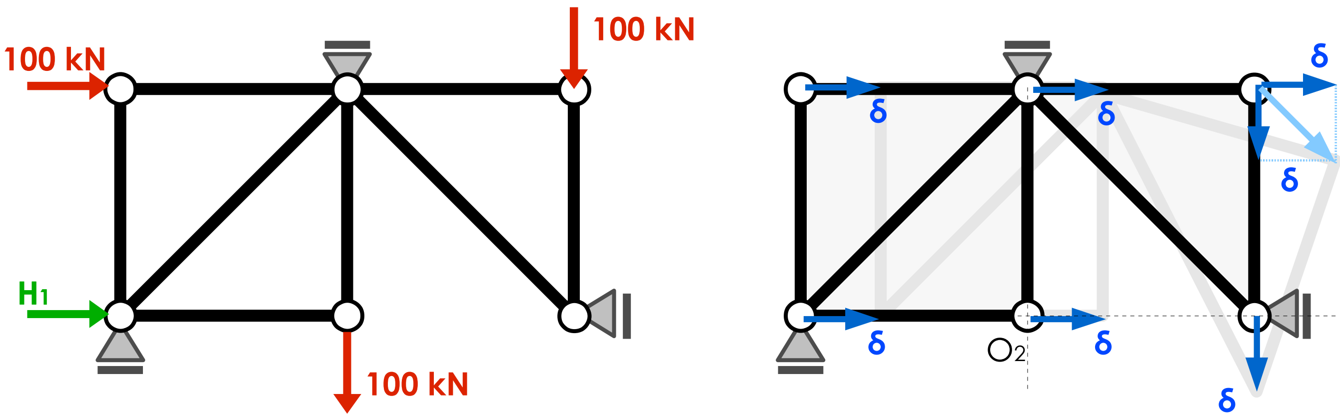
$$\delta L = 20\cdot\delta - 16\cdot\frac{\delta}{2} - 8\cdot\frac{3\delta}{2} + 24\cdot\frac{3\delta}{2} - 24\cdot 2\delta + V_D\cdot 2\delta = 0 \quad\Rightarrow\quad V_D = 6$$

EXERCISE 5

Determine reaction at support 1 force in the 1-5 strut with the use of PVW.

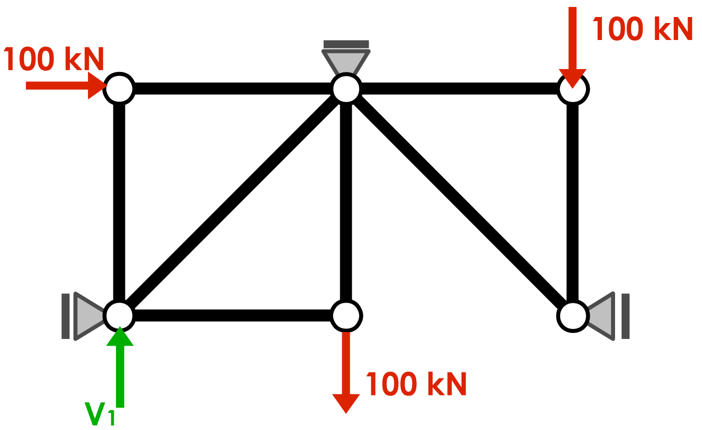


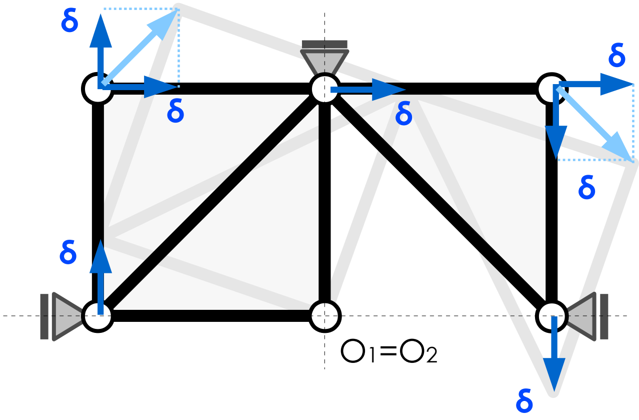
Reaction H_1:



$$\delta L = H_1\cdot\delta + 100\cdot\delta + 100\cdot\delta = 0 \quad\Rightarrow\quad H_1 = -200$$

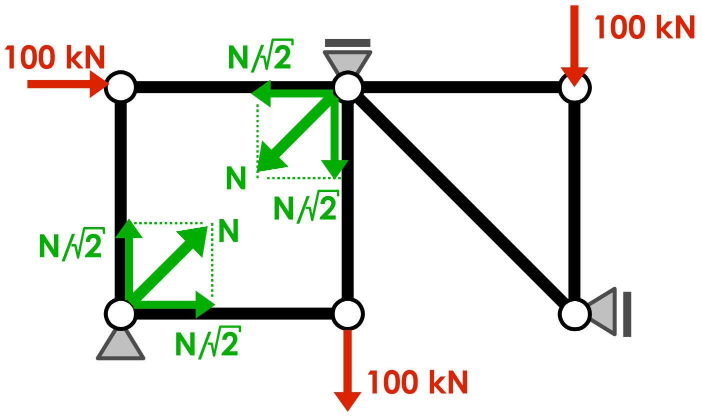
Reaction V_1:

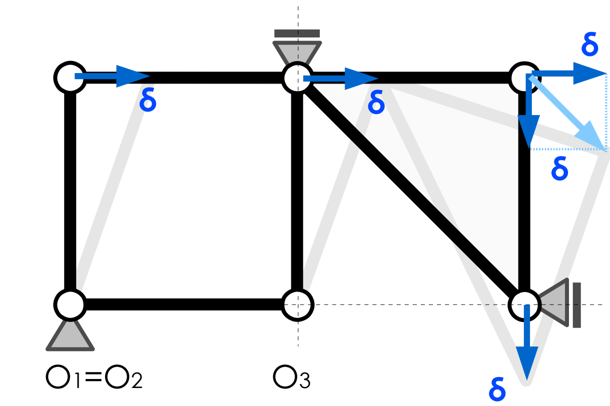




$$\delta L = V_1 \cdot \delta + 100 \cdot \delta + 100 \cdot \delta = 0 \quad \Rightarrow \quad V_1 = -200$$

Strut 1-5:

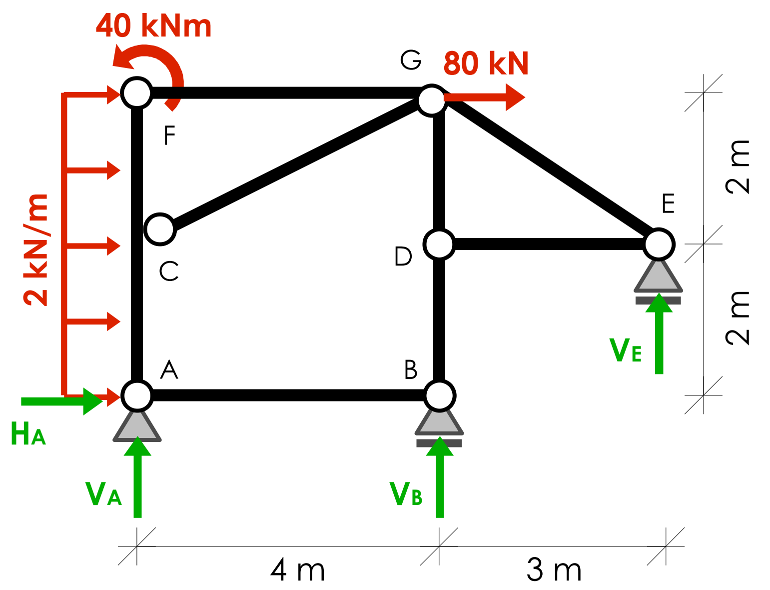




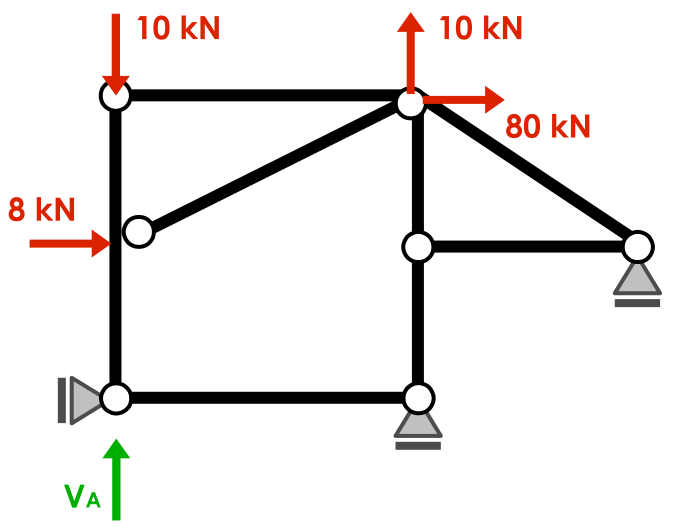
$$\delta L = 100 \cdot \delta - \frac{N}{\sqrt{2}} \cdot \delta + 100 \cdot \delta = 0 \quad \Rightarrow \quad N = 200\sqrt{2}$$

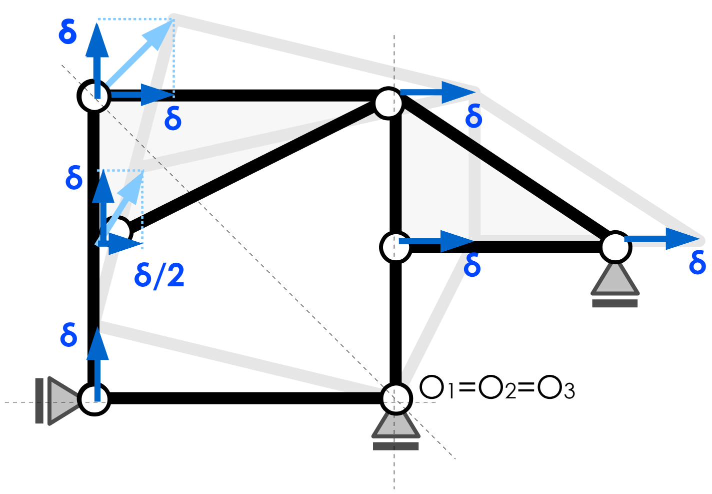
EXERCISE 6

Determine reactions V_A and V_E and force in bowstring C-G with the use of PVW.



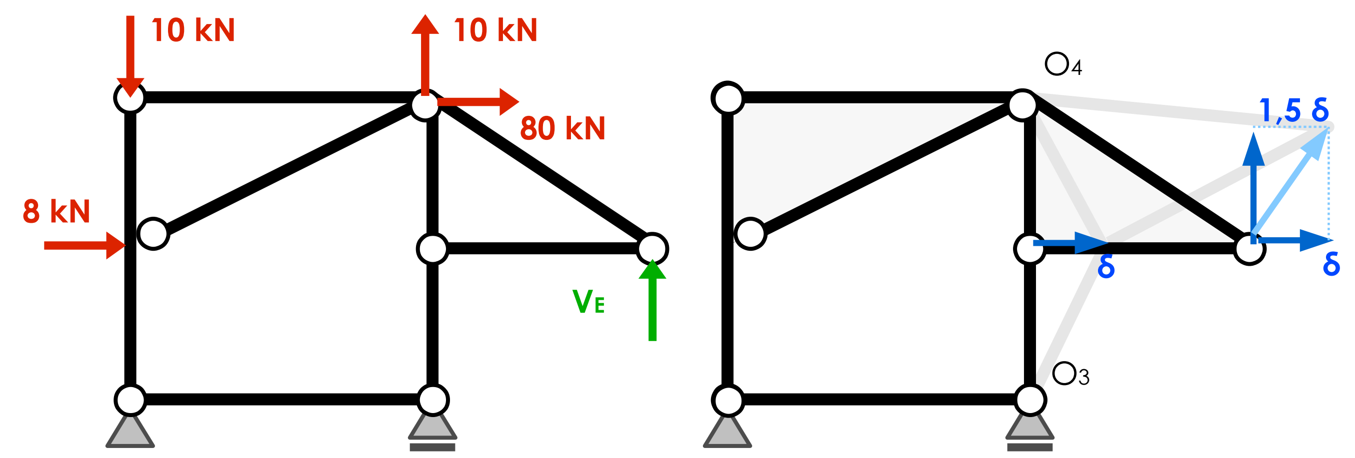
Reaction V_A:





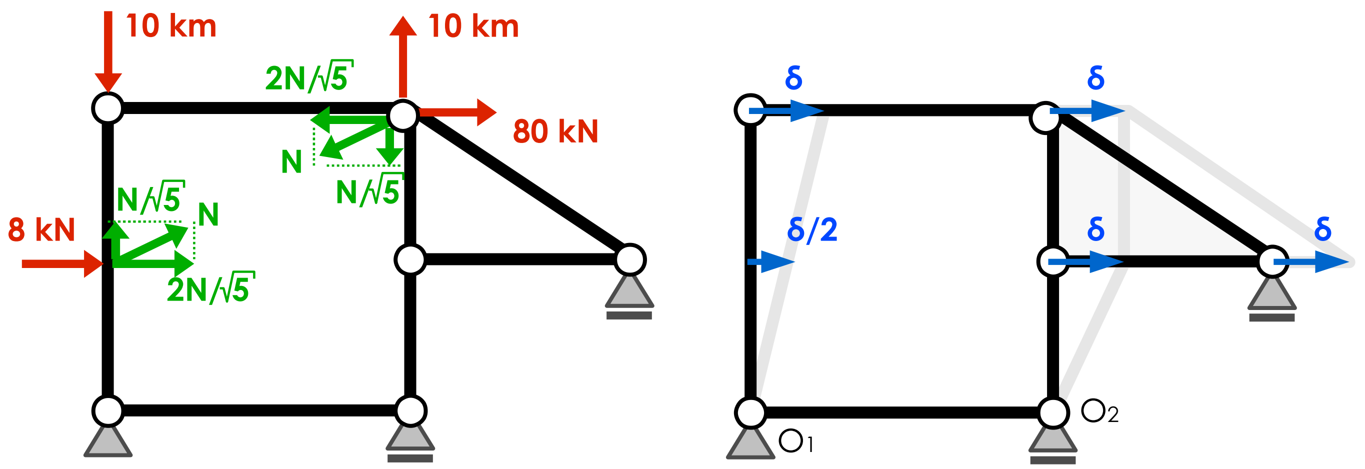
$$\delta L = V_A \cdot \delta + 8 \cdot \frac{\delta}{2} - 10 \cdot \delta + 80 \cdot \delta = 0 \quad \Rightarrow \quad V_A = -74$$

Reaction V_E:



$$\delta L = V_E \cdot 1{,}5\delta = 0 \quad \Rightarrow \quad V_E = 0$$

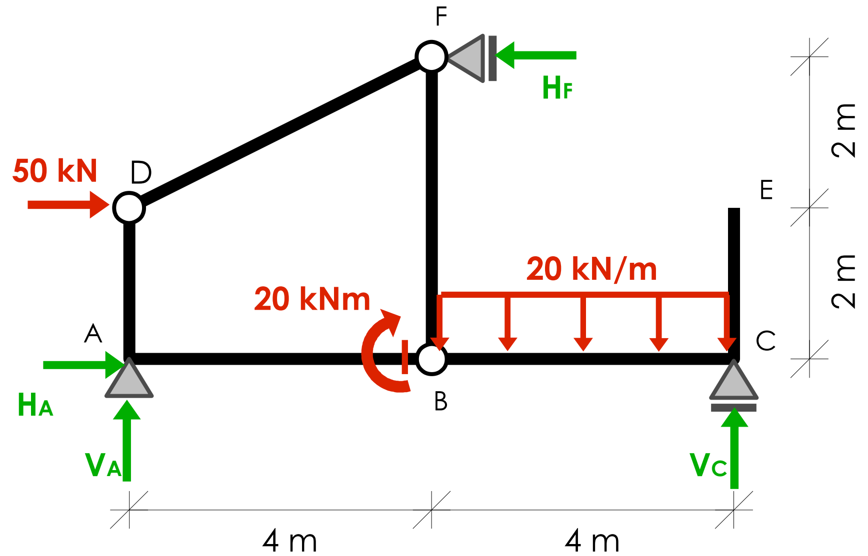
Bowstring C-G:



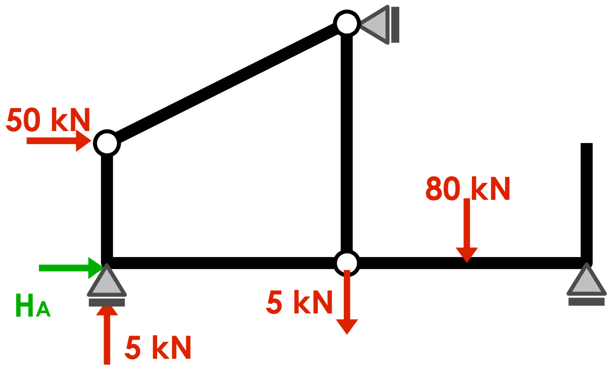
$$\delta L = 8\cdot\frac{\delta}{2} + \frac{2N}{\sqrt{5}}\cdot\frac{\delta}{2} - \frac{2N}{\sqrt{5}}\cdot\delta + 80\cdot\delta = 0 \quad \Rightarrow \quad N = 84\sqrt{5}$$

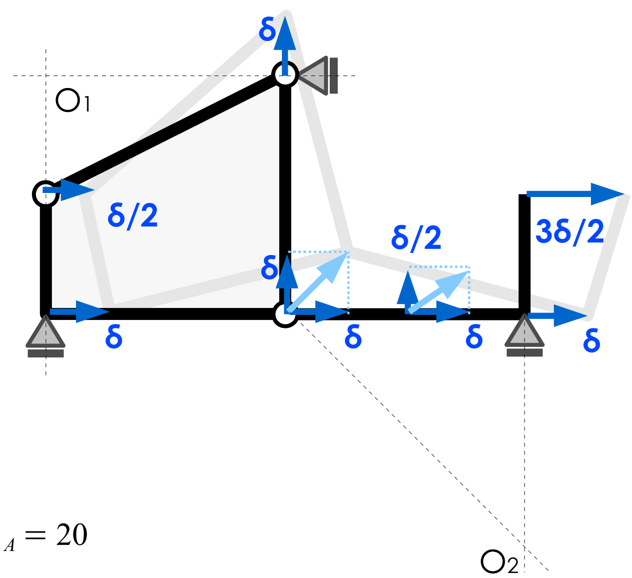
EXERCISE 7

Determine reaction H_A and force in bowstring D-F with the use of PVW.



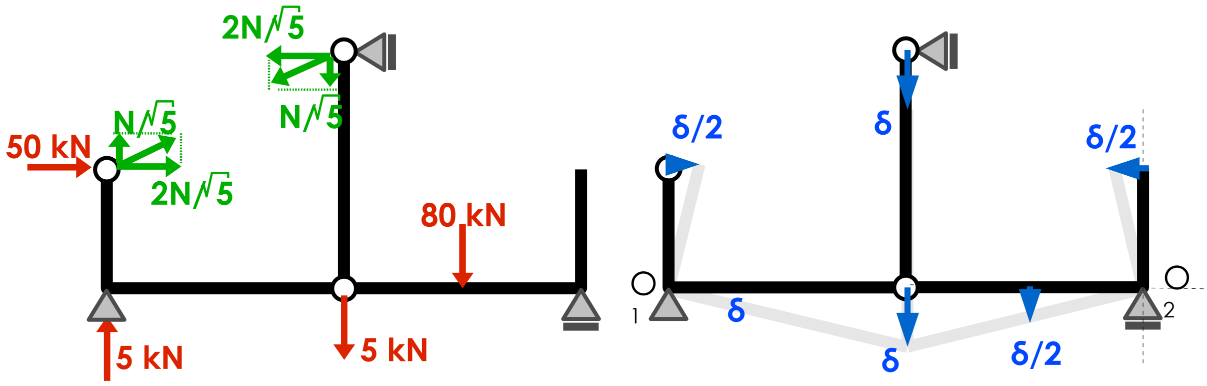
Reaction H_A:





$$\delta L = H_A \cdot \delta + 50 \cdot \frac{\delta}{2} - 5 \cdot \delta - 80 \cdot \frac{\delta}{2} = 0 \quad \Rightarrow \quad H_A = 20$$

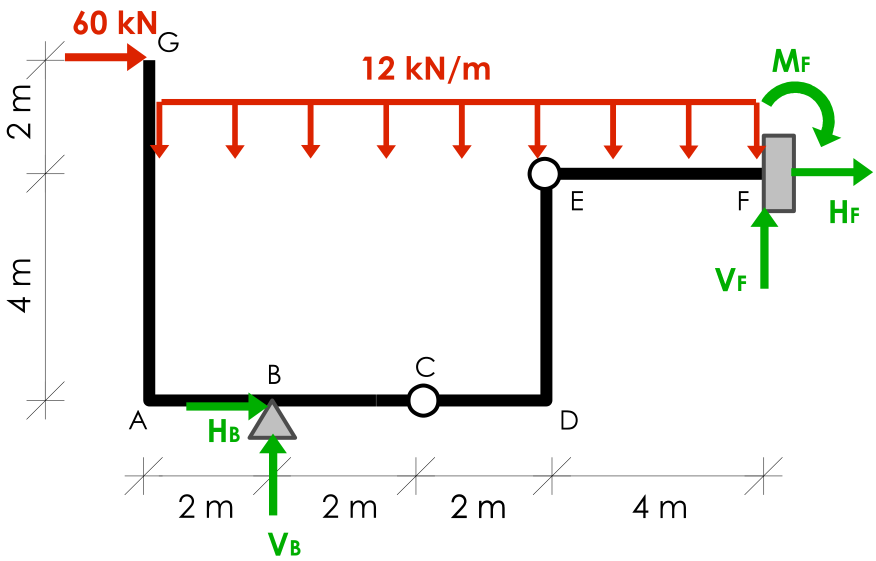
Bowstring D-F:



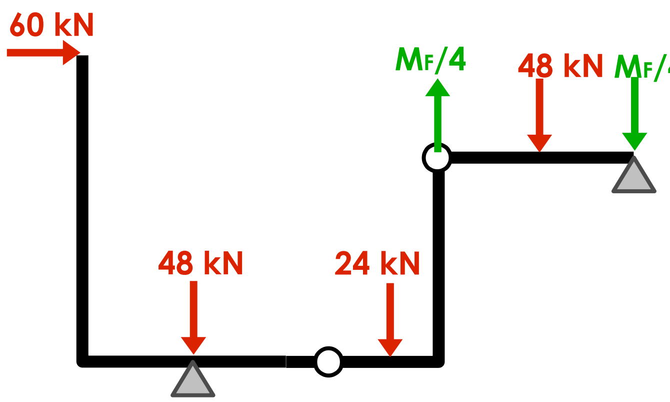
$$\delta L = 50 \cdot \frac{\delta}{2} + \frac{2N}{\sqrt{5}} \cdot \frac{\delta}{2} + 5 \cdot \delta + \frac{N}{\sqrt{5}} \cdot \delta + 80 \cdot \frac{\delta}{2} = 0 \quad \Rightarrow \quad N = -35\sqrt{5}$$

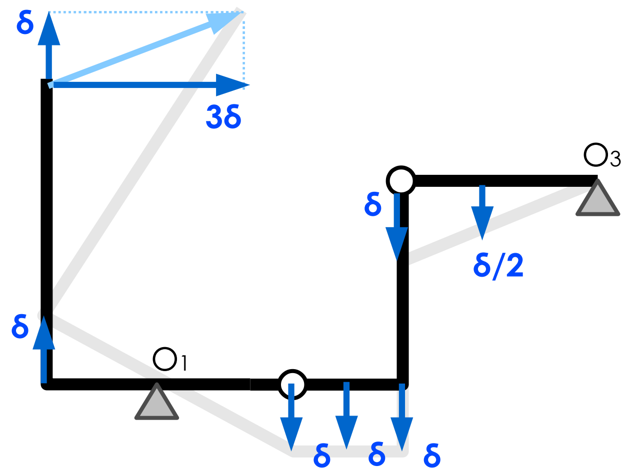
EXERCISE 8

Determine reactions V_B and M_F with the use of PVW.



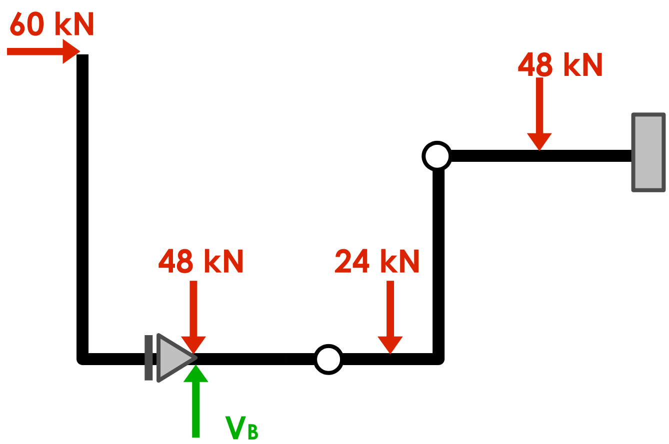
Reaction M_F:

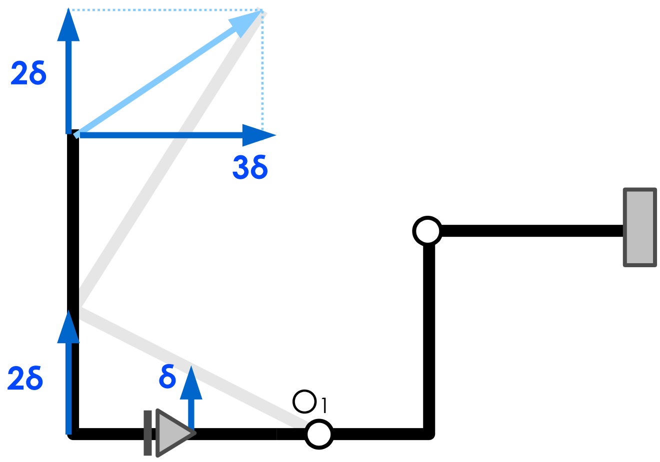




$$\delta L = 60 \cdot 3\delta + 24 \cdot \delta - \frac{M_F}{4} \cdot \delta + 48 \cdot \frac{\delta}{2} = 0 \quad \Rightarrow \quad M_F = 912$$

Reaction V_B:

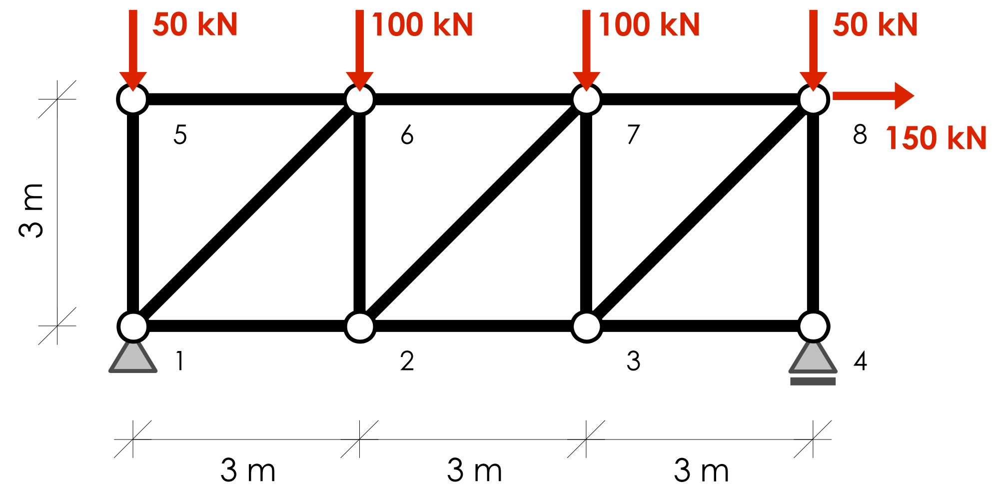




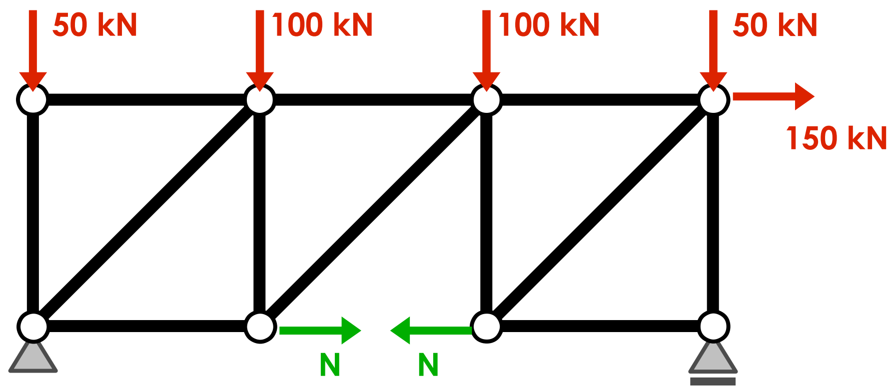
$$\delta L = V_B \cdot \delta - 48 \cdot \delta + 60 \cdot 3\delta = 0 \quad \Rightarrow \quad V_B = -132$$

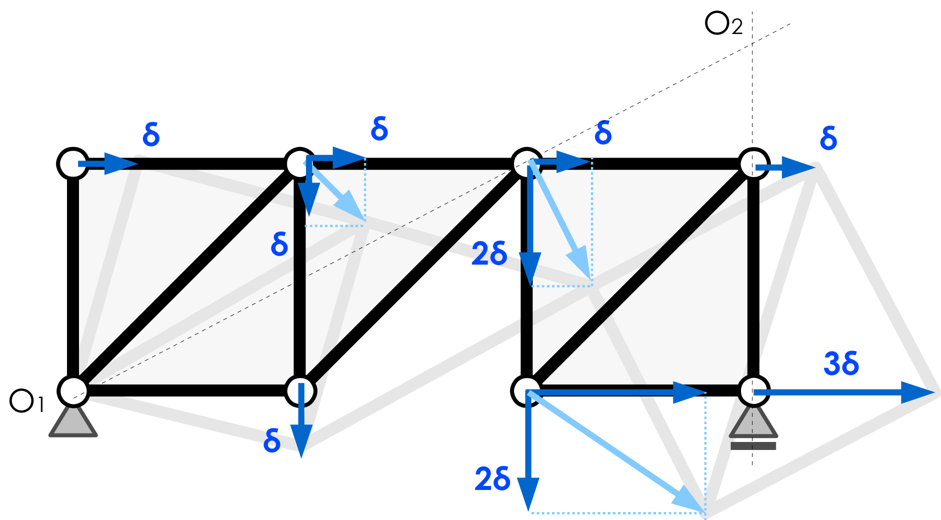
EXERCISE 9

Determine the forces in bars 2-3, 2-7 and 3-7 with the use of PVW



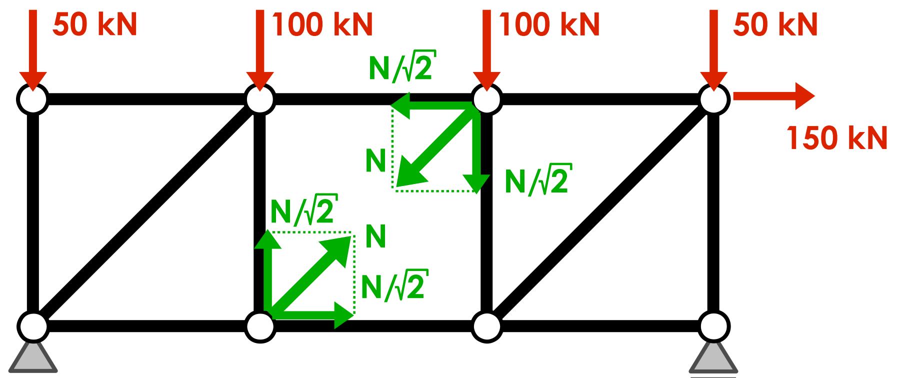
Bar 2-3:

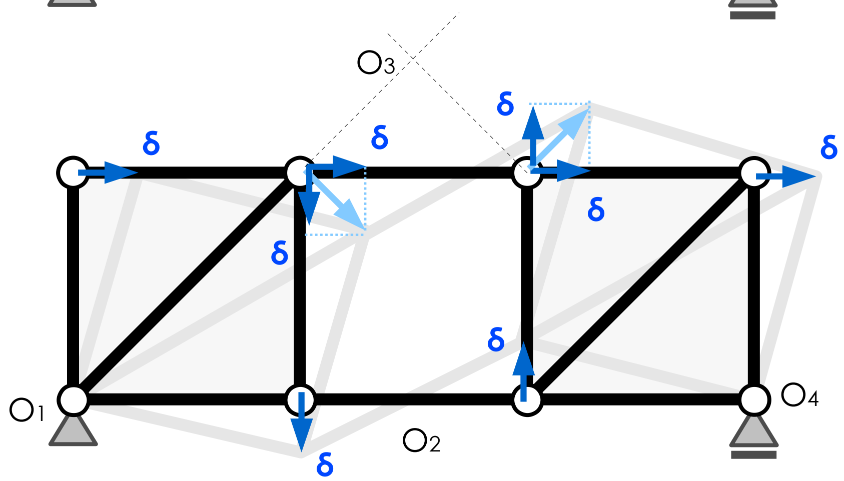




$$\delta L = 100 \cdot \delta + 100 \cdot 2\delta + 150 \cdot \delta - N \cdot 3\delta = 0 \quad \Rightarrow \quad N = 150$$

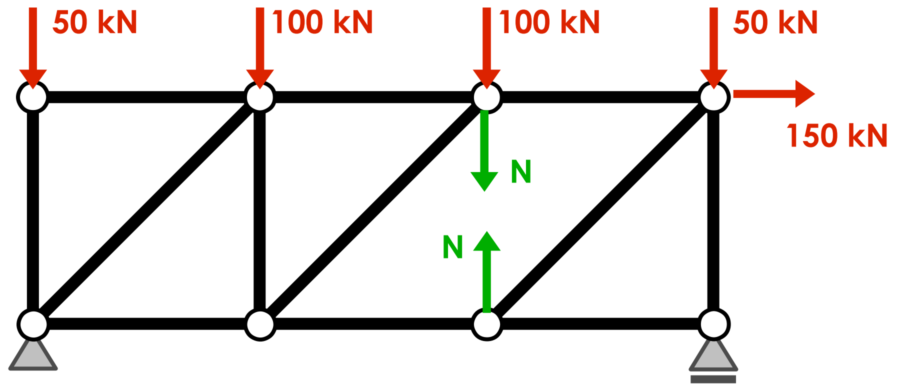
Bar 2-7:

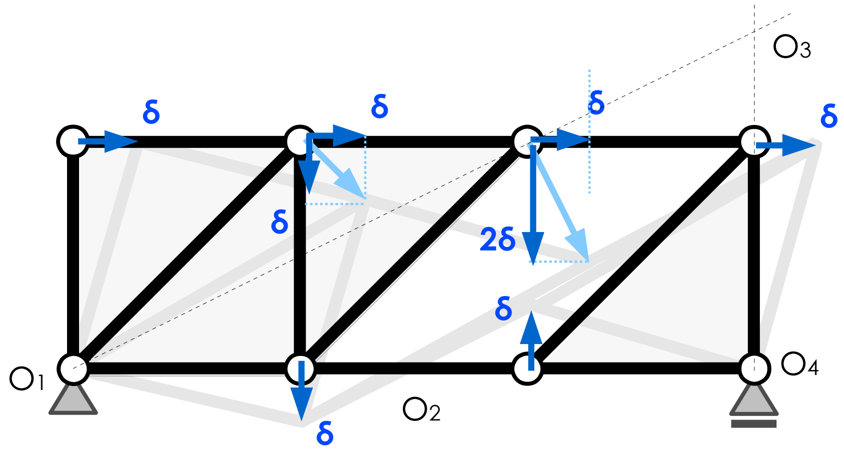




$$\delta L = 100\cdot\delta - 100\cdot\delta + 150\cdot\delta - \frac{N}{\sqrt{2}}\cdot\delta - \frac{N}{\sqrt{2}}\cdot\delta - \frac{N}{\sqrt{2}}\cdot\delta = 0 \quad \Rightarrow \quad N = 50\sqrt{2}$$

Bar 3-7:

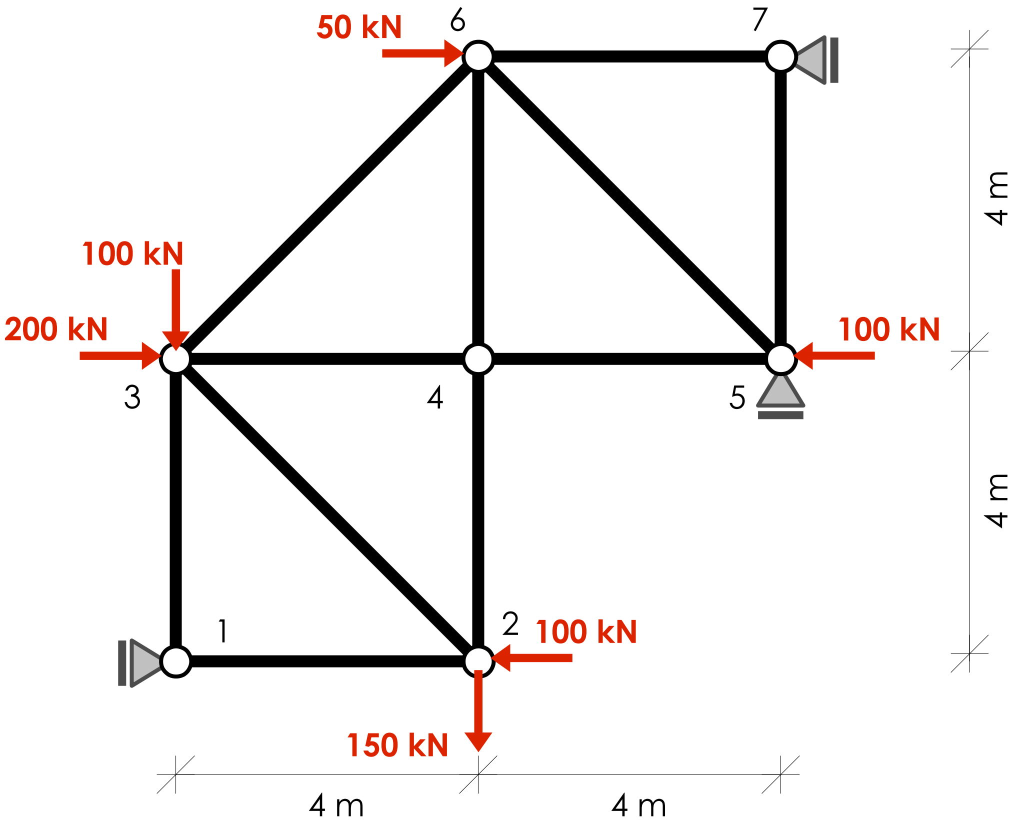




$$\delta L = 100 \cdot \delta + 100 \cdot 2\delta + N \cdot 2\delta + N \cdot \delta + 150 \cdot \delta = 0 \quad \Rightarrow \quad N = -150$$

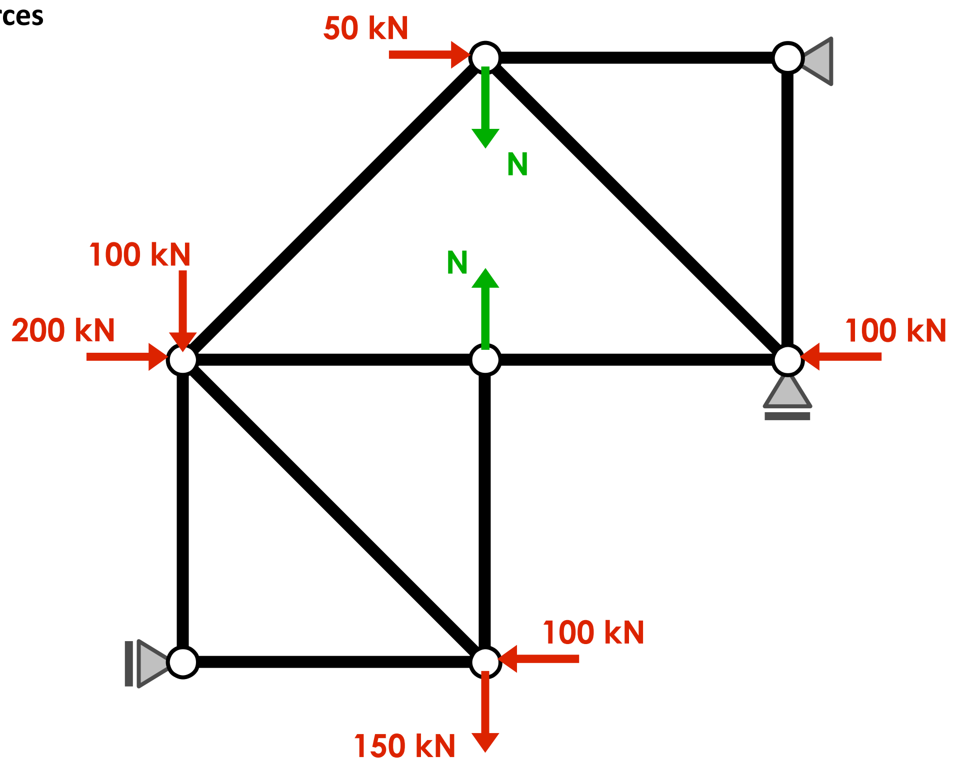
EXERCISE 10

Determine the force in bar 4-6 with the use of PVW.



After cutting through we obtain:

System of forces



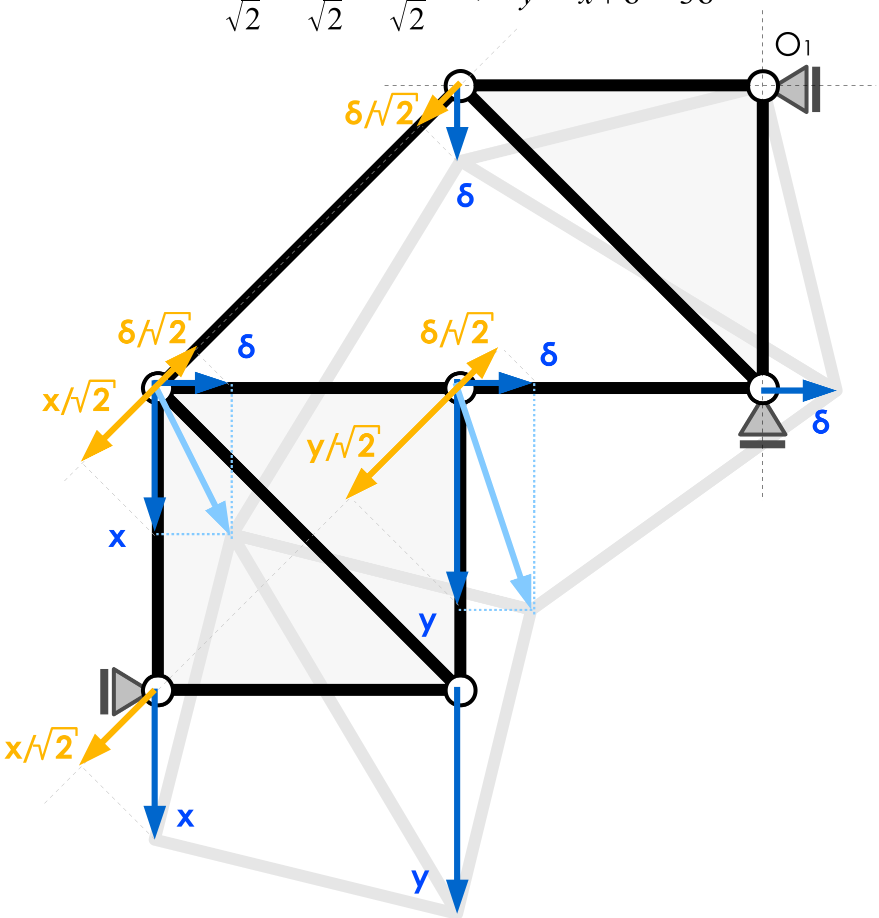
Distribution of displacements

- Suports in nodes 5 and 7 determine the center of rotation O_1 for body 5-6-7.
- Horizontal component of displacement in nodes 3 and 4 must be the same as in node 5 (they all lie on a single horizontal line)
- Unknown vertical components of displacements in nodes 3 and 4 are denoted as *x* and *y* respectively.
- Component *x* is determined with the use of condition, that projection of deisplacement vector in nodes 3 and 6 on direction connecting 3 and 6 must be the same:

$$\frac{x}{\sqrt{2}}-\frac{\delta}{\sqrt{2}}=\frac{\delta}{\sqrt{2}} \quad \Rightarrow \quad x=2\delta$$

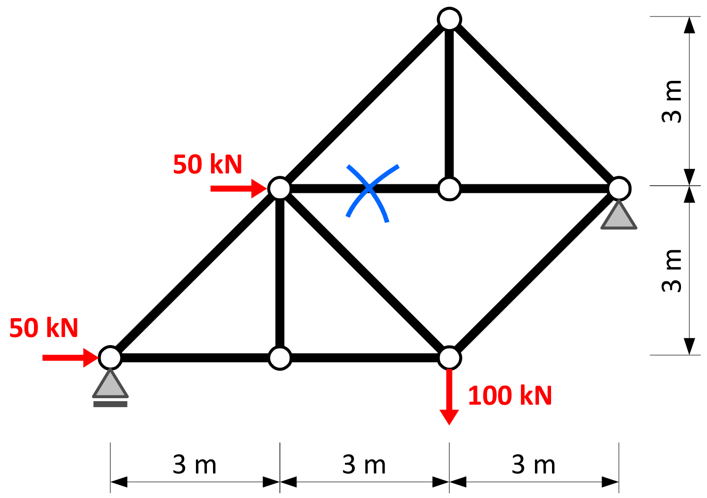
- Component *y* is determined with the use of condition, that projection of deisplacement vector in nodes 1 and 4 on direction connecting 1 and 4 must be the same:

$$\frac{x}{\sqrt{2}}=\frac{y}{\sqrt{2}}-\frac{\delta}{\sqrt{2}} \quad \Rightarrow \quad y=x+\delta=3\delta$$



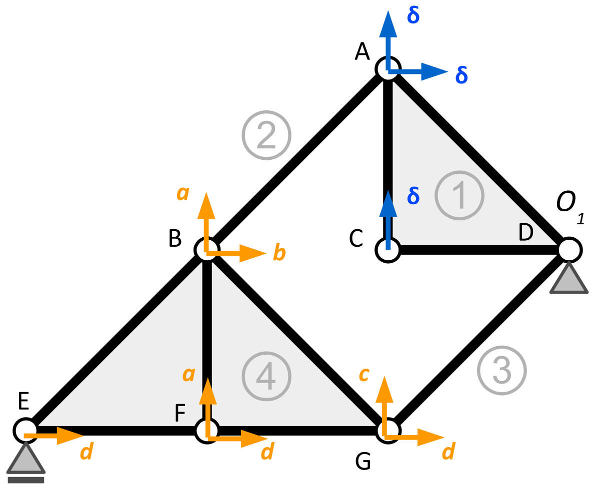
$$\delta L=200\cdot\delta+100\cdot 2\delta+150\cdot 3\delta-N\cdot 3\delta+N\cdot\delta-100\cdot\delta=0 \quad \Rightarrow \quad N=375$$

EXERCISE 11
Find the force in the bar denoted in the picture with the use of PVW.

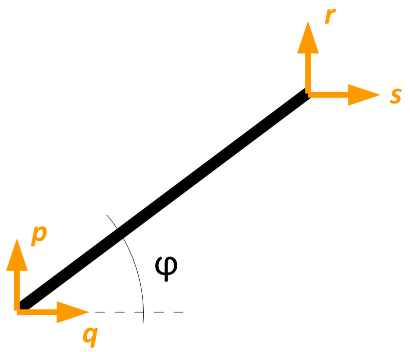


SOLUTION:

Center of rotation and distribution of displacements in body 1 may be found immediately. All other unknown displacements (their horizontal and vertical components) will be denoted with a, b, c, d . Accounting for theorems on distribution of velocitiec in a rigid body we obtain:



Values of unknown displacements may be found by the use of theorem on equal projections of velocity vectors in certain points on a line connecting those points. In case of an oblique bar we may write down:



$$p\cdot\sin\varphi+q\cdot\cos\varphi = r\cdot\sin\varphi+s\cdot\cos\varphi$$

In case of bars inclined at angle $\varphi=45^\circ$:

$$p\cdot\frac{1}{\sqrt{2}}+q\cdot\frac{1}{\sqrt{2}}=r\cdot\frac{1}{\sqrt{2}}+s\cdot\frac{1}{\sqrt{2}}$$

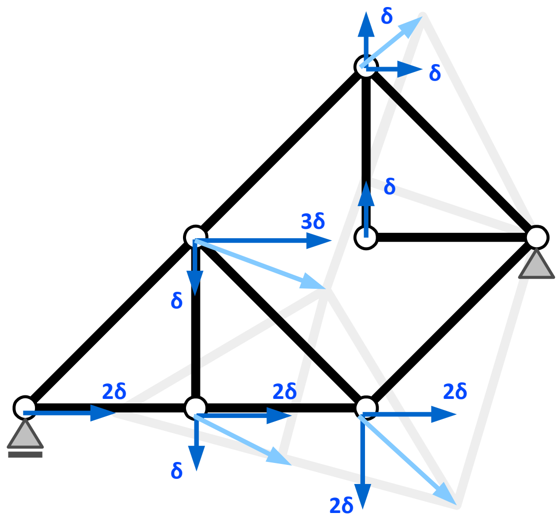
so finally:

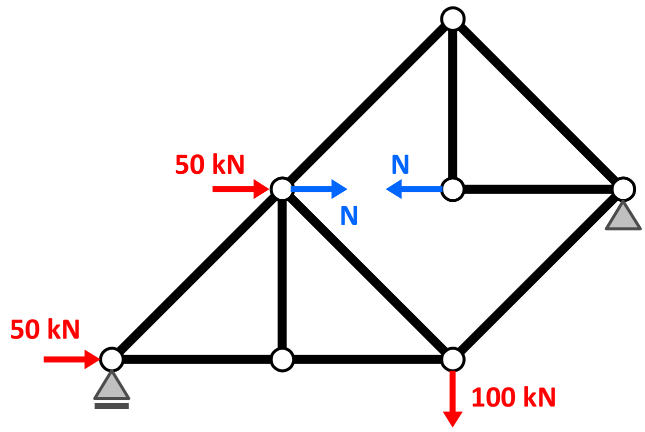
$$p + q = r + s$$

Using this simplified form (valid only for bars inclined at angle $\varphi=45^\circ$!) one must take care of the orientation of virtual displacements. Projecting on following directions gives us:

$$\begin{cases} AB: \quad a+b=\delta+\delta \\ GD: \quad c+d=0 \\ BG: -a+b=-c+d \\ EB: \quad d=b+a \end{cases} \Rightarrow \begin{cases} a=-\delta \\ b=3\delta \\ c=-2\delta \\ d=2\delta \end{cases}$$

Distribution of displacements:





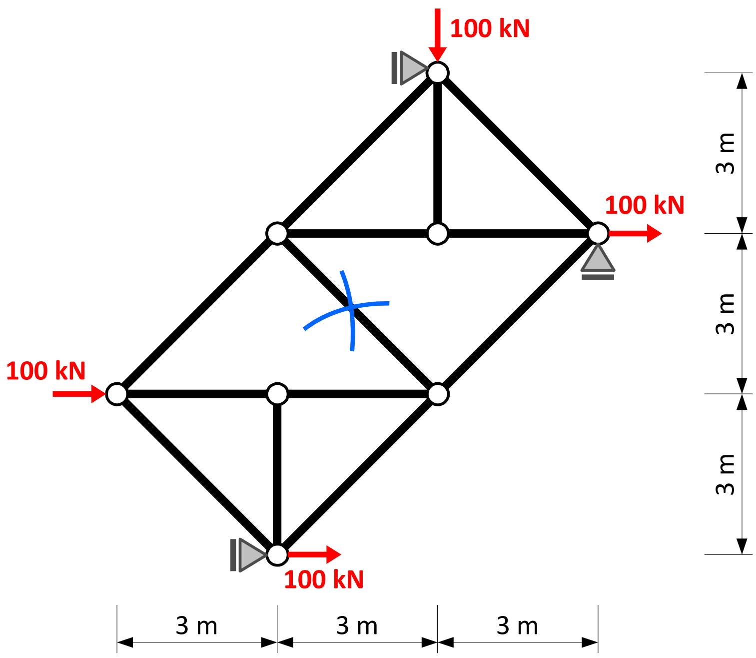
Virtual work:

$$\delta L=50\cdot3\delta + N\cdot3\delta + N\cdot0 + 50\cdot2\delta + 100\cdot2\delta = 0$$

$$\boxed{N=-150 \text{ kN}}$$

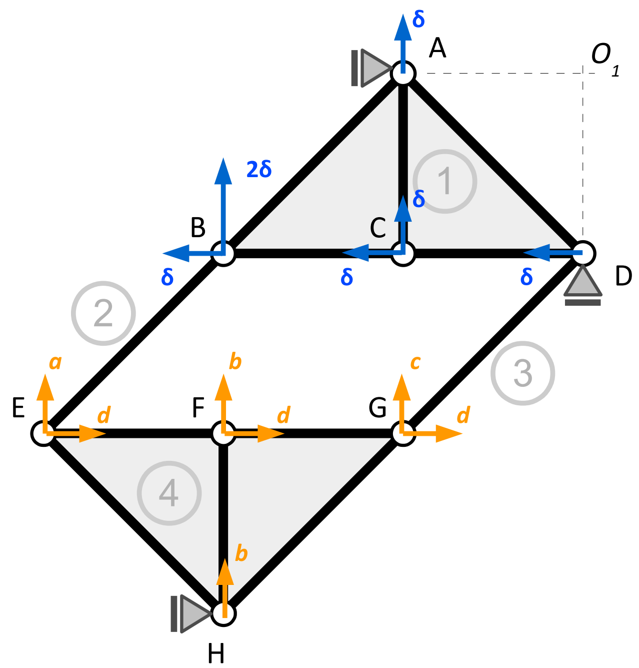
EXERCISE 12

Find the force in the bar denoted in the picture with the use of PVW.



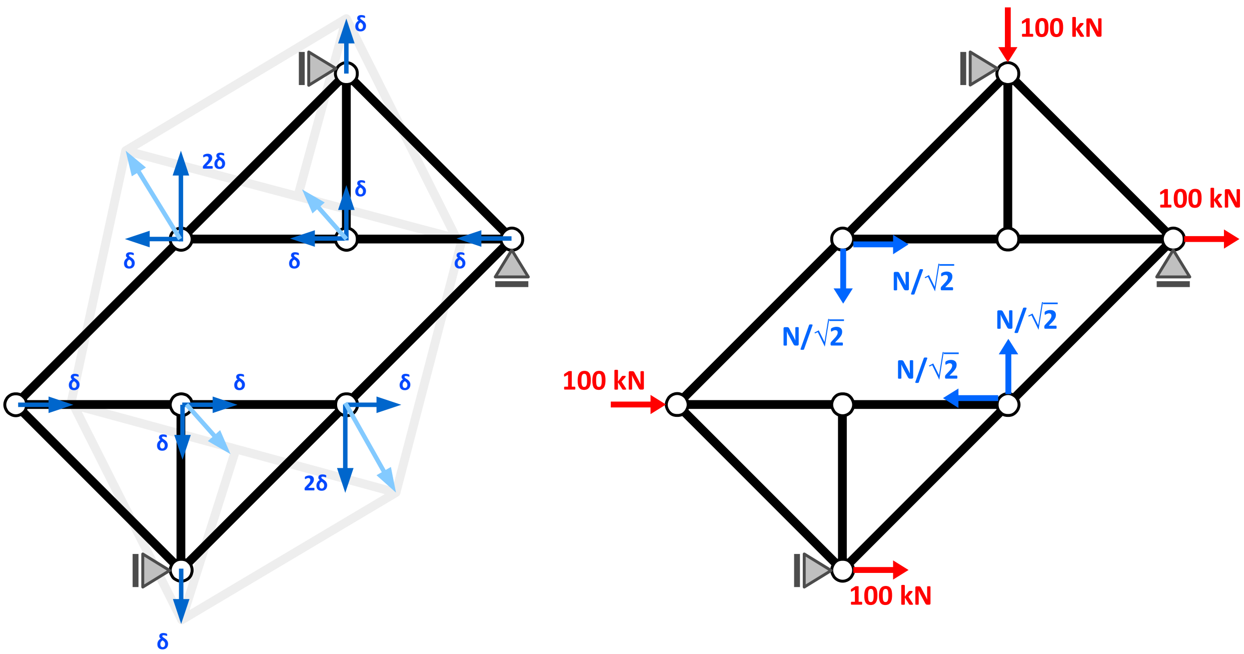
SOLUTION:

Let's determine the distribution of displacements after cutting through bar. Center of rotation and distribution of displacements for body may be found immediately. All other displacements are denoted as unknowns with the use of constraints imposed by supports and theorems on distribution of velocities in rigid body.



Projecting on following directions gives us:

$$\begin{cases} EB: \quad a+d=-\delta+2\delta \\ GD: \quad c+d=-\delta \\ EH: -a+d=-b \\ HG: \quad c+d=b \end{cases} \Rightarrow \begin{cases} a=0 \\ b=-\delta \\ c=-2\delta \\ d=\delta \end{cases}$$



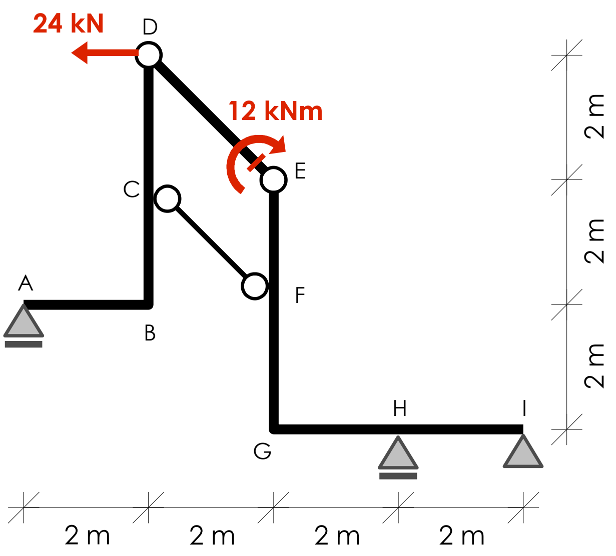
Virtual work:

$$\delta L=-100\cdot\delta-\frac{N}{\sqrt{2}}\cdot\delta-\frac{N}{\sqrt{2}}\cdot 2\delta-100\cdot\delta+100\cdot\delta-\frac{N}{\sqrt{2}}\cdot\delta-\frac{N}{\sqrt{2}}\cdot 2\delta+100\cdot 0=0$$

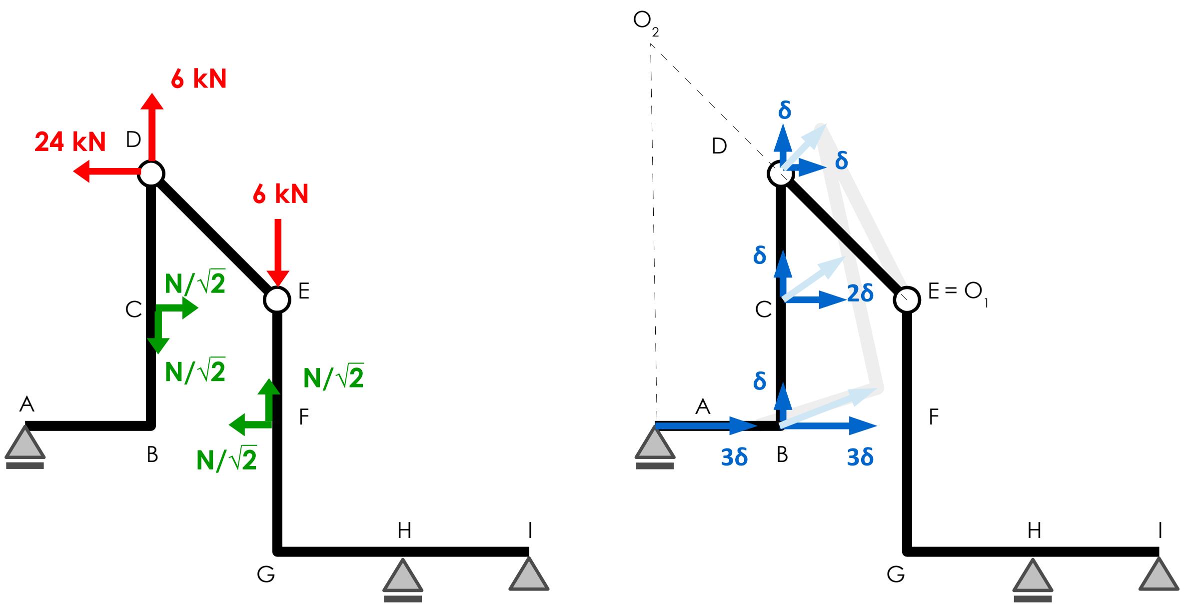
$$\boxed{N=-\frac{50}{3}\sqrt{2}\ \text{kN}\approx-23{,}57\ \text{kN}}$$

EXERCISE 13

Find the force in bar CF with the use of PVW:



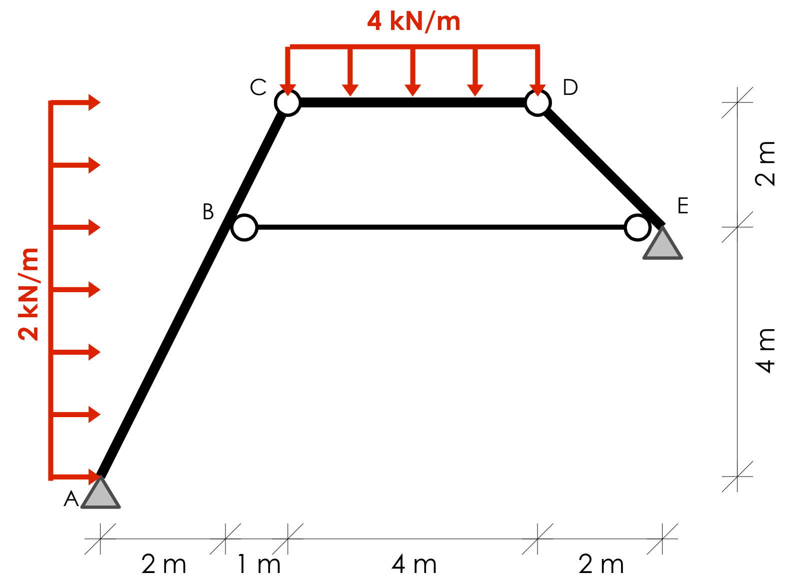
SOLUTION:



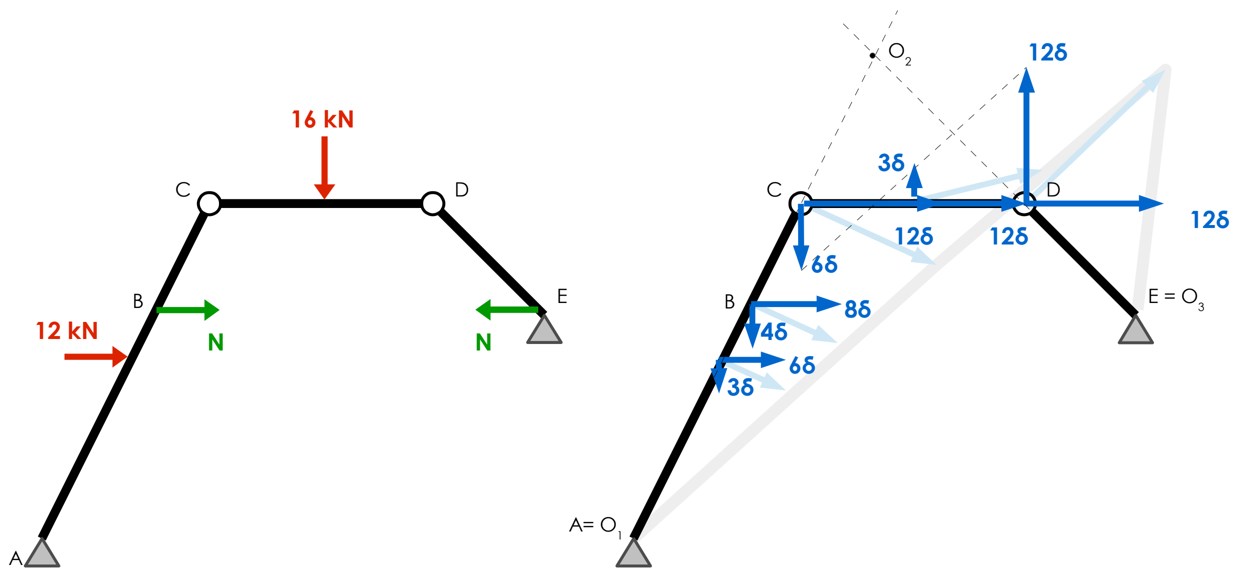
$$\delta L = -24\cdot\delta + 6\cdot\delta + \frac{N}{\sqrt{2}}\cdot 2\delta - \frac{N}{\sqrt{2}}\cdot\delta = 0 \qquad \Rightarrow \qquad N = 18\sqrt{2}\ \text{kN}$$

EXERCISE 14

Find the force in bar BE with the use of PVW:



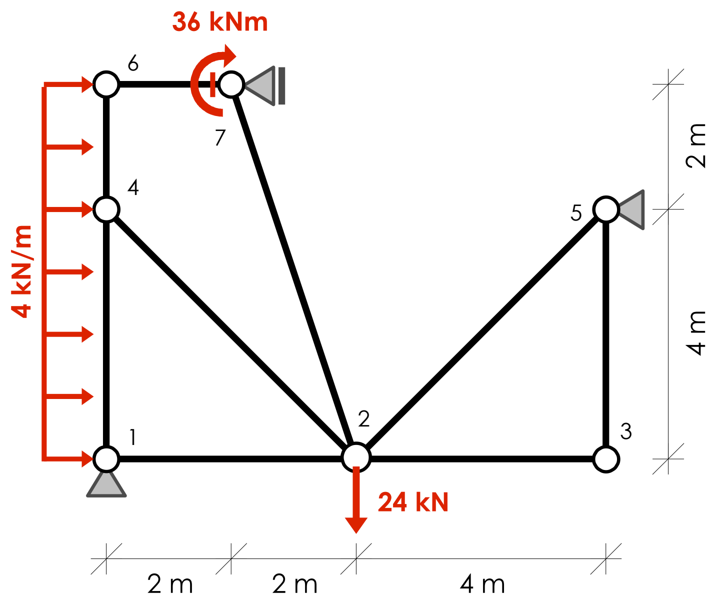
SOLUTION:



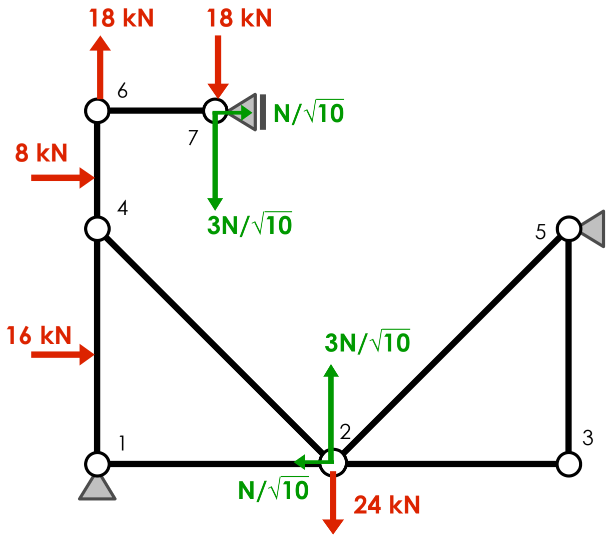
$$\delta L = 12 \cdot 6\delta + N \cdot 8\delta - 16 \cdot 3\delta = 0 \quad \Rightarrow \quad N = -3 \text{ kN}$$

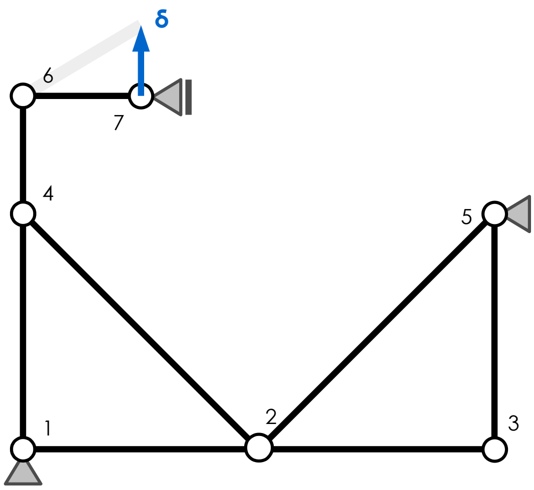
EXERCISE 15

Find the force in bar 2-7 with the use of PVW::



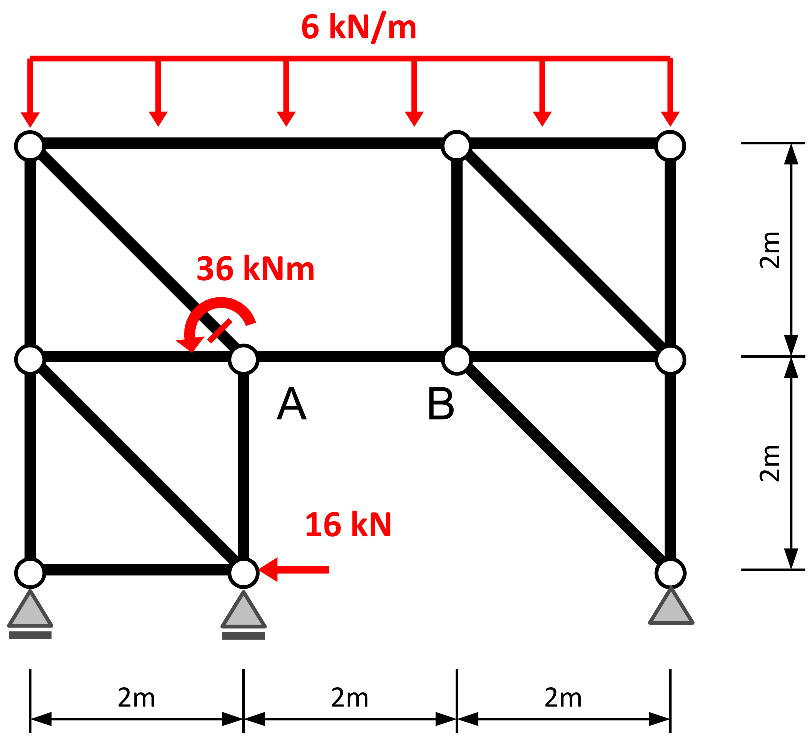
SOLUTION:



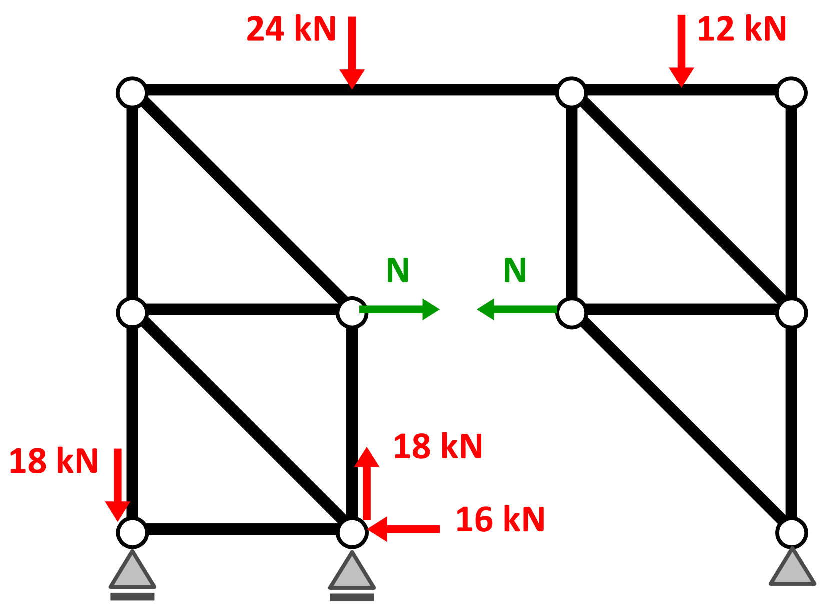


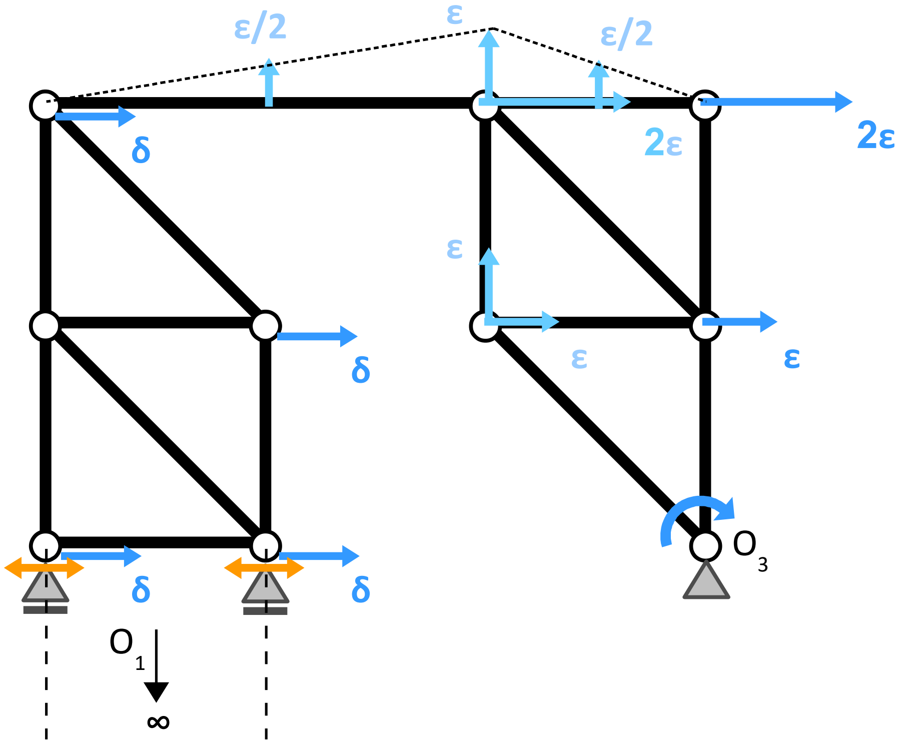
$$\delta L = -18 \cdot \delta - \frac{3N}{\sqrt{10}} \cdot \delta = 0 \qquad \Rightarrow \qquad N = -6\sqrt{10}\ \text{kN}$$

EXERCISE 16
Find an axial force in bar AB with the use of Principle of Virtual Works.



SOLUTION:





Due to equality of projections of virtual displacements on a line connecting two points in a rigid body:

$$\delta = 2\,\varepsilon \quad \Rightarrow \quad \varepsilon = \frac{1}{2}\delta$$

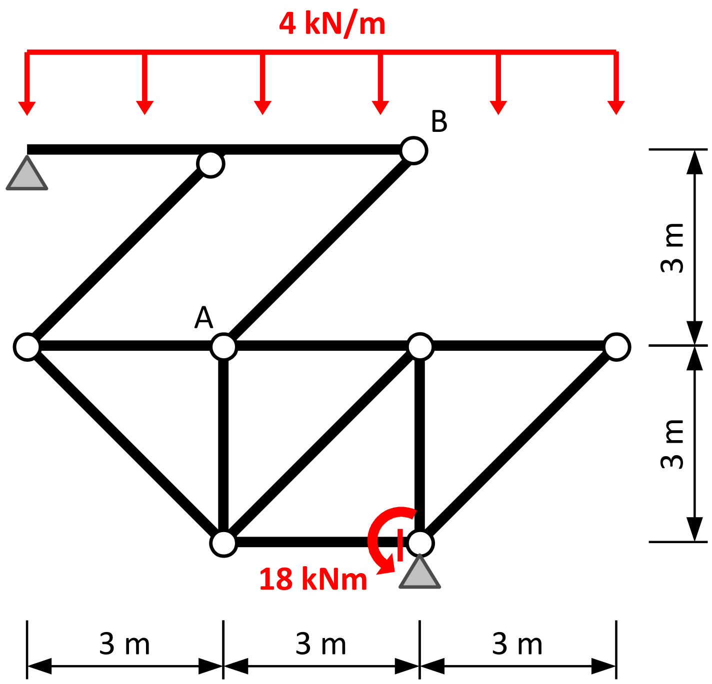
Virtual work:

$$\delta L = -16\cdot\delta + N\cdot\delta - N\cdot\frac{\delta}{2} - 24\cdot\frac{1}{2}\left(\frac{\delta}{2}\right) - 12\cdot\frac{1}{2}\left(\frac{\delta}{2}\right) = 0 \quad \underset{\delta}{\forall}$$

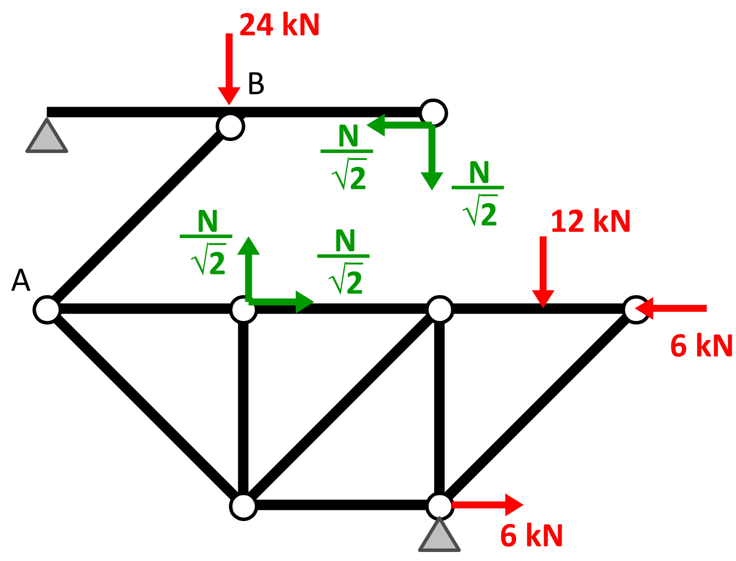
$$N = 50 \text{ kN}$$

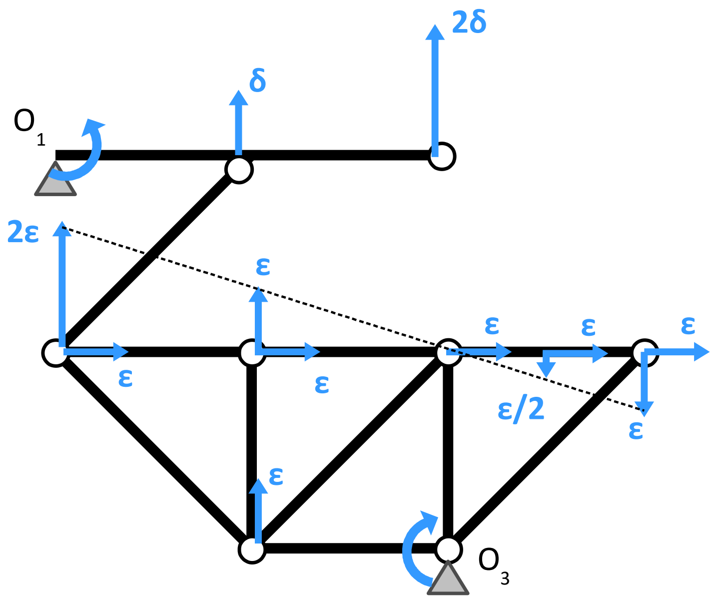
EXERCISE 17

Find an axial force in bar AB with the use of Principle of Virtual Works.

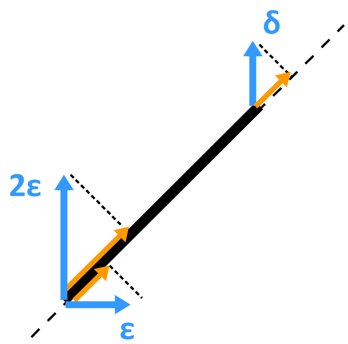


SOLUTION:





Due to equality of projections of virtual displacements on a line connecting two points in a rigid body:



$$\frac{2\,\varepsilon}{\sqrt{2}} + \frac{\varepsilon}{\sqrt{2}} = \frac{\delta}{\sqrt{2}} \quad \Rightarrow \quad \varepsilon = \frac{\delta}{3}$$

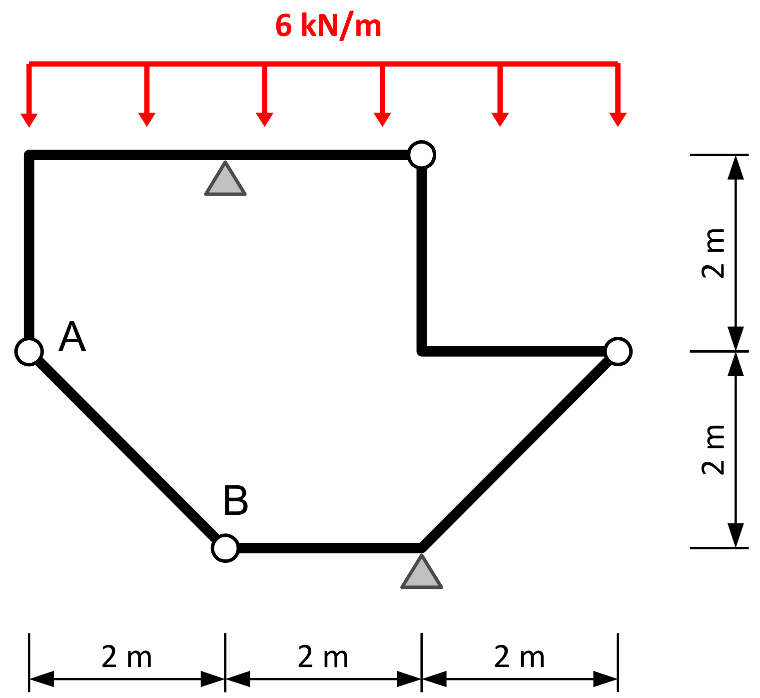
Virtual work:

$$-24\cdot\delta + 12\cdot\frac{1}{2}\left(\frac{\delta}{3}\right) - 6\cdot\frac{\delta}{3} - \frac{N}{\sqrt{2}}\cdot 2\,\delta + \frac{N}{\sqrt{2}}\cdot\frac{\delta}{3} + \frac{N}{\sqrt{2}}\cdot\frac{\delta}{3} = 0 \quad \forall_{\delta}$$

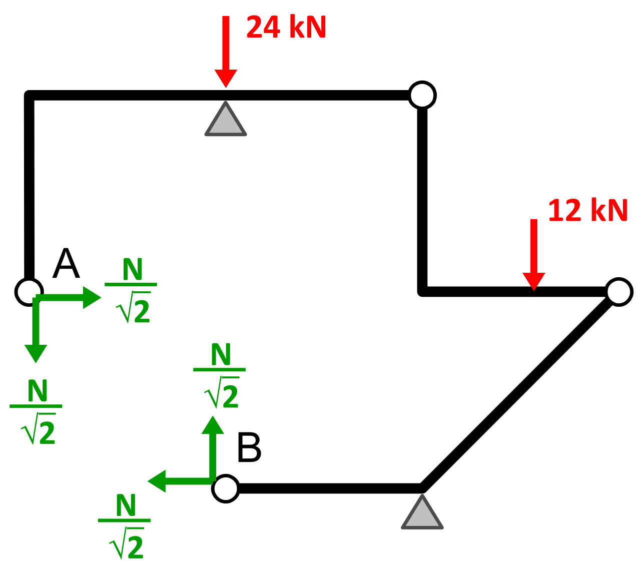
$$N = 18\sqrt{2}\ \mathrm{kN}$$

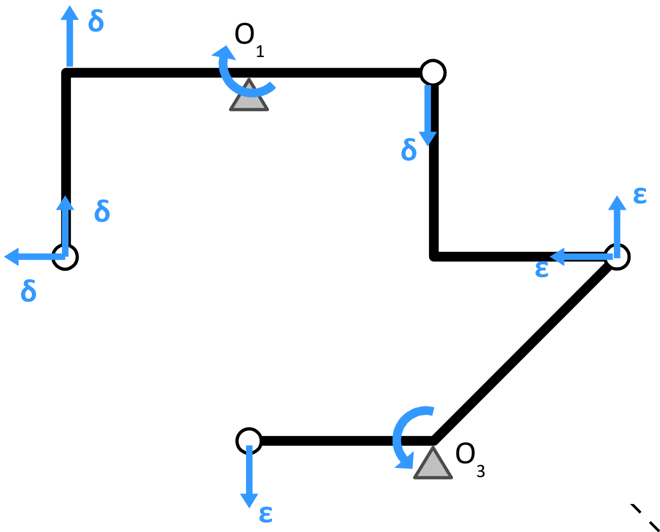
EXERCISE 18

Find an axial force in bar AB with the use of Principle of Virtual Works.

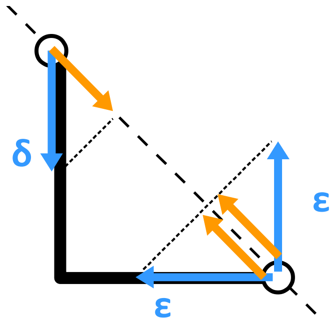


SOLUTION:

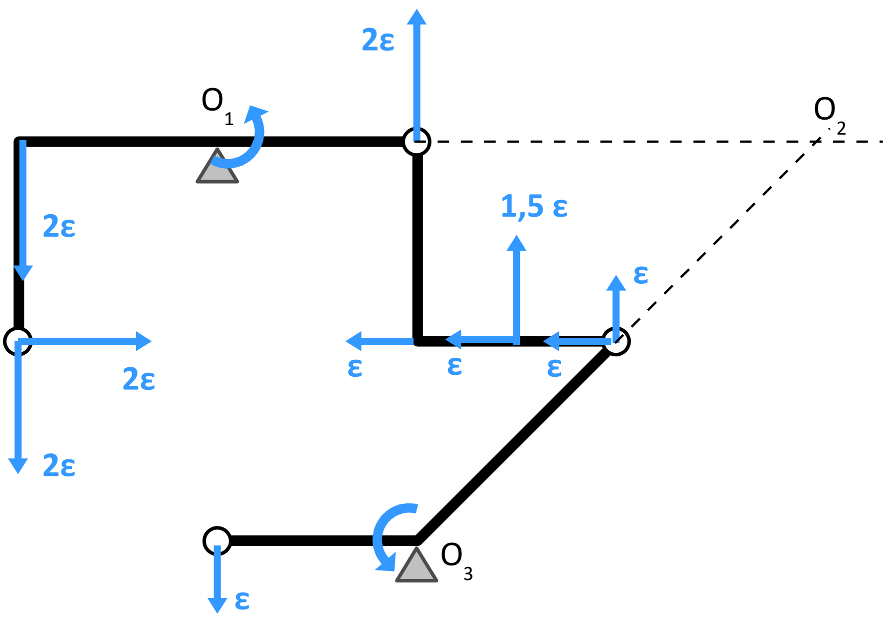




Due to equality of projections of virtual displacements on a line connecting two points in a rigid body:



$$\frac{\varepsilon}{\sqrt{2}} + \frac{\varepsilon}{\sqrt{2}} = -\frac{\delta}{\sqrt{2}} \quad \Rightarrow \quad \varepsilon = -\frac{\delta}{2}$$



Virtual work:

$$\delta L = -12 \cdot 1{,}5\varepsilon - \frac{N}{\sqrt{2}} \cdot \varepsilon + \frac{N}{\sqrt{2}} \cdot 2\varepsilon + \frac{N}{\sqrt{2}} \cdot 2\varepsilon = 0 \quad \forall_{\varepsilon}$$

$$N = 6\sqrt{2}\ \mathrm{kN}$$