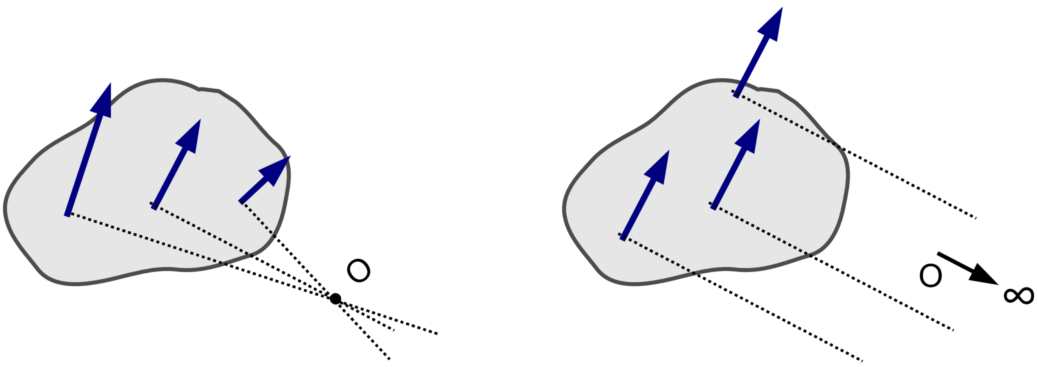
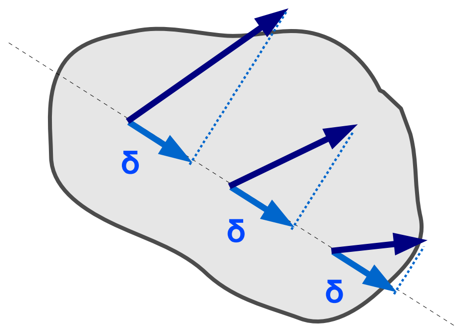
PRINCIPLE OF VIRTUAL WORKS

THEOREMS ON DISTRIBUTION OF VELOCITIES IN A RIGID BODY

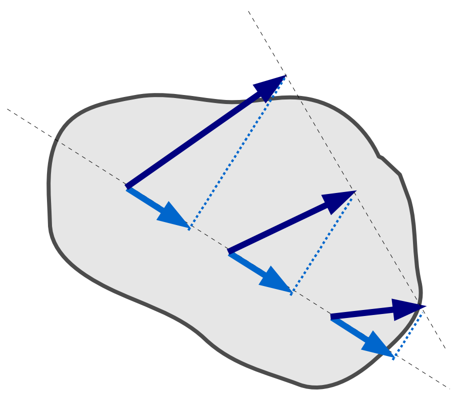
1. **Any planar motion of a rigid body in every moment *t* may be interpreted as a rotation about an instant center of rotation.**
   - Instant center of rotation in each moment *t* is in general in a different place – it moves as well.
   - Instant center of rotation may be placed – in particular - in infinity, however, in a certain direction – rotation becomes then a parallel translation in a perpendicular direction.



2. **In case of points lying on a single straight line, projections of their velocity vectors on this line are all equal.**



3. **In case of points lying on a single straight line, ends of their velocity vectors also lie on a single straight line (in general, a different one)**

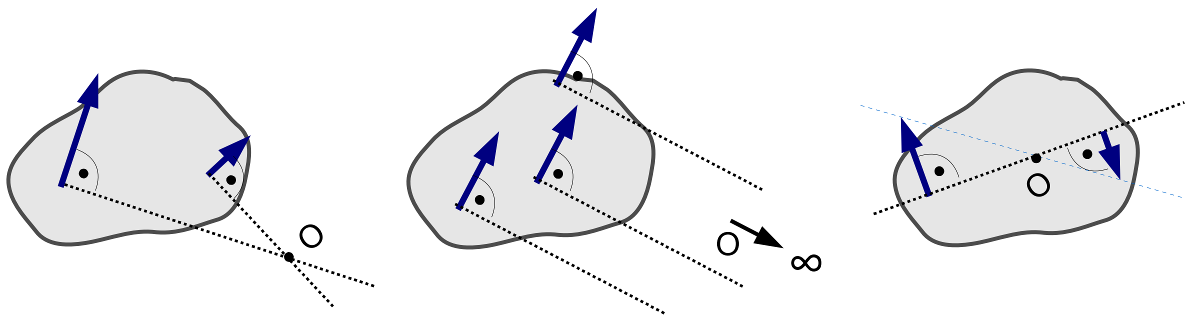


Following conclusions may be derived:

1. **Velocity vector in a given point is always perpendicular to a line connecting this point with an instant center of rotation.**

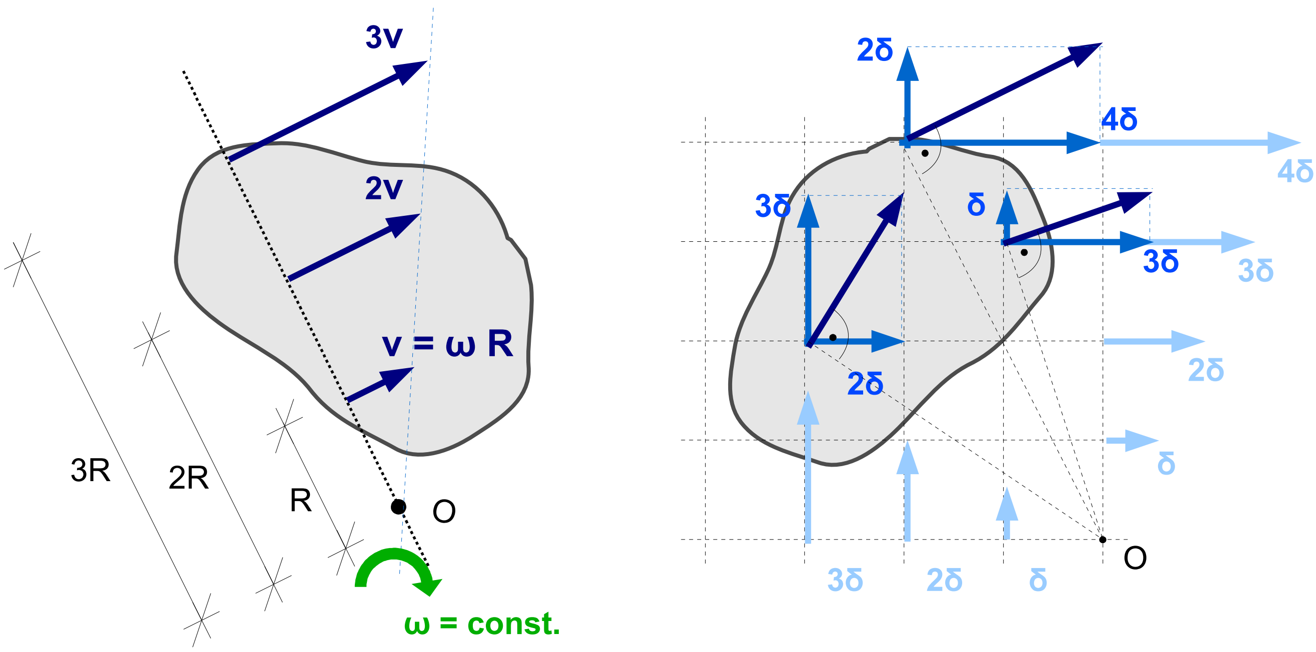
2. **If we know at least directions of velocity vectors (not the vectors themselves) in two points which do not lie on a line perpendicular to those directions, then the instant center of rotation lies at intersection of lines which are perpendicular to those direction of velocities** (in particular – when those lines are parallel, their intersection lies in infinity).

3. **If we know velocity vectors in two points lying on a line which is perpendicular to those vectors, then the instant center of rotation lies at intersection of this line with a line connecting ends of velocity vectors.**



4. **Velocity vectors lying ona single straight line connecting them with the instant center of rotation have lengths which is proportional to the distance from the center of rotation.**

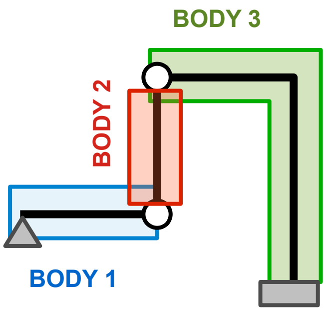
5. **If we determine the instant center of rotation for a plane rigid body, then we are able to determine (relative) velocities in every point of the body.**

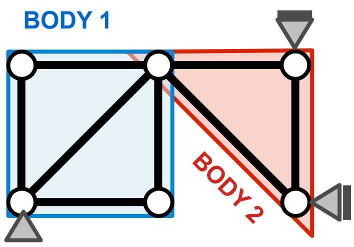


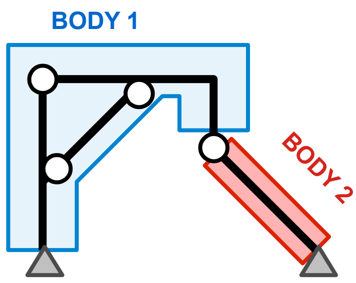
GENERAL METHODOLOGY OF SOLVING EXCERCISES CONCERNING PVW

1. DETERMINE RIGID BODIES

- A rigid body may consist of a singe bar (straight ar curved one) and any three such rigid bodies which are connected together with the use of three non-collinear joints.

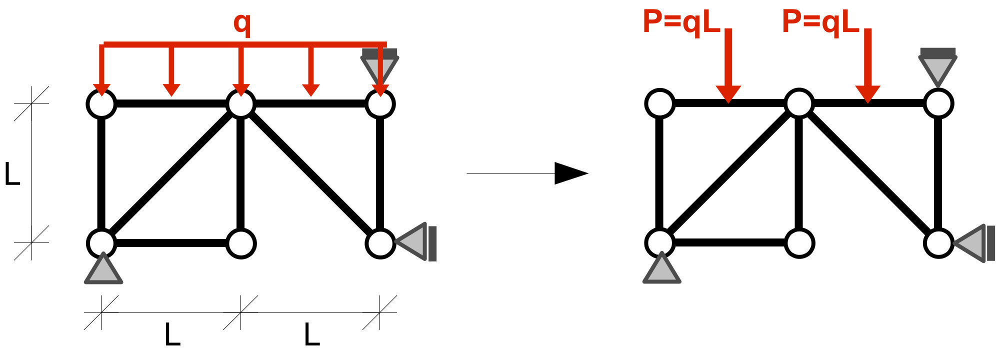




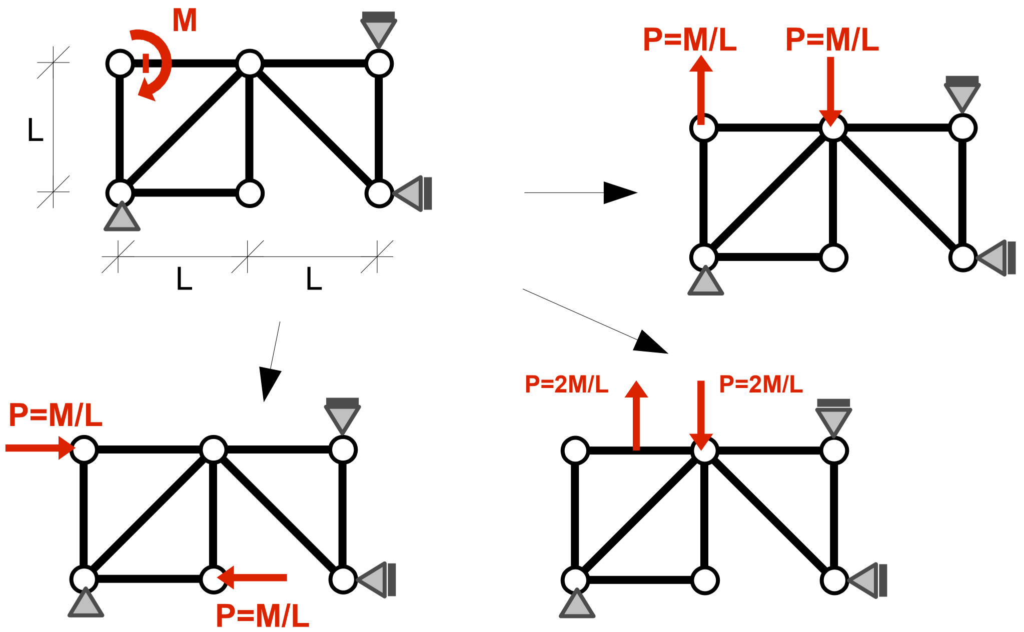


2. REPLACE EACH EXTERNAL LOAD WITH POINT FORCES

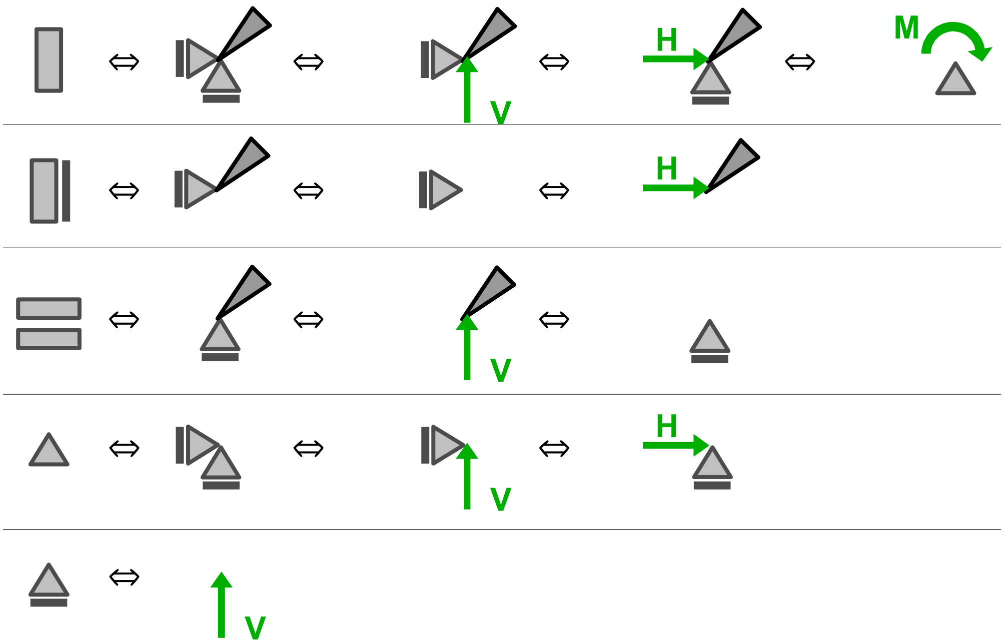
- The distributed load must be decomposed into parts loading separately each of determined rigid bodies. Each one of those parts is then replaced with an appropriate resultant point force.



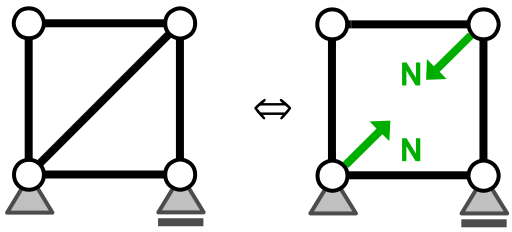
- Point moments are decomposed into a couple of point forces (two point forces of the same magnitude, acting along different but parallel lines and oriented in an opposite way). The forces constituting a couple may be applied anywhere on the body, to which the point force is applied. The forces must be equal *P* = *M* / *d*, where *M* is the value of point moment and *d* the distance between the lines of actions of those forces.



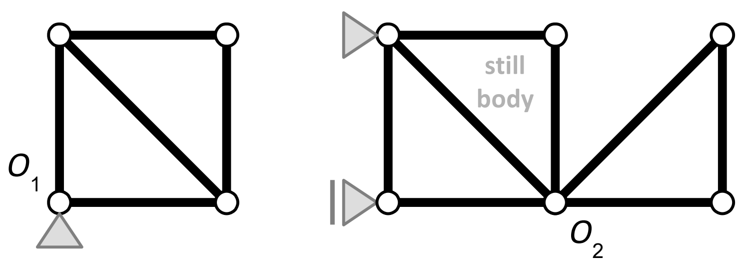
3. **RELEASE A CONSTRAINT CORRESPONDING TO A FORCE WHICH WE ARE LOOKING FOR**
   - In case of reaction at a support, we replace the support with an appropriate released support with additional unknown force



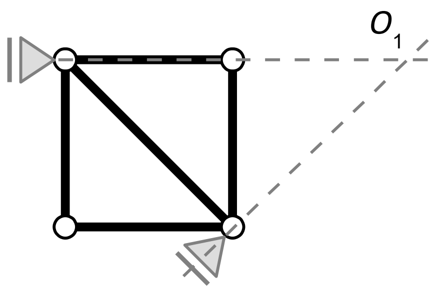
   - In case of an axial force in a truss bar we replace the bar with a system of two opposite unknown point forces parallel to the axis of removed bar and applied to the nodes of this bar



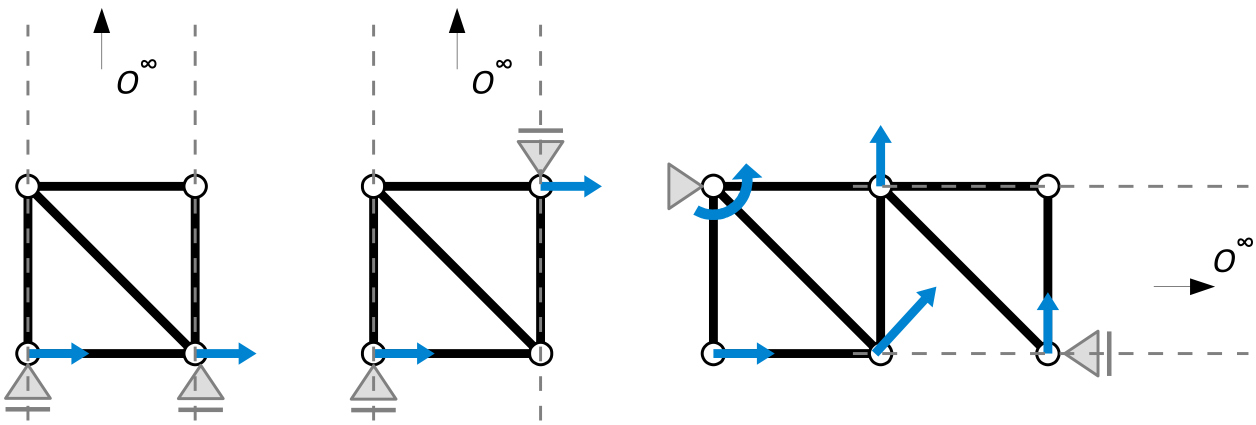
4. **DETERMINE INSTANT CENTER OF ROTATION FOR EVERY BODY WHICH MAY PERFORM ANY MOTION**
   - If in a body, for which not all degrees of freedom are constrained, there is a single immovable point (e.g. Pinned support or a joint connection to a separate immovable body), then this point is instant center of rotation.



- Instant center of rotation lies at intersection of lines which are perpendicular to two directions of permissible displacement.



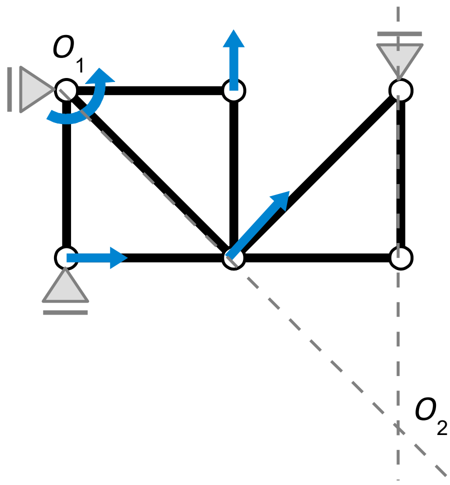
- In particular there may exist an improper instant center of rotation, lying in infinity – the body performs a parallel translation. Such situation occurs when there are two point which have parallel directions of permissible displacement and they do not lie on a line perpendicular to this direction.



5. **INTRODUCE A VIRTUAL DISPLACEMENT** (of any value or orientation) **OF A SINGLE BODY** (chosen arbitrary), **WHICH IS IN ACCORDANCE WITH PERMISSIBLE DIRECTIONS OF DISPLACEMENT.**

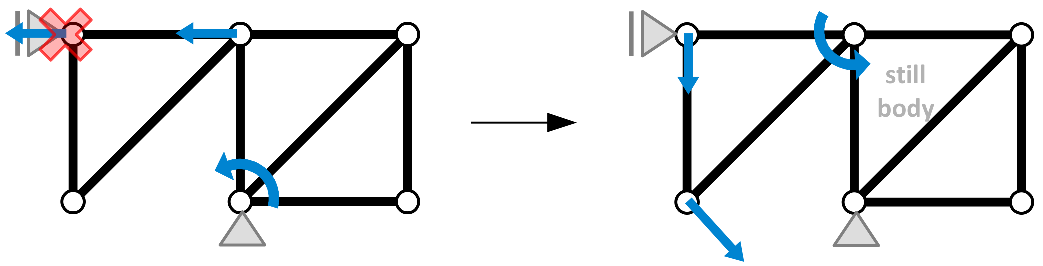
6. DETERMINE VIRTUAL DISPLACEMENTS OF THE WHOLE SYSTEM ACCORDING TO THE THEOREMS ON DISTRIBUTION OF VELOCITIES IN A RIGID BODY.

- In particular, having determined virtual displacements of point of the first body, the displacement of joint connection of this body with another body gives a permissible direction of displacement for this second body. If the permissible direction of displacement in another point of the second body is found (e.g. due to presence of certain support or joint connection with another body), then instant center of rotation may be determined.

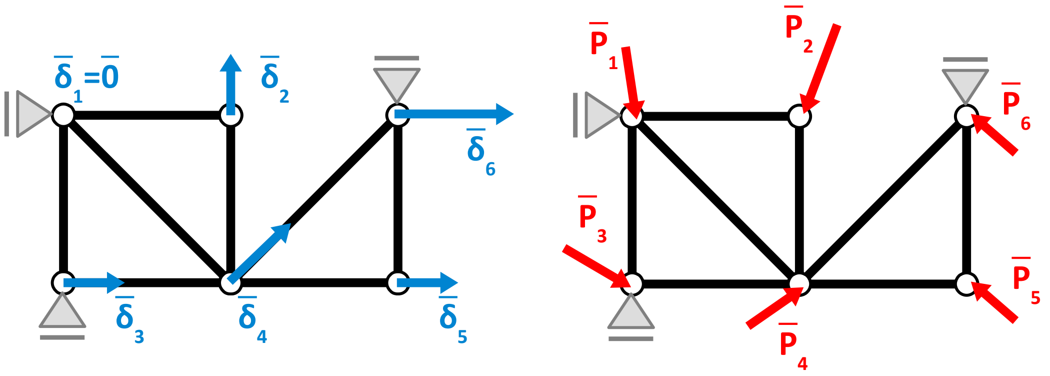


7. **HAVING ASSUMED THE FIRST VIRTUAL DISPLACEMENT, DISPLACEMENTS OF OTHER BODIES CANNOT BE CHOSEN ARBITRARY** – if the system was immovable and allowed a single degree of freedom, then the allowed motion is the one, which was chosen at the beginning – **ALL OTHER DISPLACEMENTS MUST FOLLOW FROM THIS ASSUMPTION.**

8. **IF DURING DETERMINING VIRTUAL DISPLACEMENTS WE OBTAIN AN INCONSISTENCY** (e.g. according to the theorems on distribution of velocities in a rigid body it concludes that there should exist a component displacement along direction which is not permissible in certain point), **THEN PROPOSED DISTRIBUTION OF DISPLACEMENTS IS WRONG**. We should go back to the first chosen displacement and if it was the only permissible displacement, then we must assume that the whole body must remain still. According to this new assumption we determine new displacements in the rest of bodies.



9. **HAVING DETERMINED FULL DISTRIBUTION OF DISPLACEMENTS WE CALCULATE VARIATION OF WORK δL FOR INFINITELY SMALL DISPLACEMENT AS A SUM OF SCALAR PRODUCTS OF DISPLACEMENT VECTORS AND LOAD VECTORS IN EVERY POINT IN WHICH LOAD IS APPLIED**.



$$\delta L = \sum_{i=1}^{N} \bar{\boldsymbol{\delta}}_i \cdot \bar{\mathbf{P}}_i = \sum_{i=1}^{N} \left(\delta_{i,x} P_{i,x} + \delta_{i,y} P_{i,y} \right)$$

10. **OBTAINED VALUE MUST BE EQUAL ZERO – THIS GIVES US AN EQUATION FOR UNKNOWN FORCE**. The equation may be divided by the value of the first displacement as all other displacements are proportional to it.