KINEMATIC ANALYSIS OF STRUCTURAL SYSTEM

The aim of **kinematic analysis** of a structural system is to answer the question is the system a mechanism (it may perform a motion as a rigid body, it has at elast 1 degree of freedom) or not. In particular, in case of systems, for which preliminary estimate of degrees of freedom suggest, that they are stable, kinematic analysis is to prove that **indeed the system cannot move in any way**. It may be proven by **assumption of a motion which is compatible with constraints applied to the system and then showing that it leads to inconsistency** , e.g.:

- Show that assumed motion require a **velocity in a supported point which is not admissible by that support** (Fig. 1, Fig. 3).
- Show that assumed motion of two rigid bodies require **in a common point (hinge) two incompatible directions of velocity** (Fig. 2).
- Show that assumed motion require a **non-zero velocity in the only possible location of instant center of rotation** (Fig. 3).

Fig. 1

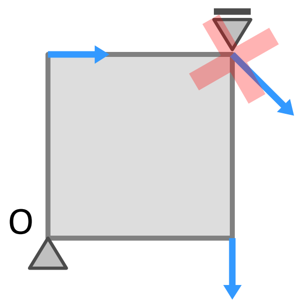


Fig. 2

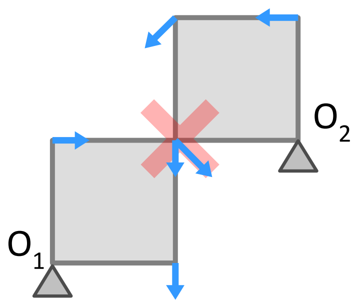
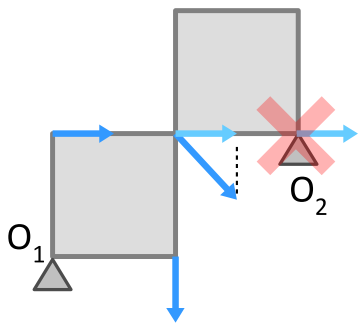
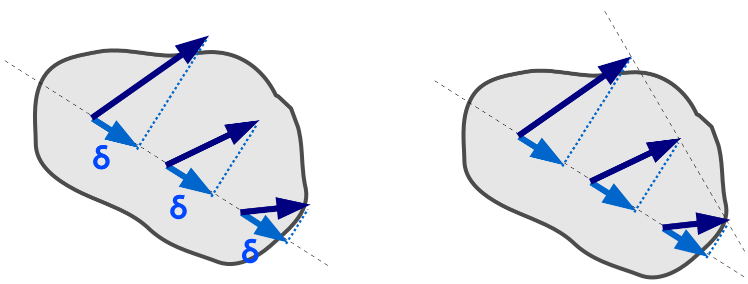


Fig. 3



- Show that assumed motion requires a **distribution of velocities which violates theorems on distribution of velocities in a rigid body**, namely:
  - *Projections of velocity vectors of points lying on a single straight line on that line must be the same.*
  - *Ends of velocity vectors of points lying on a single straight line must line on a straight line.*



NUMBER OF DEGREES OF FREEDOM

Preliminary information on the number of degrees of freedom is given by the formula below. For a plane motion:

$$NDOF = 3 \cdot B - 2 \cdot H - R$$

gdzie:

B – **number of rigid bodies**. Each rigid body has 3 DOF on a plane – horizontal translation, vertical translation and rotation. Each bar (straight or curved) and each 3 such bars connected with 3 non-colinear hinges (in particular a truss made of triangular areas) is considered to be a rigid body.

P – **number of one-fold hinges**. Each one-fold hinge corresponds with two constraints on motion of one of the bodies connected in it – horizontal and vertical motion must be the same in that point for both connected bodies. A hinge in which N+1 bodies are connected is called an N-fold hinge and it is equivalent to N one-fold hinges. In case of hinges that are contained in a rigid body (trusses) this number must be reduced to a number corresponding with the number of *distinct* rigid bodies (not contained in a single rigid body) connected in that hinge.

R – **number of constraints** applied to the system (in particular: **number of support reactions**).

OUTLINE OF KINEMATIC ANALYSIS

1. **Determine the number of rigid bodies.** For each body it is necessary to find possible location of instant center of rotation (in general, it will be different for different bodies) and corresponding distribution of velocities. Our goal is to show that such a distribution is not admissible by the supports.

2. **Analysis starts with a body, to which the greatest number of constraints is applied** (supports of greatest number of reactions). If there are few bodies of the same number of constraints we may chose any of those.

3. **Find the instant center of rotation fo the 1st body and find corresponding distribution of velocities – orientation** and **magnitude** of velocity may be chosen **arbitrary –** it will depend on a parameter v .

4. **We chose the next body, determine its instant center of rotation and corresponding distribution of velocities** depending on the case:

   - If the **previously analyzed body is immovable,** the next body will be **any neighboring body** (connected with the previous one with a hinge)

     - **Instant center of rotation of that next body will be located in the hinge connecting it with the immovable body (**unless it cannot move itself**)**

     - Orientation and magnitude of motion is chosen arbitrary.

   - If the **previously analyzed body can move,** the next body will be the neighboring body, to which any constraints are applied

     - Instant center of rotation for that next body will be located at **intersection of lines which are perpendicular to the direction which is admissible by constraints and to direction of assumed velocity in the hinge**, which which it is connected to the previously analyzed body (this line passes through that hinge and through instant center of rotation of previously analyzed body).

     - Orientation and magnitude of motion is determined according to the just found location of instant center of rotation and to the velocity in a hinge connecting that body with the one analyzed previously, so that also for that next body the components of velocities could be expressed with the use of the parameter v assumed earlier.

   - If the **previously analyzed body can move,** and **no constraints are applied to any of the neighboring bodies**, then the next body will be the body to which a second greatest number of constraints is applied, which is not a neighboring one – this means that between that next body and that analyzed previously there are some other bodies connected with hinges.

- Orientation and magnitude of motion is chosen arbitrary ,however we make it dependent of some other paramter u .

5. We repeat step 4 for any sequence of rigid bodies in which the distribution of velocities depend on different parameters until such distributions are determined in bodies which are separated only with a single body connected with them by hinges. We determine the velocities in those hinges and apply a theorem on equality of projections of velocities of points lying on a single line in a rigid body – the theorem is applied for the line passing through mentioned hinges. In such a way we find dependence between v i u .

6. **Repeat steps 4 and 5 until an inconsistency is found that will prove that assumed motion is not admissible.**

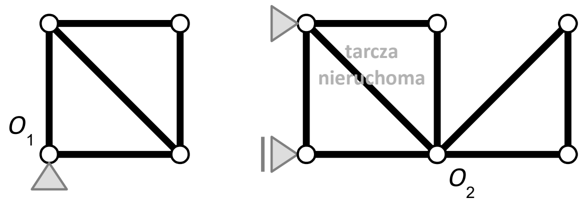
7. **After finding the inconsistency we reject the initial assumption** that led us to inconsistency (namely the assumed motion of the 1st body) and **we assume that the 1st body is immovable**. Having made this assumption another kinematic analysis is performed – we assume motion of a neighboring body that will rotate about an immovable hinge connecting that body with the immovable one. We perform the whole outline again until new inconsistency is found. Then we assume that also this neighboring body cannot move. We repeat the analysis until all possible motions are proved to be inconsistent.

FINDING THE INSTANT CENTER OF ROTATION

Instant center of rotation is a point, in which the velocity field has zero value. There is only one such point – this means that **if for a certain body an immovable point is found, then either it is an instant center of rotation or the body cannot move at all**.

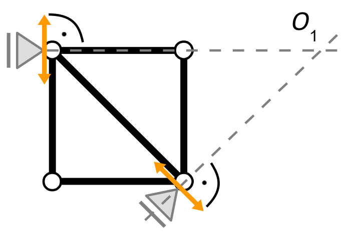
- If any point in a body is **pinned** then it is the only possible location of instant center of rotation.

- If a body is **connected in a hinge with an immovable body**, then the hinge is the only possible location of instant center of rotation.

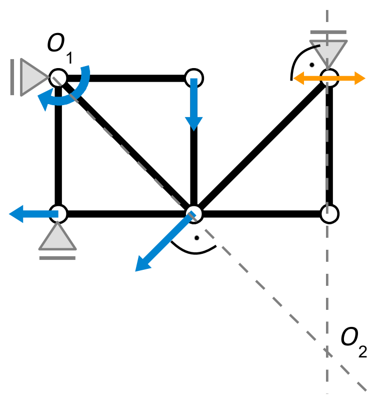


In all other cases **instant center of rotation lies at intersection of lines which are perpendicular to the admissible directions of velocities**. It is enough to know **two such directions**.

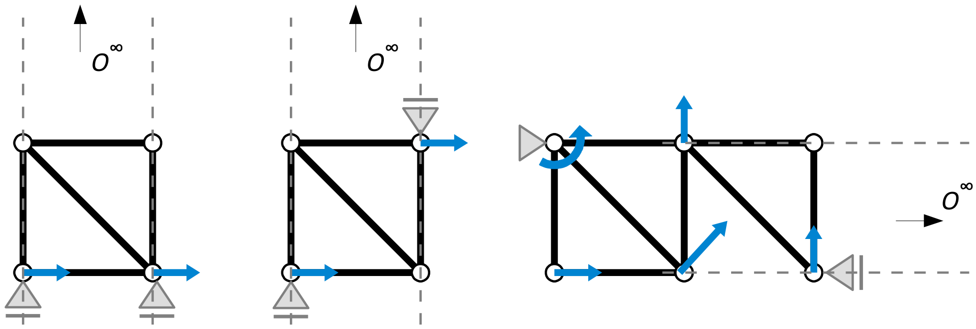
- Admissible velocity directions are determined by supports.



- Admissible velocity directions may be determined by a support and by an a already known direction of velocity in a hinge. Since the latter must be perpendicular to a line connecting the hinge with instant center of rotation (both for body 1 and 2), so **instant center of rotation of body 2 must lie on a line connecting instant center of rotation of body 1 and hinge**.



- If lines perpendicular to admissible velocity directions are parallel, then intant center of rotation is an improper point in infinity – the body performs only **translation**, rigid move without rotation. In such case **all points in that body wave exactly the same velocity vector**.

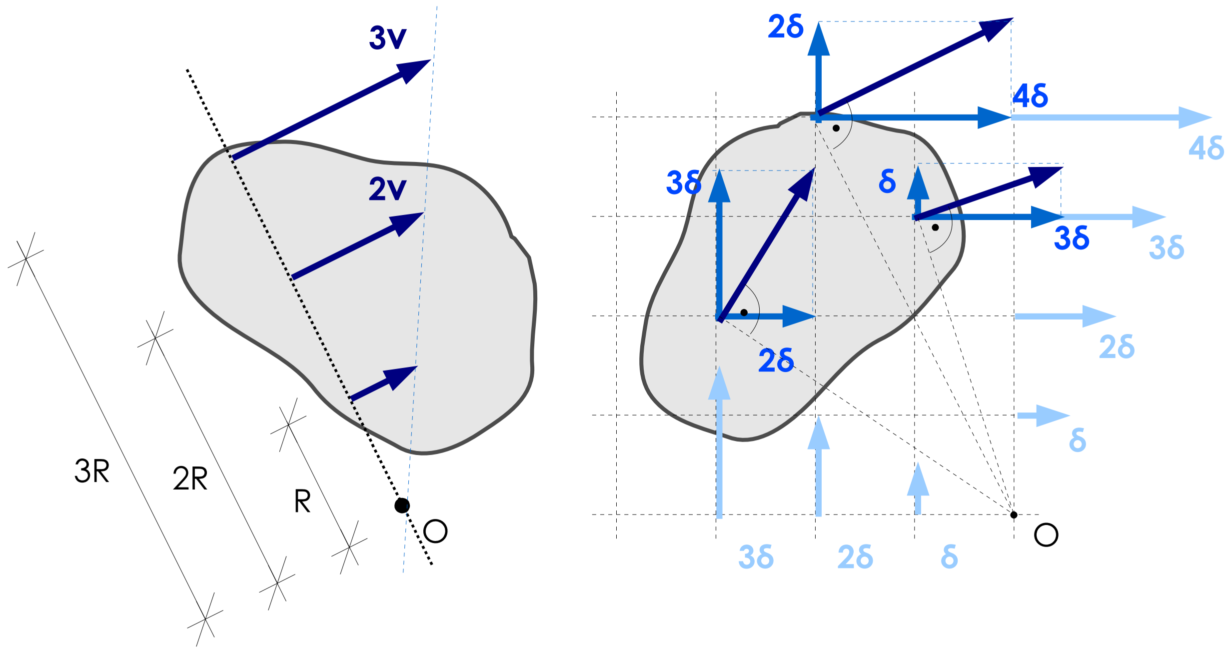


FINDING THE DISTRIBUTION OF VELOCITIES

- **Velocity vector** in a given point of a rigid body **is always perpendicular to the line connecting that point with instant center of rotation** of that body.

- **Velocities of points** lying on a single line connecting them with instant center of rotation **are proportional to the distance form the instant center of rotation.**

- According to the theorem on equality of projections of velocities on a line connecting two points **within a single body horizontal projections may be shifted horizontally, vertical projections may be shifted vertically**.

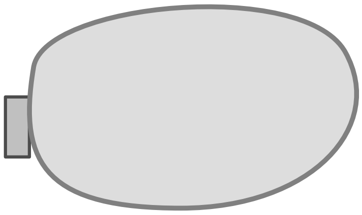


IMMOVABLE BODIES

Simple kinematic analysis of few basic cases is discussed below. In the future we won't perform an analysis of such a body – we will state immediately that it is immovable.

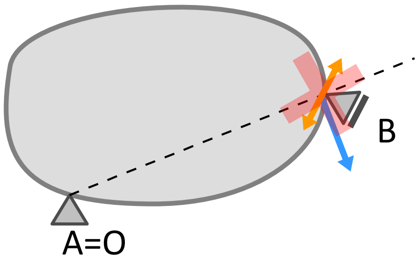
FIXED BODY

1. Fixed support reduces the number of DOF with 3 – the body is thus immovable.

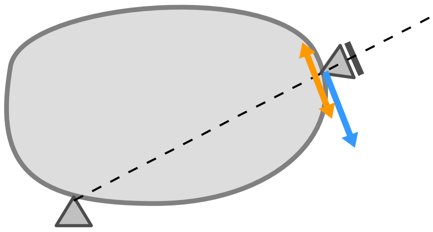


PIN AND ROLLER SUPPORT

1. Point A is immovable, so – unless the body does not move at all – it is the only possible location of instant center of rotation O.
2. Velocity in B must be perpendicular to the line OB.
3. Such direction of velocity is not admissible due to roller support in B. Such a motion is not possible.
4. It is the only possible motion due to pin support in A. The body is thus immovable.

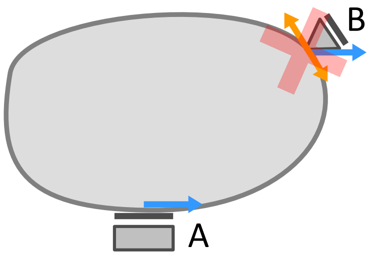


CONCLUSION: The body could perform motion if only admissible velocity direction in a point supported by a roller support was perpendicular to the line connecting the supported point with instant center of rotation.

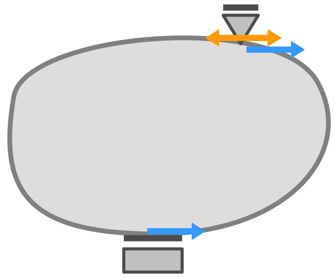


FIXED SUPPORT WITH ADMISSIBLE DISPLACEMENT AND ROLLER SUPPORT

1. Support at A allows only for translation so all point in the body must have the same velocity – its direction is the one admissible by the support.
2. Such direction of velocity is not admissible due to roller support in B. Such a motion is not possible.
3. It is the only possible motion due to pin support in A. The body is thus immovable.

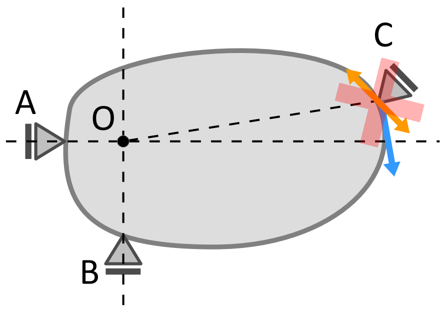


CONCLUSION: The body could perform motion if only admissible velocity direction in a point supported by a roller support was the same as the one allowed at the fixed support.

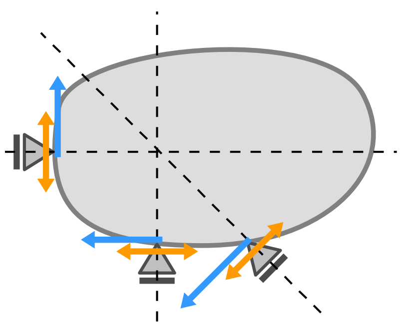


THREE ROLLER SUPPORTS

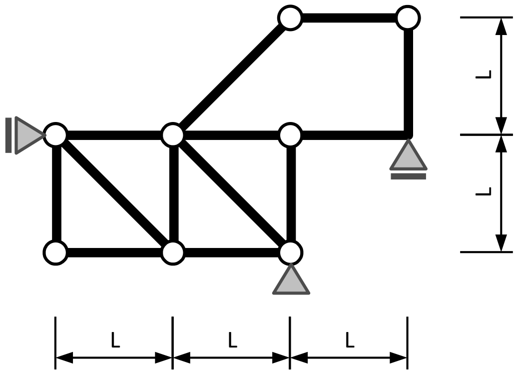
1. Possible location of instant center of rotation is found at intersection of lines which are perpendicular to admissible directions of velocities at roller supports at A and B.
2. Velocity in C must be perpendicular to the line OC.
3. Such direction of velocity is not admissible due to roller support in C. Such a motion is not possible.
4. In the same way rotations about two other possible locations of instant center of rotation are proved to be inconsistent with the third support. Since these are the only possible motions allowed by those supports, so the body is immovable.



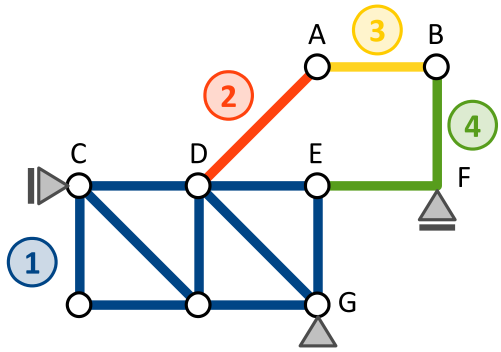
CONCLUSION: The body could perform motion if only admissible velocity directions at roller supports intersected in a single point.



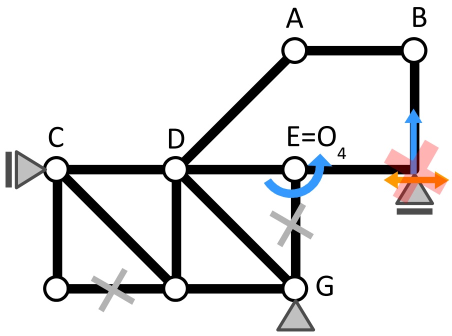
EXAMPLE 1.



1. The system consists of 4 rigid bodies

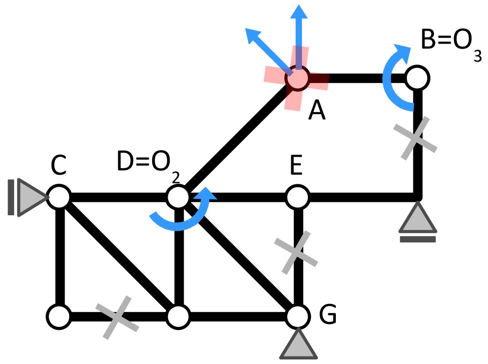


2. Body (1) is immovable.
3. Point E is immovable – it is a possible location of instant center of rotation for body (4).
4. Direction of velocity in F due to rotation about E is inconsistent with admissible velocity direction in F due to presence of support. Such a motion is impossible, so body (4) is immovable.



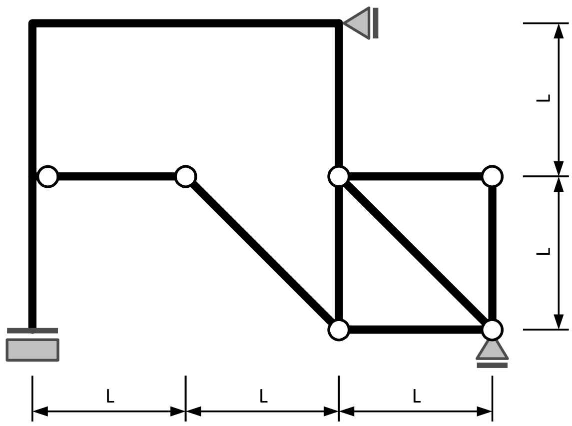
5. Point D is immovable – it is a possible location of instant center of rotation for body (2).
6. Point B is immovable – it is a possible location of instant center of rotation for body (3).

7. Direction of velocity in A due to rotation of (2) about D is inconsistent with velocity in A due to rotation of (3) about B. Such a motion is impossible, so bodies (2) and (3) are immovable.



8. The whole system is immovable.

EXAMPLE 2.



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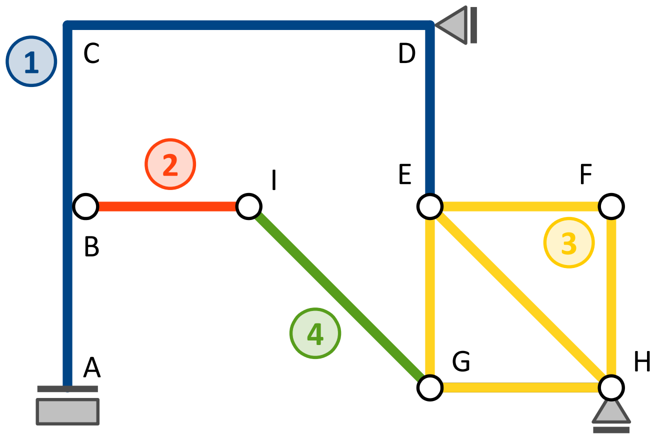
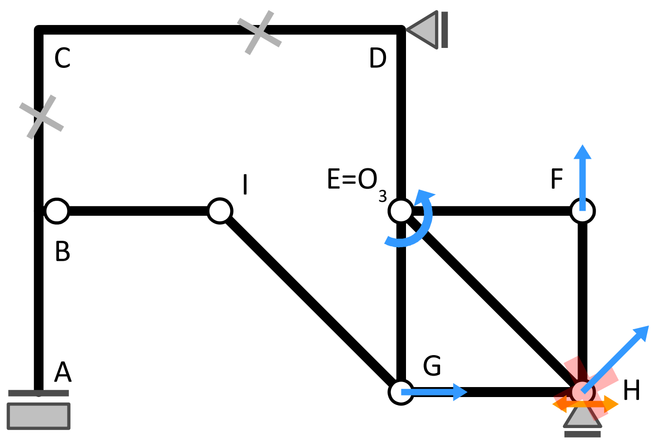
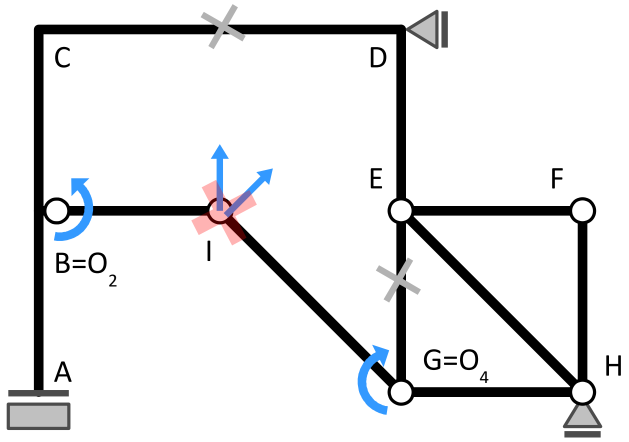


2. Body (1) is immovable.
3. Point E is immovable – it is a possible location of instant center of rotation for body (3).
4. Direction of velocity in H due to rotation about E is inconsistent with admissible velocity direction in H due to presence of support. Such a motion is impossible, so body (3) is immovable.

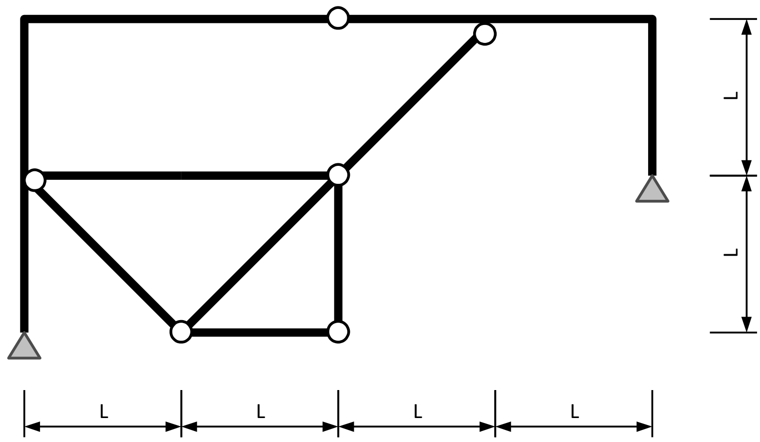


5. Point B is immovable – it is a possible location of instant center of rotation for body (2).
6. Point G is immovable – it is a possible location of instant center of rotation for body (4).
7. Direction of velocity in I due to rotation of (2) about B is inconsistent with velocity in I due to rotation of (4) about G. Such a motion is impossible, so bodies (2) and (4) are immovable.

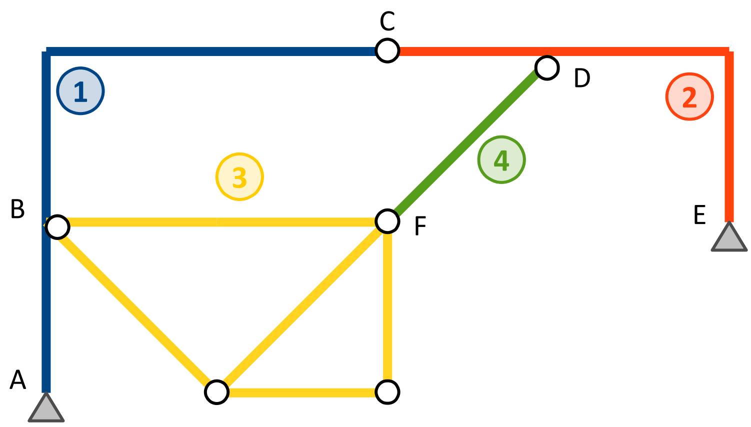


8. The whole system is immovable.

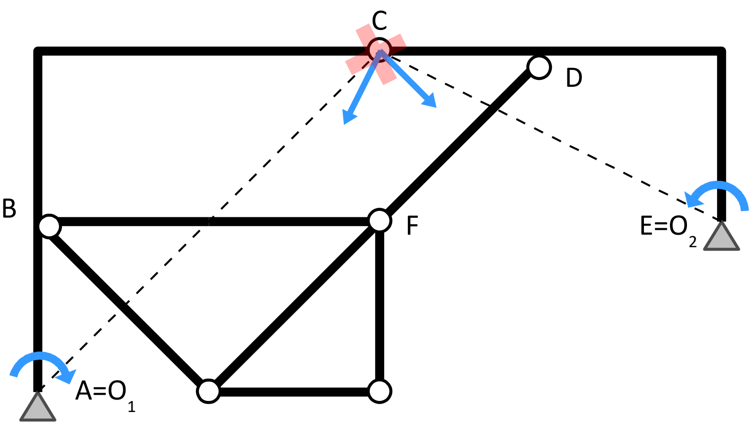
EXAMPLE 3.



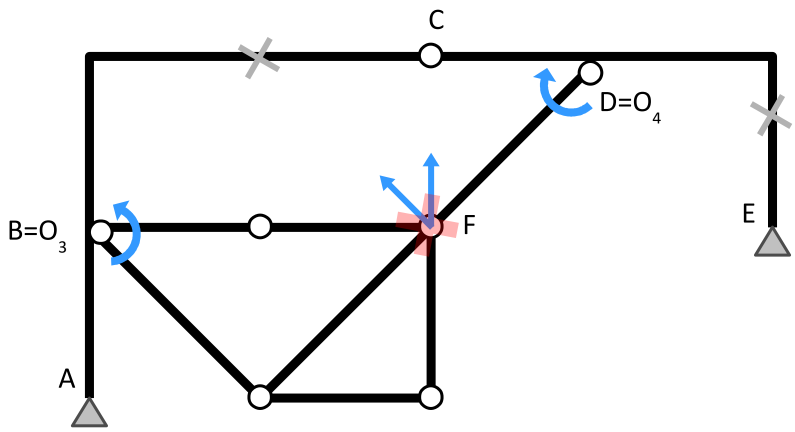
1. The system consists of 4 rigid bodies



2. Body (1) may only rotate about A.
3. Body (2) may only rotate about E.
4. Direction of velocity in C due to rotation of (1) about A is inconsistent with velocity in C due to rotation of (2) about E. Such a motion is impossible, so bodies (1) and (2) are immovable.

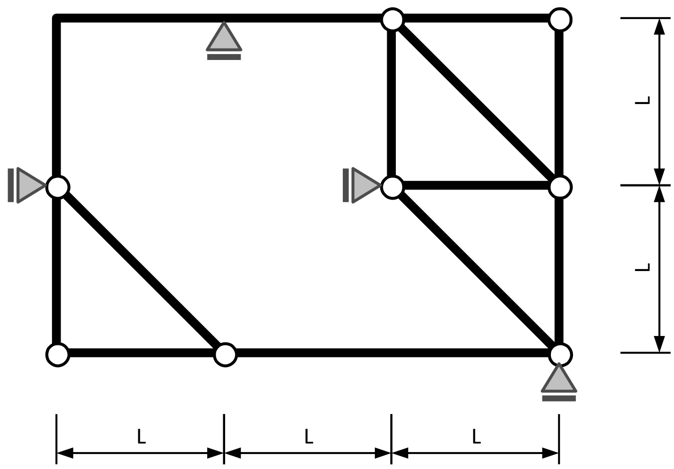


5. Point B is immovable – it is a possible location of instant center of rotation for body (3).
6. Point D is immovable – it is a possible location of instant center of rotation for body (4).
7. Direction of velocity in F due to rotation of (3) about B is inconsistent with velocity in F due to rotation of (4) about D. Such a motion is impossible, so bodies (3) and (4) are immovable.

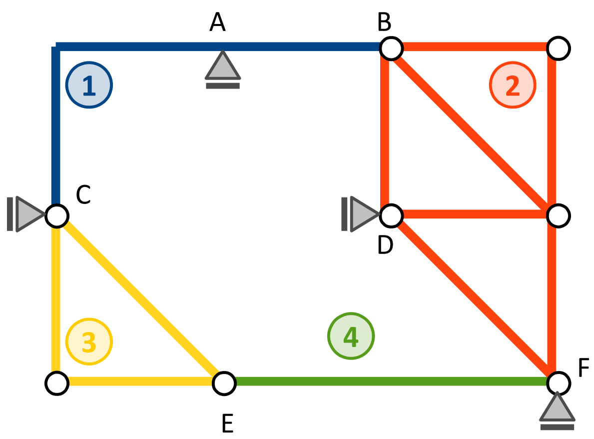


8. The whole system is immovable.

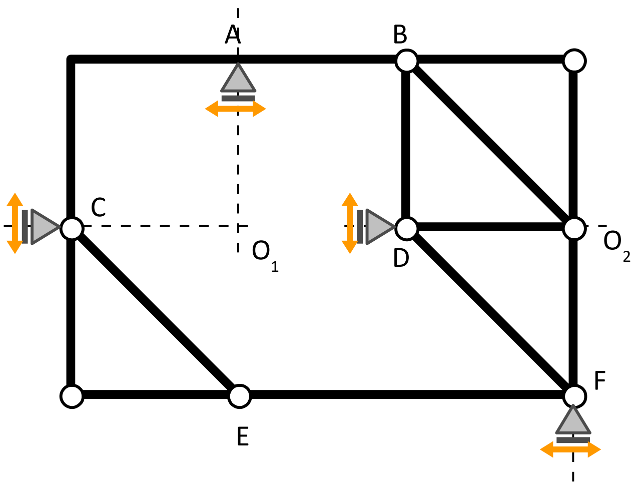
EXAMPLE 4.



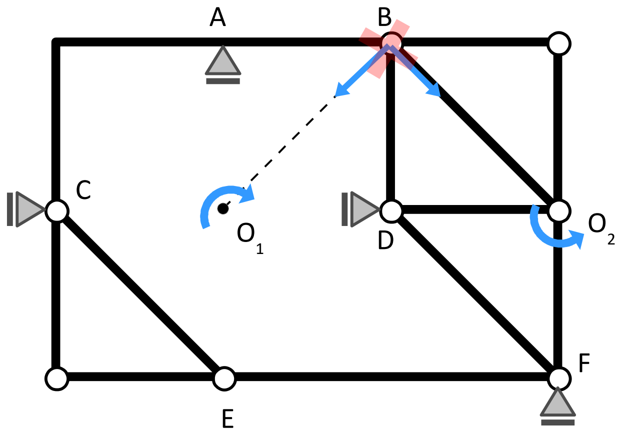
1. The system consists of 4 rigid bodies



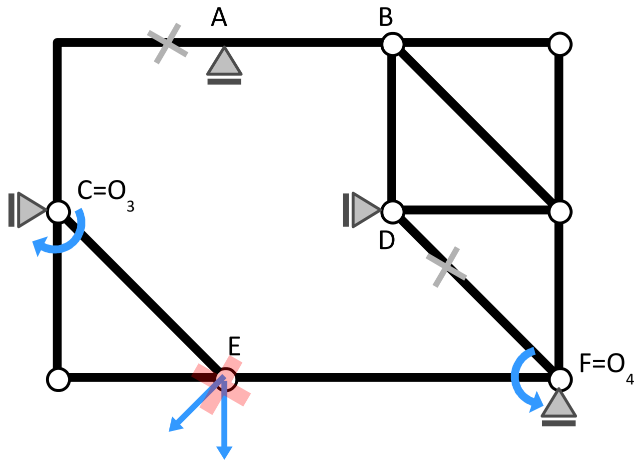
2. Instant center of rotation of body (1) is at intersection of lines which are perpendicular to the admissible velocity directions at supported point.
3. Instant center of rotation of body (2) is at intersection of lines which are perpendicular to the admissible velocity directions at supported point.



4. Direction of velocity in B due to rotation of (1) is inconsistent with velocity in B due to rotation of (2). Such a motion is impossible, so bodies (1) and (2) are immovable.

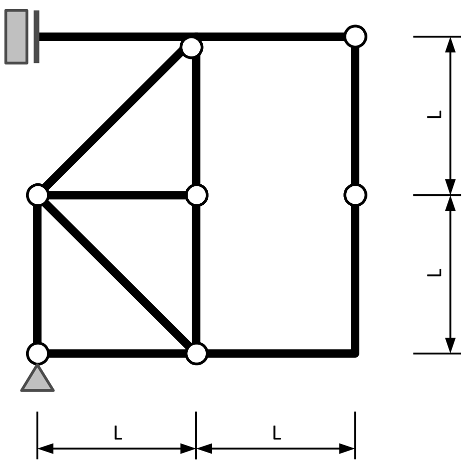


5. Point C is immovable – it is a possible location of instant center of rotation for body (3).
6. Point F is immovable – it is a possible location of instant center of rotation for body (4).
7. Direction of velocity in E due to rotation of (3) about C is inconsistent with velocity in E due to rotation of (4) about F. Such a motion is impossible, so bodies (3) and (4) are immovable.

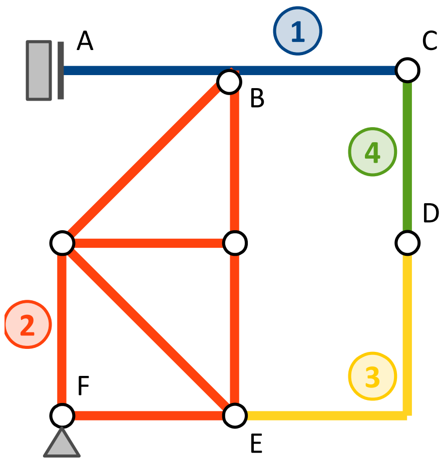


8. The whole system is immovable

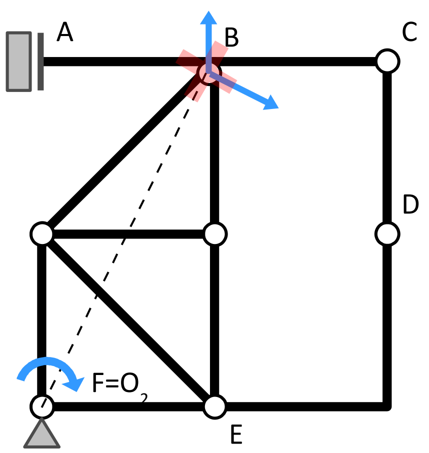
EXAMPLE 5.



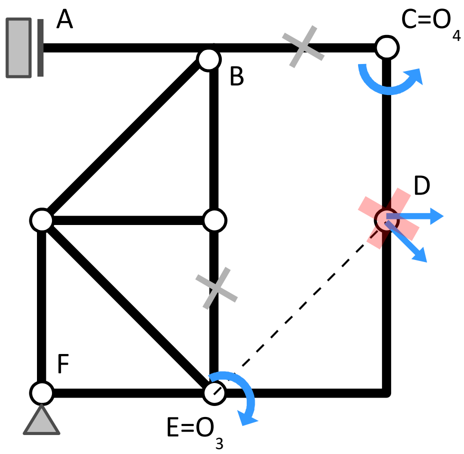
1. The system consists of 4 rigid bodies.



2. Body (1) may only perform vertical translation.
3. Body (2) may only rotate about F.
4. Direction of velocity in B due to translation of (1) is inconsistent with velocity in B due to rotation of (2) about F. Such a motion is impossible, so bodies (1) and (2) are immovable.

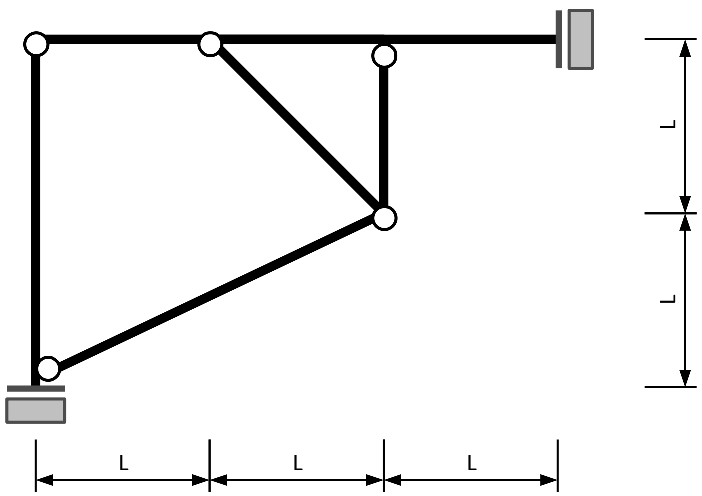


5. Point C is immovable – it is a possible location of instant center of rotation for body (4).
6. Point E is immovable – it is a possible location of instant center of rotation for body (3).
7. Direction of velocity in D due to rotation of (3) about E is inconsistent with velocity in D due to rotation of (4) about C. Such a motion is impossible, so bodies (3) and (4) are immovable.

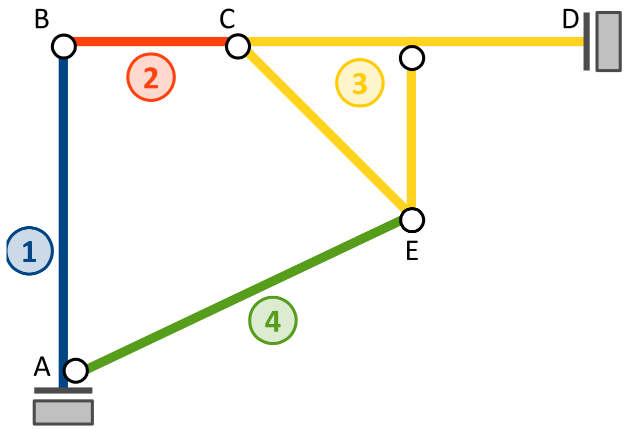


8. The whole system is immovable.

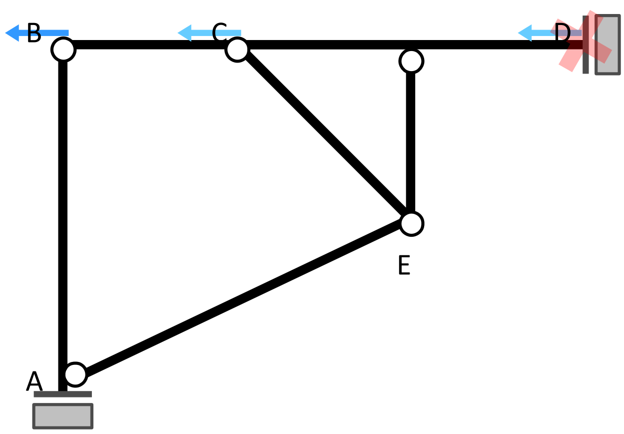
EXAMPLE 6.



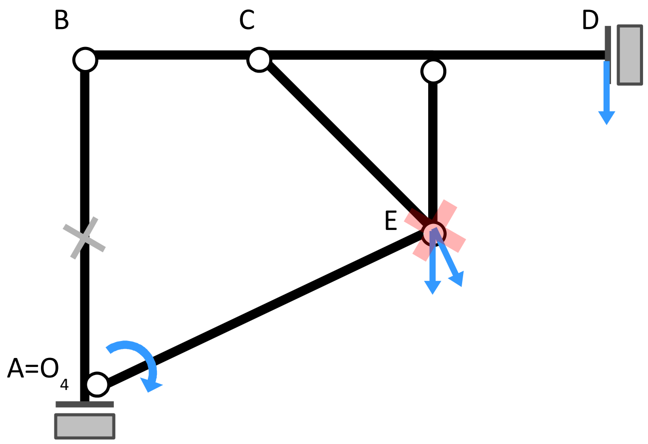
1. The system consists of 4 rigid bodies.



2. Body (1) may only perform horizontal translation.
3. Horizontal projection of velocity in B must be shifted to C and D. Horizontal translation of (1) requires horizontal component of velocity at D what is not allowed by the presence of suport in D. Such a motion is not possible. Body (1) is immovable.

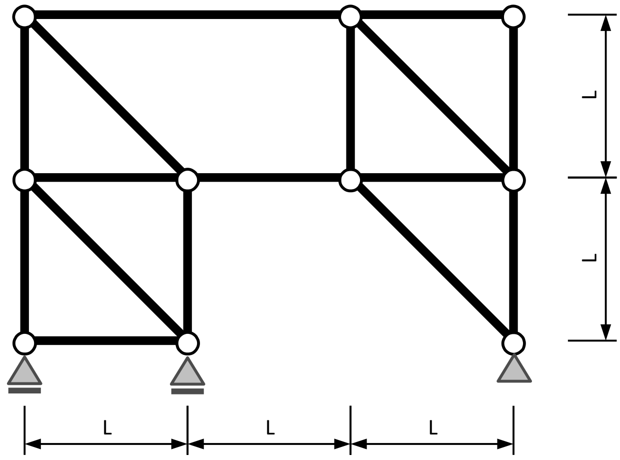


4. Point A is immovable – it is a possible location of instant center of rotation for body (4).
5. Body (3) may only perform vertical translation.
6. Direction of velocity in E due to rotation of (4) about A is inconsistent with velocity in E due to translation of (3). Such a motion is impossible, so bodies (3) and (4) are immovable.

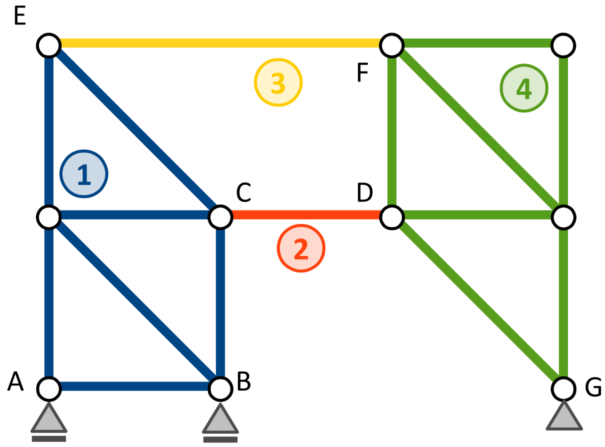


7. Points B and C are immovable. Body (2) is pinned in two point so it is immovable.
8. The whole system is immovable.

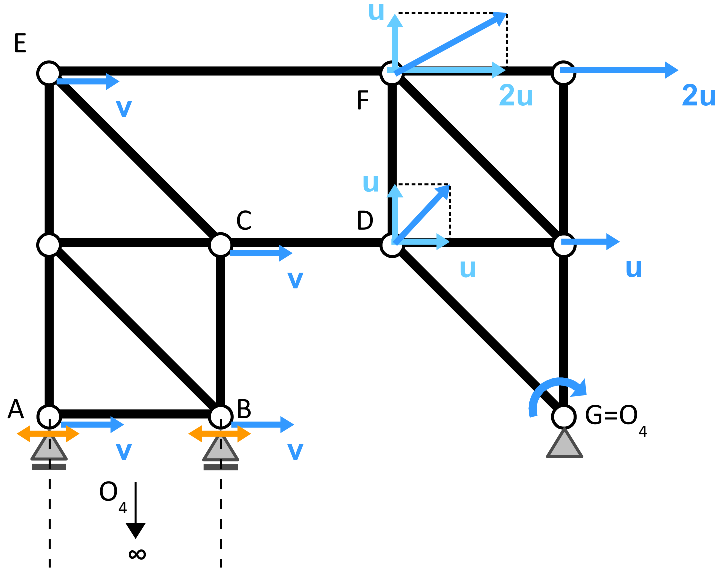
EXAMPLE 7.



1. The system consists of 4 rigid bodies.



2. Body (1) may only perform horizontal translation.
3. Body (4) may only rotate about G.

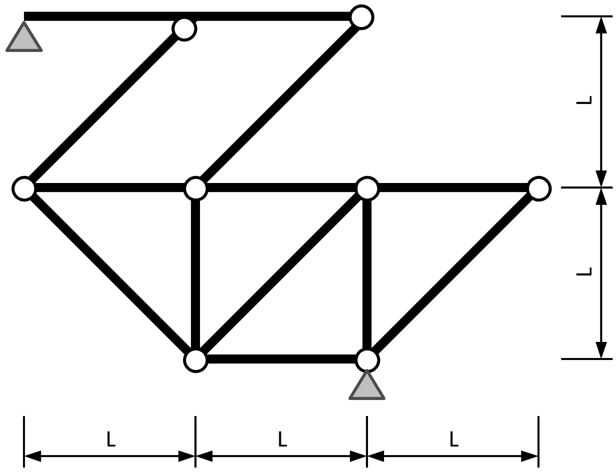


4. Projection of velocities in C and D on CD must be the same.
5. Projection of velocities in E and E on EF must be the same.
6. We obtain a system of equations:

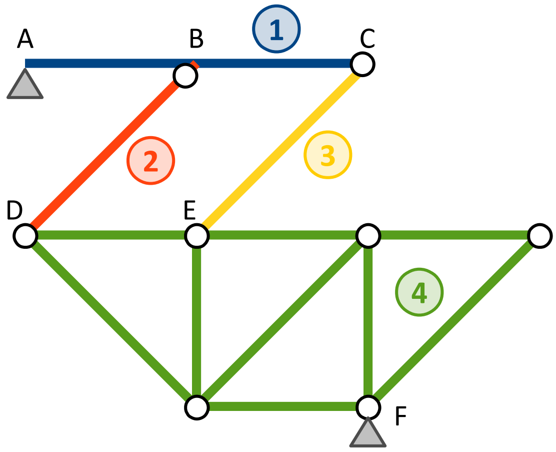
$$\begin{cases} v = 2u \\ v = u \end{cases} \quad \Rightarrow \quad \begin{cases} v = 0 \\ u = 0 \end{cases}$$

7. Bodies (1) and (4) are immovable.
8. Bodies (2) and (3) are both pinned in two point so they are immovable.
9. The whole system is immovable.

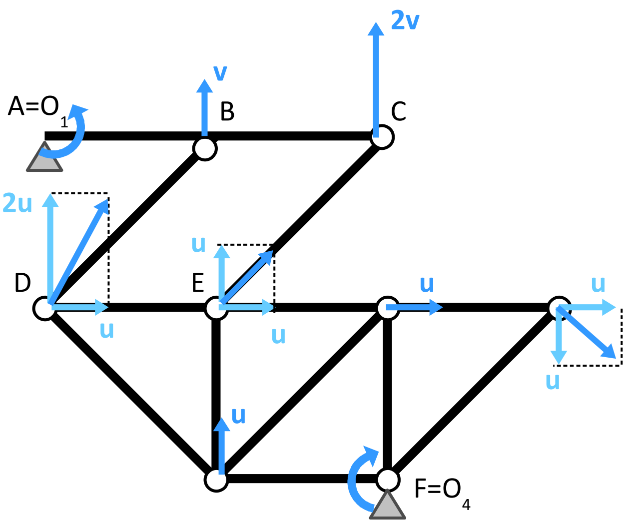
EXAMPLE 8.



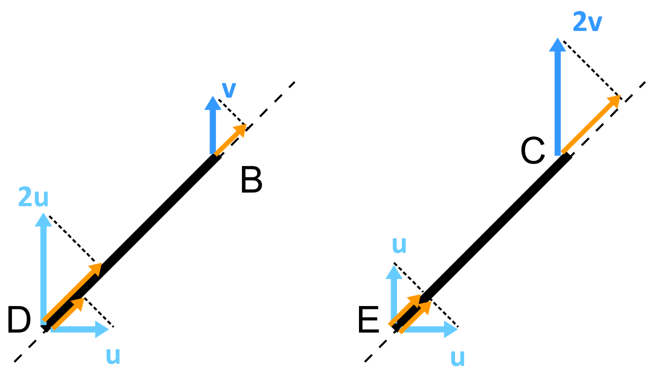
1. The system consists of 4 rigid bodies.



2. Body (1) may only rotate about A.
3. Body (4) may only rotate about F.



4. Projection of velocities in B and D on BD must be the same.
5. Projection of velocities in C and E on CE must be the same.

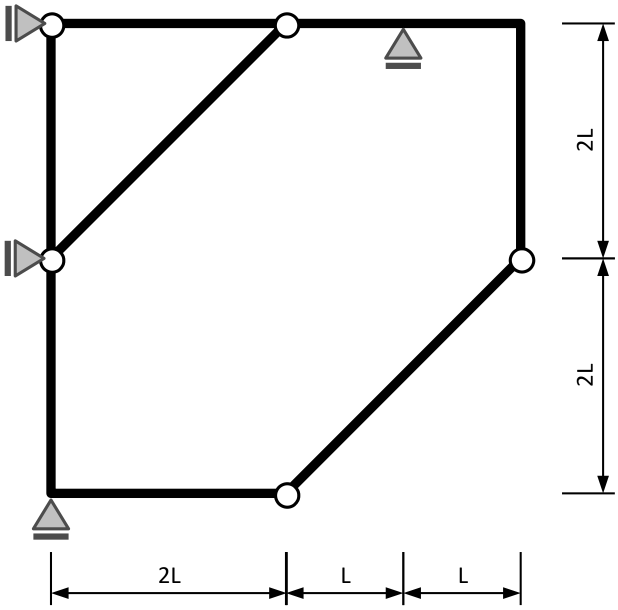


6. We obtain a system of equations:

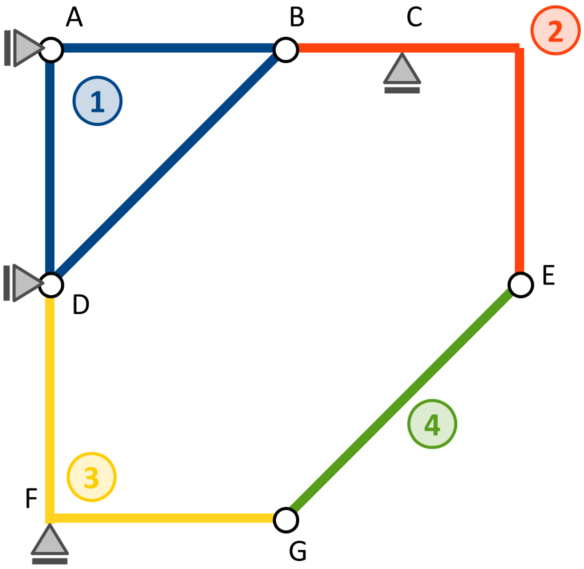
$$\begin{cases} \dfrac{v}{\sqrt{2}} = \dfrac{2u}{\sqrt{2}} + \dfrac{u}{\sqrt{2}} \\ \dfrac{2v}{\sqrt{2}} = \dfrac{u}{\sqrt{2}} + \dfrac{u}{\sqrt{2}} \end{cases} \Rightarrow \begin{cases} v = 0 \\ u = 0 \end{cases}$$

7. Bodies (1) and (4) are immovable.
8. Bodies (2) and (3) are both pinned in two point so they are immovable.
9. The whole system is immovable.

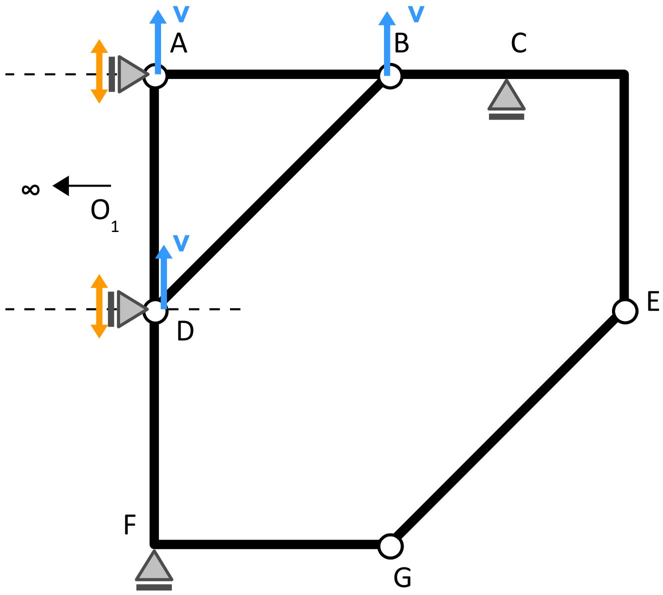
EXAMPLE 9.



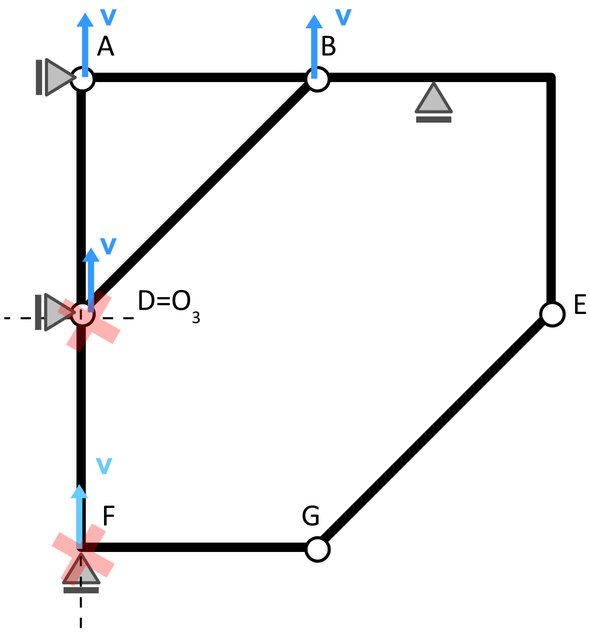
1. The system consists of 4 rigid bodies.



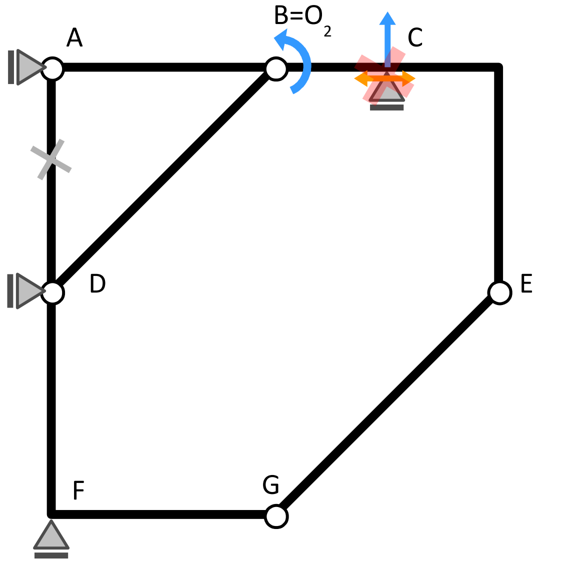
2. Body (1) may only perform vertical translation.



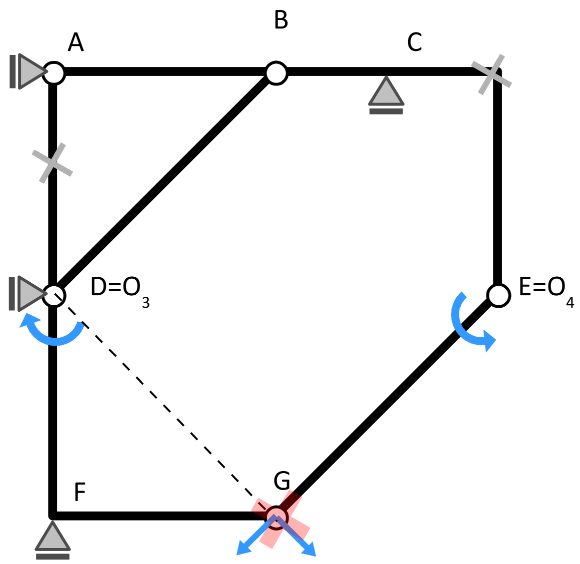
3. Instant center of rotation of (3) is at intersection of lines perpendicular to admissible velocity direction at supported point and to direction of velocity in hinge.
4. In the possible location of instant center of rotation for (3) there is a non-zero velocity, so such a motion is impossible. What's more, vertical component must be shifted along a vertical line what requires vertical velocity at F and this is not allowed by the presence of a support. Motion of body (1) is impossible. Body (1) is immovable.



5. Point B is immovable – it is a possible location of instant center of rotation for body (2).
6. Direction of velocity in C due to rotation about B is inconsistent with admissible velocity direction in C due to presence of support. Such a motion is impossible, so body (2) is immovable.



7. Point D is immovable – it is a possible location of instant center of rotation for body (3).
8. Point E is immovable – it is a possible location of instant center of rotation for body (4).
9. Direction of velocity in G due to rotation of (3) about D is inconsistent with velocity in G due to rotation of (4) about E. Such a motion is impossible, so bodies (3) and (4) are immovable.



10. The whole system is immovable.

