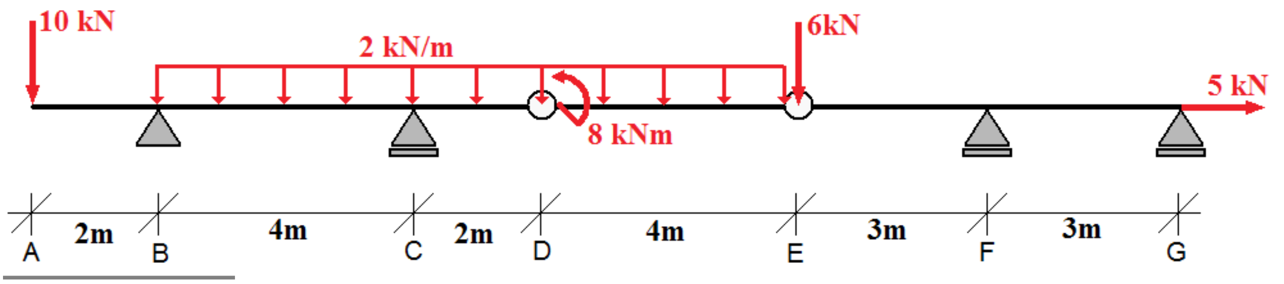
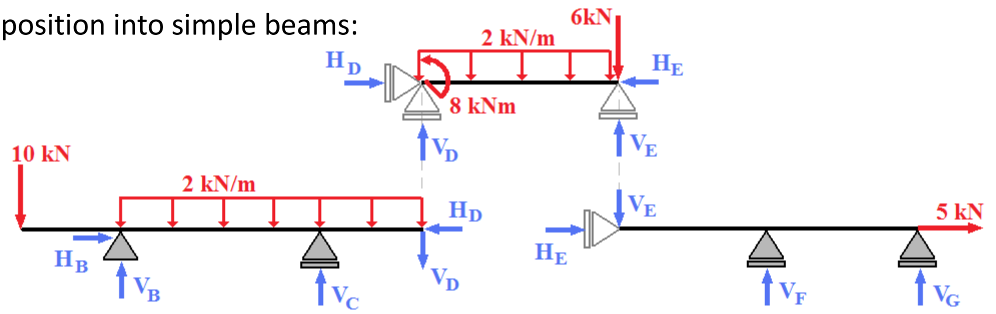
EXCERCISE 4.14

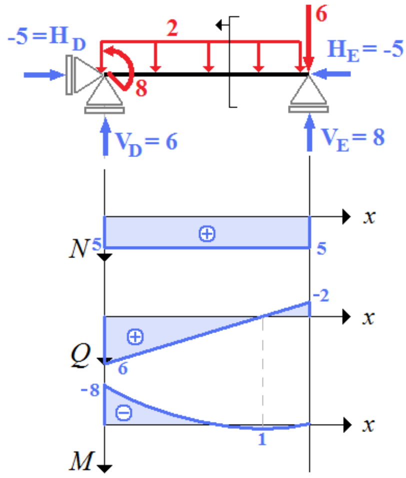
Determine cross-sectional forces in a multispan beam shown below. Decompose the beam into simple beams:



Decomposition into simple beams:



- **Point moment may not be applied to a joint but only to a certain beam.**
- **Point forces applied to joint must be subordinated to a single beam** (it may be any of beams connected in the joint).
- Horizontal load is beared by the only pinned support in point B – we may instantly write down horizontal reactions:: $H_B = H_D = H_E = -5$



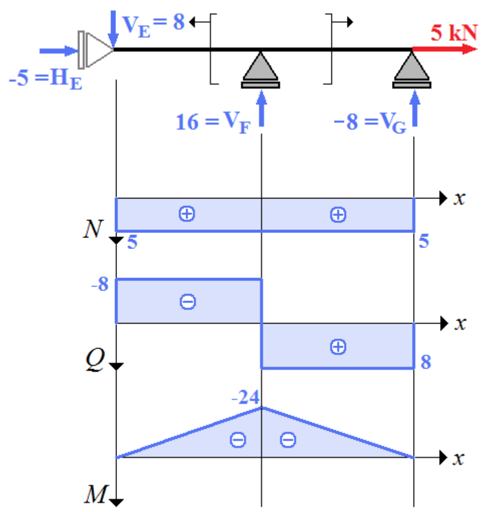
Beam DE $x \in (0,4)$

Reactions:

$$\Sigma X = 0 \quad \Rightarrow \quad H_D - (-5) = 0 \quad \Rightarrow \quad H_D = -5$$
$$\Sigma M_D = 0 \quad \Rightarrow \quad -2 \cdot 4 \cdot 2 - 6 \cdot 4 + 8 + V_E \cdot 4 = 0 \quad \Rightarrow \quad V_E = 8$$
$$\Sigma Y = 0 \quad \Rightarrow \quad V_D + V_E - 6 - 2 \cdot 4 = 0 \quad \Rightarrow \quad V_D = 6$$

Cross-sectional forces:

$$DE,\ x \in (0,4)$$
$$\begin{cases} N(x) = 5 \\ Q(x) = 6 - 2 \cdot x \\ M(x) = 6 \cdot x - 2 \cdot x \cdot \dfrac{x}{2} - 8 \end{cases} \qquad \frac{dM}{dx} = Q(x) = 0 \quad \Rightarrow \quad x = 3$$
$$M_{max} = M(3) = 1$$



Beam EG $x \in (0,6)$

Reactions:

$$\Sigma X = 0 \quad \Rightarrow \quad H_E + 5 = 0 \quad \Rightarrow \quad H_E = -5$$
$$\Sigma M_F = 0 \quad \Rightarrow \quad 8 \cdot 3 + V_G \cdot 3 = 0 \quad \Rightarrow \quad V_G = -8$$
$$\Sigma Y = 0 \quad \Rightarrow \quad V_F + V_G - 8 = 0 \quad \Rightarrow \quad V_D = 16$$

Cross-sectional forces:

$$EF,\ x \in (0,3) \qquad \begin{cases} N(x) = 5 \\ Q(x) = -8 \\ M(x) = -8 \cdot x \end{cases}$$

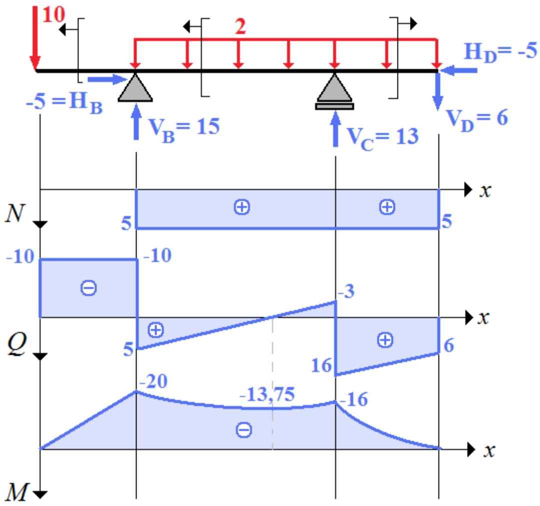
$$FG,\ x \in (3,6) \qquad \begin{cases} N(x) = 5 \\ Q(x) = 8 \\ M(x) = -8 \cdot (6 - x) \end{cases}$$

REMARK: *For every simple beam we take a separate local coordinate system (x,z). Variable x appearing in functions N, Q, M is **different** for **different beams**.*

Beam AD $x\in(0,8)$

Reactions:

$$\Sigma X=0 \Rightarrow H_B-(-5)=0 \Rightarrow H_B=-5$$
$$\Sigma M_B=0 \Rightarrow 10\cdot 2-2\cdot 6\cdot 3+V_C\cdot 4-6\cdot 6=0 \Rightarrow V_C=13$$
$$\Sigma Y=0 \Rightarrow V_B+V_C-10-6-2\cdot 6=0 \Rightarrow V_B=15$$



Cross-sectional forces:

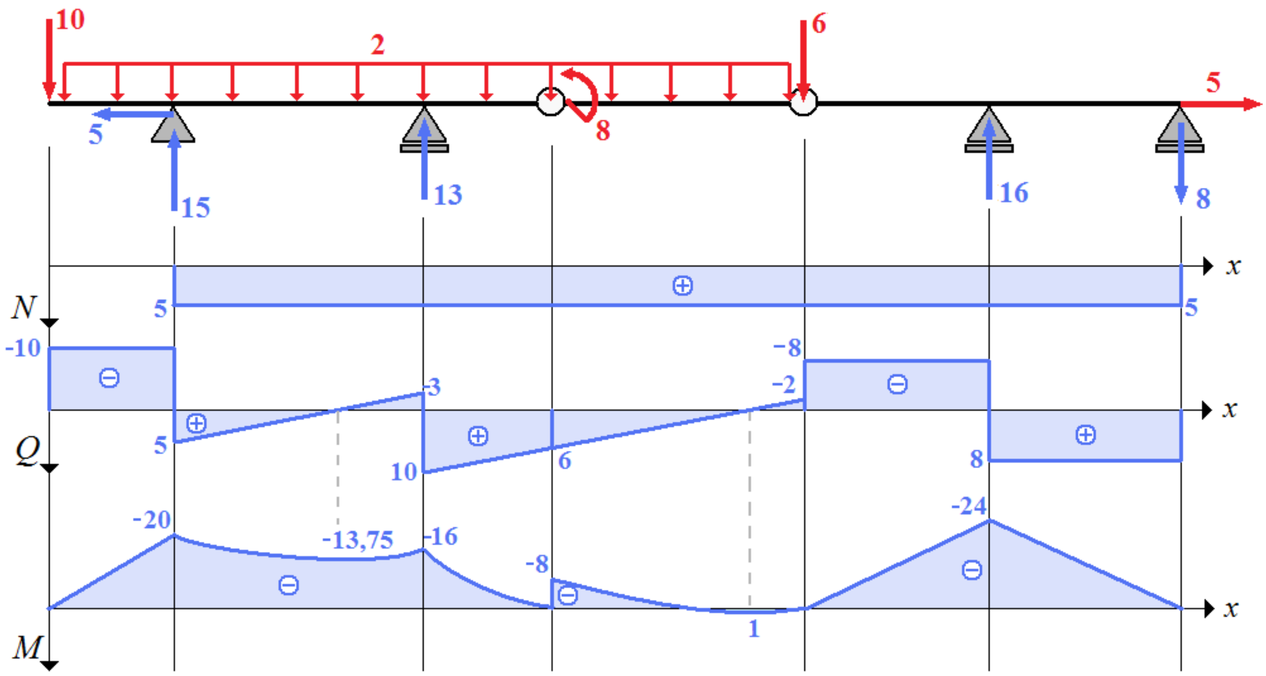
$AB,\ x\in(0,2)$
$$\begin{cases} N(x)=0 \\ Q(x)=-10 \\ M(x)=-10\cdot x \end{cases}$$

$BC,\ x\in(2,6)$
$$\begin{cases} N(x)=5 \\ Q(x)=-10+15-2\cdot(x-2) \\ M(x)=-10\cdot x+15\cdot(x-2)-2\cdot(x-2)\cdot\dfrac{(x-2)}{2} \end{cases}$$

$CD,\ x\in(6,8)$
$$\begin{cases} N(x)=5 \\ Q(x)=6+2\cdot(8-x) \\ M(x)=-6\cdot(8-x)-2\cdot(8-x)\cdot\dfrac{(8-x)}{2} \end{cases}$$

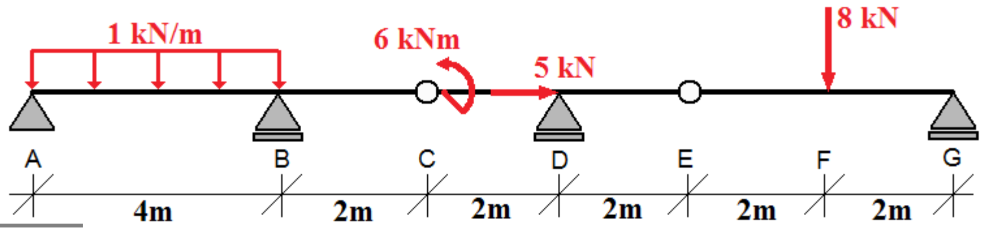
$$\frac{dM}{dx}=Q(x)=0 \Rightarrow x=4,5$$
$$M_{max}=M(4,5)=13,75$$

Reactions and cross-sectional forces:

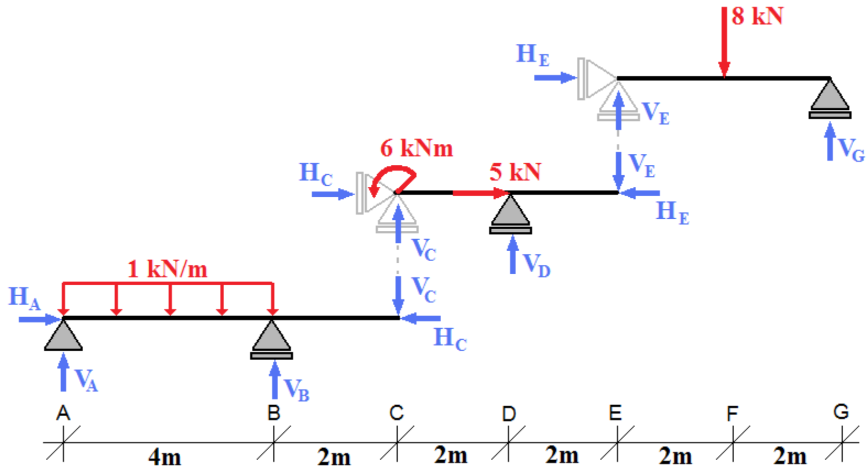


EXCERCISE 4.15

Determine cross-sectional forces in a multispan beam shown below. Decompose the beam into simple beams::



Decomposition into simple beams:



Beam EG $x \in (0,4)$

Reactions:

$$\Sigma X=0 \;\Rightarrow\; H_E=0$$
$$\Sigma M_E=0 \;\Rightarrow\; -8\cdot 2+V_G\cdot 4=0 \;\Rightarrow\; V_G=4$$
$$\Sigma Y=0 \;\Rightarrow\; V_E+V_G-8=0 \;\Rightarrow\; V_E=4$$

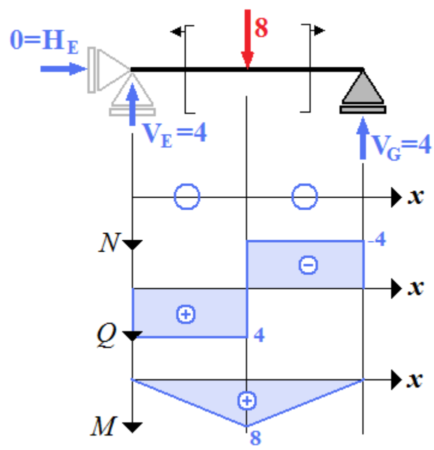
Cross-sectional forces:

$EF,\ x\in(0,2)$

$$\begin{cases} N(x)=0 \\ Q(x)=4 \\ M(x)=4\cdot x \end{cases}$$

$FG,\ x\in(3,6)$

$$\begin{cases} N(x)=0 \\ Q(x)=-4 \\ M(x)=4\cdot(4-x) \end{cases}$$



Beam CE $x \in (0,4)$

Reactions:

$$\Sigma X=0 \;\Rightarrow\; H_C+5=0 \;\Rightarrow\; H_C=-5$$
$$\Sigma M_D=0 \;\Rightarrow\; -4\cdot 2-V_C\cdot 2+6=0 \;\Rightarrow\; V_C=-1$$
$$\Sigma Y=0 \;\Rightarrow\; V_C+V_D-4=0 \;\Rightarrow\; V_D=5$$

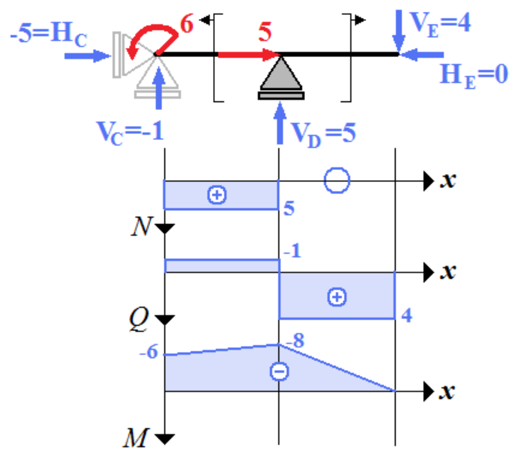
Cross-sectional forces:

$CD,\ x\in(0,2)$

$$\begin{cases} N(x)=5 \\ Q(x)=-1 \\ M(x)=-6-1\cdot x \end{cases}$$

$DE,\ x\in(2,4)$

$$\begin{cases} N(x)=0 \\ Q(x)=4 \\ M(x)=-4\cdot(4-x) \end{cases}$$



<u>Beam AC</u> $x \in (0,6)$

Reactions:

$$\Sigma X = 0 \quad \Rightarrow \quad H_A - (-5) = 0 \quad \Rightarrow \quad H_A = -5$$
$$\Sigma M_A = 0 \quad \Rightarrow \quad -1 \cdot 4 \cdot 2 + 4 \cdot V_B - 6 \cdot (-1) = 0 \quad \Rightarrow \quad V_B = 0{,}5$$
$$\Sigma Y = 0 \quad \Rightarrow \quad V_A + V_B - (-1) - 1 \cdot 4 = 0 \quad \Rightarrow \quad V_A = 2{,}5$$

Cross-sectional forces:

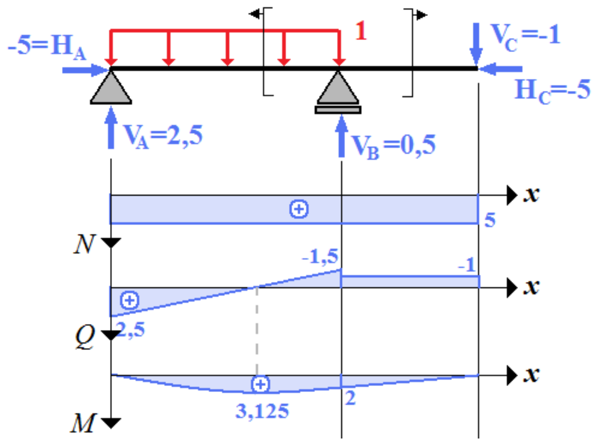
$AB,\ x \in (0,4)$

$$\begin{cases} N(x) = 5 \\ Q(x) = 2{,}5 - 1 \cdot x \\ M(x) = 2{,}5 \cdot x - 1 \cdot x \cdot \dfrac{x}{2} \end{cases}$$

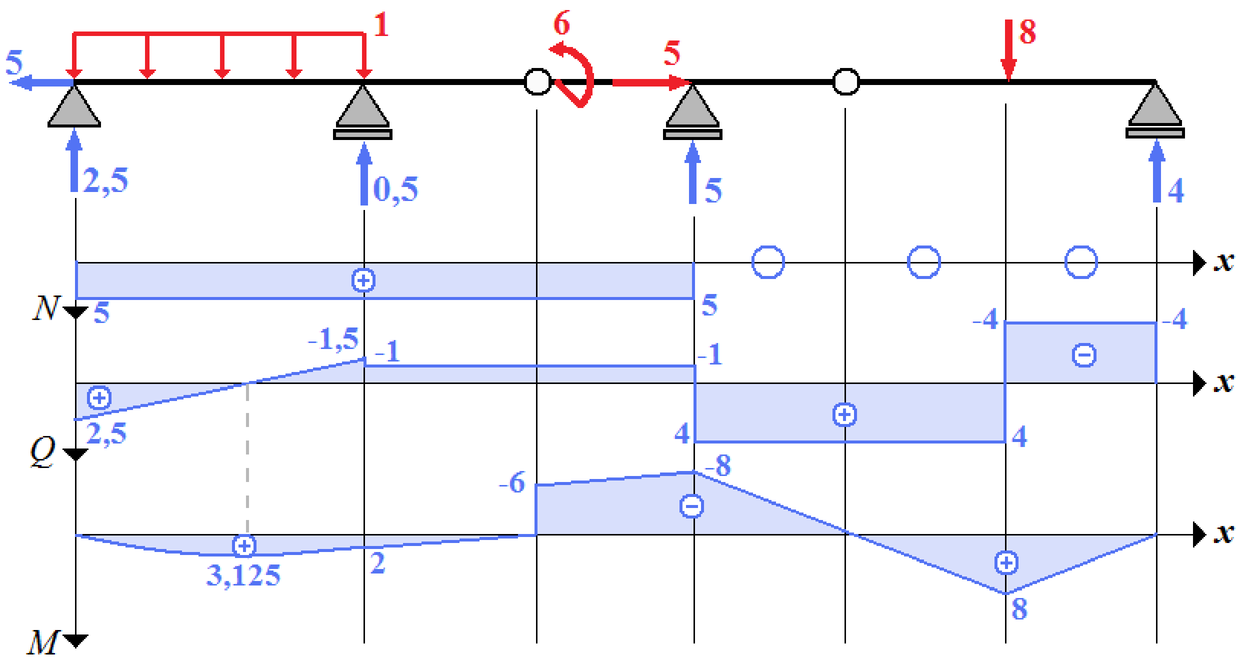
$$\frac{dM}{dx} = Q(x) = 0 \quad \Rightarrow \quad x = 2{,}5$$
$$M_{max} = M(2{,}5) = 3{,}125$$

$BC,\ x \in (4,6)$

$$\begin{cases} N(x) = 5 \\ Q(x) = -1 \\ M(x) = -(-1) \cdot (6 - x) \end{cases}$$

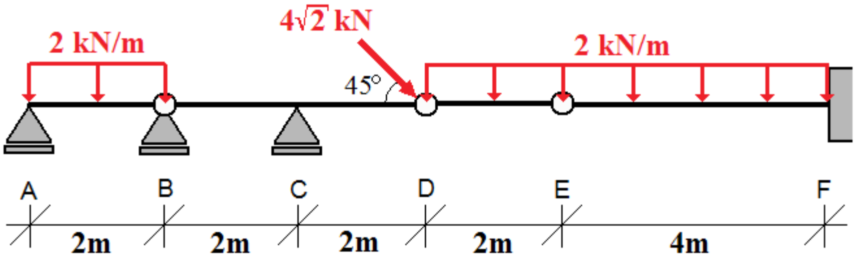


Reactions and cross-sectional forces:

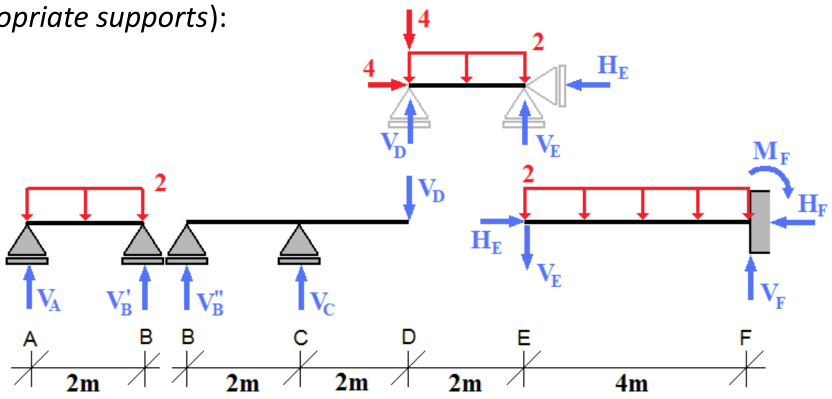


EXCERCISE 4.16

Determine cross-sectional forces in a multispan beam shown below. Decompose the beam into simple beams:



Decomposition into simple beams: (**REMARK:** *Beam AB and beam BD lie on the same support, however, due to presence of joint, they are bent independently. They may be treated as two separate beams lying on different supports – reaction at support in point B will be a sum of reactions at appropriate supports*):



Beam AB $x \in (0,2)$

Reactions (no load along *X* direction)

$$\Sigma M_A = 0 \quad \Rightarrow \quad -2 \cdot 2 \cdot 1 - V_B^I \cdot 2 = 0 \quad \Rightarrow \quad V_B^I = 2$$

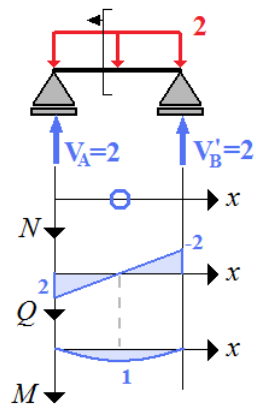
$$\Sigma Y = 0 \quad \Rightarrow \quad V_A + V_B^I - 2 \cdot 2 = 0 \quad \Rightarrow \quad V_A = 2$$

Cross-sectional forces:

$AB,\, x \in (0,2)$

$$\begin{cases} N(x) = 0 \\ Q(x) = 2 - 2 \cdot x \\ M(x) = 2 \cdot x - 2 \cdot x \cdot \dfrac{x}{2} \end{cases} \qquad \frac{dM}{dx} = Q(x) = 0 \quad \Rightarrow \quad x = 1$$

$$M_{max} = M(1) = 1$$



Beam DE $x \in (0,2)$

Reactions:

$$\Sigma X = 0 \quad \Rightarrow \quad 4 - H_E = 0 \quad \Rightarrow \quad H_C = 4$$

$$\Sigma M_D = 0 \quad \Rightarrow \quad -2 \cdot 2 \cdot 1 + V_E \cdot 2 = 0 \quad \Rightarrow \quad V_E = 2$$

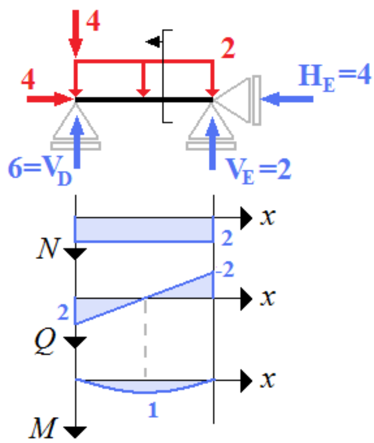
$$\Sigma Y = 0 \quad \Rightarrow \quad V_D + V_E - 4 - 2 \cdot 2 = 0 \quad \Rightarrow \quad V_D = 6$$

Cross-sectional forces:

$DE,\, x \in (0,2)$

$$\begin{cases} N(x) = -4 \\ Q(x) = 6 - 4 - 2 \cdot x \\ M(x) = 6 \cdot x - 4 \cdot x - 2 \cdot x \cdot \dfrac{x}{2} \end{cases} \qquad \frac{dM}{dx} = Q(x) = 0 \quad \Rightarrow \quad x = 1$$

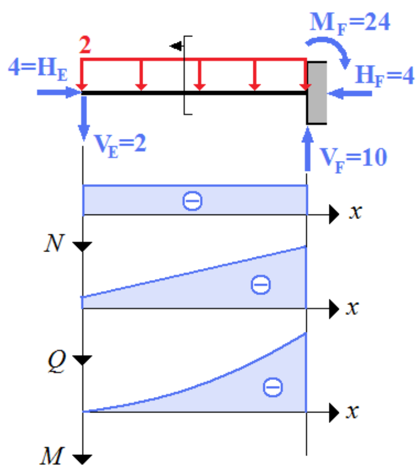
$$M_{max} = M(1) = 1$$



<u>Beam EF</u> $x\in(0,4)$

Reactions:

$$\Sigma X=0 \quad\Rightarrow\quad 4-H_F=0 \quad\Rightarrow\quad H_F=4$$
$$\Sigma Y=0 \quad\Rightarrow\quad -2-2\cdot4+V_F=0 \quad\Rightarrow\quad V_F=10$$
$$\Sigma M_F=0 \quad\Rightarrow\quad 2\cdot4+2\cdot4\cdot2-M_F=0 \quad\Rightarrow\quad M_F=24$$



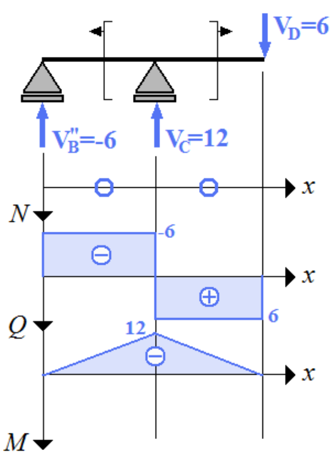
Cross-sectional forces:

$EF,\ x\in(0,4)$
$$\begin{cases} N(x)=-4 \\ Q(x)=-2-2\cdot x \\ M(x)=-2\cdot x-2\cdot x\cdot\dfrac{x}{2} \end{cases}$$

<u>Beam BD</u> $x\in(0,4)$

Reactions (no load along *X* direction):

$$\Sigma M_B=0 \quad\Rightarrow\quad -6\cdot4+V_C\cdot2=0 \quad\Rightarrow\quad V_C=12$$
$$\Sigma Y=0 \quad\Rightarrow\quad V_C+V_B^{II}-6=0 \quad\Rightarrow\quad V_B^{II}=-6$$

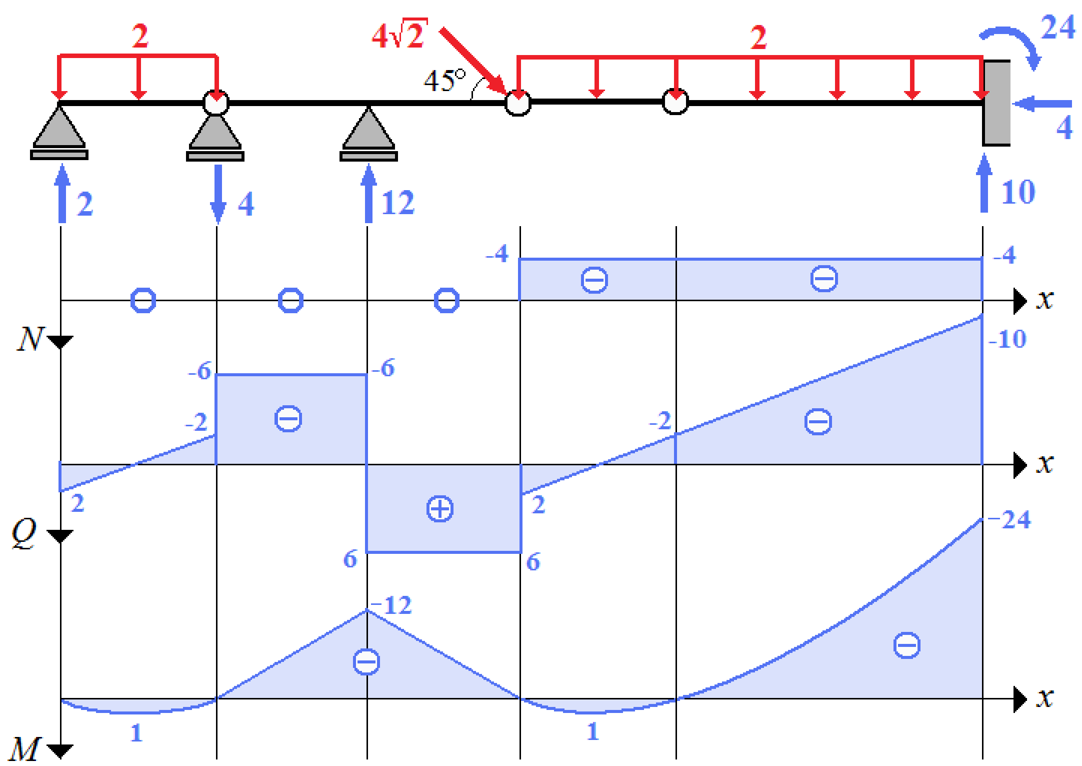


Cross-sectional forces:

$BC,\ x\in(0,2)$
$$\begin{cases} N(x)=0 \\ Q(x)=-6 \\ M(x)=-6\cdot x \end{cases}$$

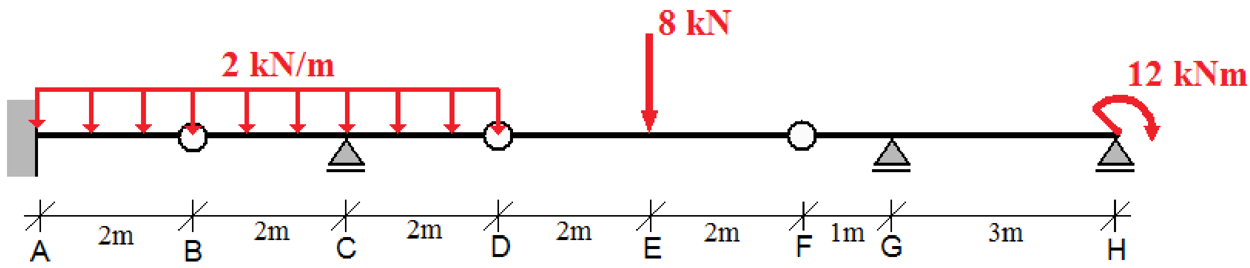
$cD,\ x\in(2,4)$
$$\begin{cases} N(x)=0 \\ Q(x)=6 \\ M(x)=-6\cdot(4-x) \end{cases}$$

Reactions and cross-sectional forces:

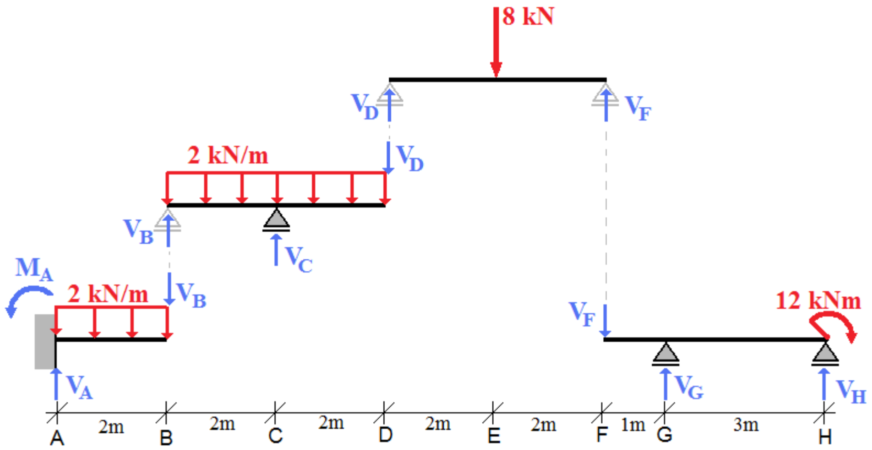


ZADANIE 4.17

Determine cross-sectional forces in a multispan beam shown below. Decompose the beam into simple beams:



There is no horizontal load in the system, so there are also no axial forces and no horizontal reactions. Decomposition into simple beams:



<u>Beam DF</u> $x\in(0,4)$

Reactions:

$$\Sigma M_D=0 \quad\Rightarrow\quad 4\cdot V_F-8\cdot 2=0 \quad\Rightarrow\quad V_F=4$$

$$\Sigma Y=0 \quad\Rightarrow\quad V_D+V_F-8=0 \quad\Rightarrow\quad V_D=4$$

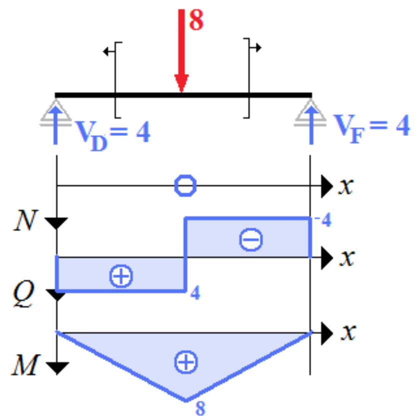
Cross-sectional forces:

$DE,\, x\in(0,2)$

$$\begin{cases} N(x)=0 \\ Q(x)=4 \\ M(x)=4\cdot x \end{cases}$$

$EF,\, x\in(2,4)$

$$\begin{cases} N(x)=0 \\ Q(x)=-4 \\ M(x)=4\cdot(4-x) \end{cases}$$



<u>Beam FH</u> $x\in(0,4)$

Reactions:

$$\Sigma M_G=0 \quad\Rightarrow\quad 4\cdot 1+V_H\cdot 3-12=0 \quad\Rightarrow\quad V_H=\frac{8}{3}$$

$$\Sigma Y=0 \quad\Rightarrow\quad -4+V_G+V_H=0 \quad\Rightarrow\quad V_G=\frac{4}{3}$$

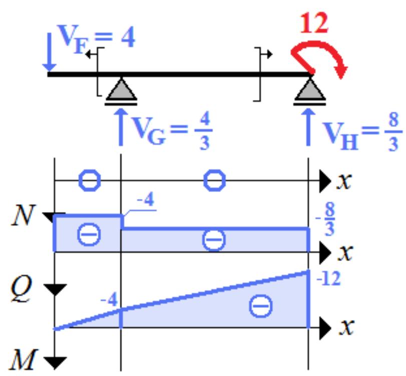
Cross-sectional forces:

$FG,\, x\in(0,1)$

$$\begin{cases} N(x)=0 \\ Q(x)=-4 \\ M(x)=-4\cdot x \end{cases}$$

$GH,\, x\in(1,4)$

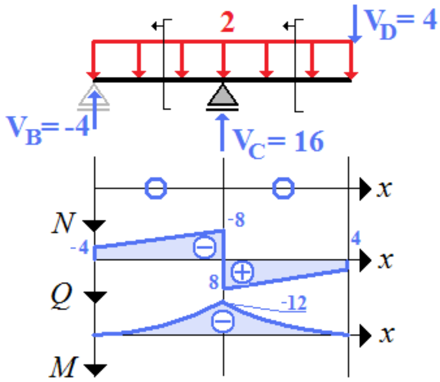
$$\begin{cases} N(x)=0 \\ Q(x)=-\frac{8}{3} \\ M(x)=-12+\frac{8}{3}\cdot(4-x) \end{cases}$$



Beam BD $x\in(0,4)$

Reactions:

$$\Sigma M_B=0 \Rightarrow -2\cdot4\cdot2+V_c\cdot2-4\cdot4=0 \Rightarrow V_C=16$$
$$\Sigma Y=0 \Rightarrow V_B+V_C-4-2\cdot4=0 \Rightarrow V_B=-4$$



Cross-sectional forces:

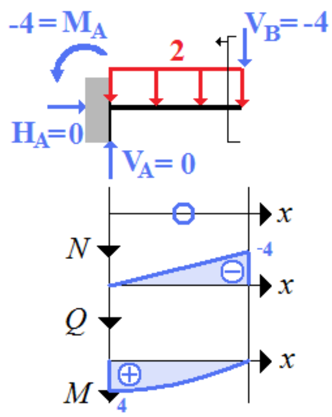
$BC, x\in(0,2)$
$$\begin{cases} N(x)=0 \\ Q(x)=-4-2\cdot x \\ M(x)=-4\cdot x-2\cdot x\cdot\frac{x}{2} \end{cases}$$

$CD, x\in(2,4)$
$$\begin{cases} N(x)=0 \\ Q(x)=-4-2\cdot x+16 \\ M(x)=-4\cdot x-2\cdot x\cdot\frac{x}{2}+16\cdot(x-2) \end{cases}$$

Beam AB $x\in(0,2)$

Reactions:

$$\Sigma X=0 \Rightarrow H_A=0$$
$$\Sigma Y=0 \Rightarrow V_A-2\cdot2+4=0 \Rightarrow V_A=0$$
$$\Sigma M_A=0 \Rightarrow M_A+4\cdot2-2\cdot2\cdot1=0 \Rightarrow M_A=-4$$



Cross-sectional forces:

$AB, x\in(0,2)$
$$\begin{cases} N(x)=0 \\ Q(x)=-2\cdot x \\ M(x)=-(-4)-2\cdot x\cdot\frac{x}{2} \end{cases}$$

Reactions and cross-sectional forces: