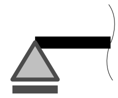
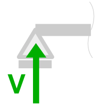
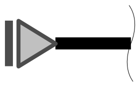
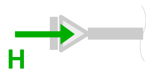
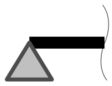
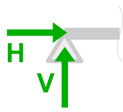
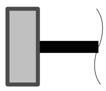
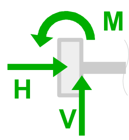
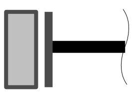
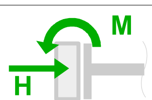
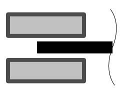
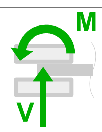
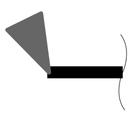
REACTION FORCES IN PLANE STRUCTURES

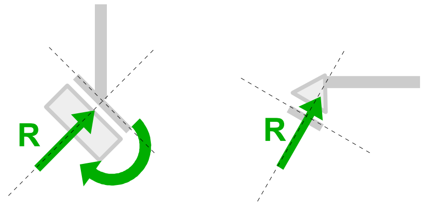
Rigid body on plane has **3 degrees of freedom**:

- **horizontal displacement**
- **vertical displacement**
- **in-plane rotation**

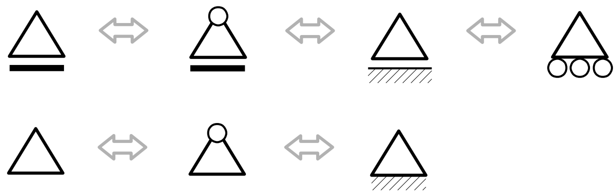
Any motion of such a body is a composition of these three elementary motions. In order to make a body immovable, one has to make those three motions impossible. We do it by applying appropriate **constraints** – **supports** – which limit the freedom of displacement. Every support provide a **reaction force**. It is a **passive foce**, so it does not exsist alone and it appears only as a response ("reaction") on applied external load. **It acts along the direction of a displacement that it should block** – its presence provides lack of motion along this directions. We distinguish following **supports**:

NAME	SYMBOL	REACTION FORCES	PERMISSED DISPLACEMENTS		
			Ux	Uy	φ
Roller support			✓	✗	✓
			✗	✓	✓
Pinned support			✗	✗	✓
Fixed support			✗	✗	✗
Fixed support with vertical displacement			✗	✓	✗
Fixed support with hotizontal displacement			✓	✗	✗
„Wedge"			✓	✓	✗

Roller supports and fixed supports with displacements may permiss a displacement inclined at any angle. Appropriate reaction will always be parallel to the direction of blocked displacement (perpendicular to direction of the permissed displacement)



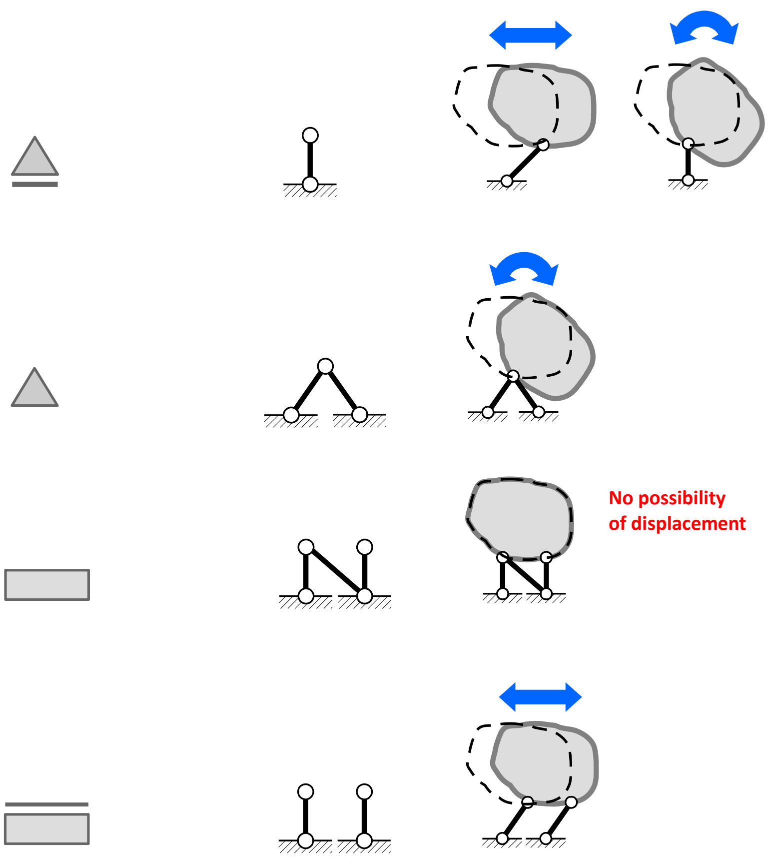
Symbols used for denoting of supports may vary in different elaborations. Few alternative symbols are presented below:



BAR DIAGRAMS OF SUPPORTS

Supports in plane systems are sometimes symbolized by a system of bars connected with joints (hinges) which block certain displacements.

Support **Bar diagram** **Displacements**



SUPPORTS IN SPATIAL SYSTEMS

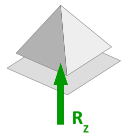
Supports for three-dimensional structures may be defined in an analogous way as for plane systems. The difference is the number of degrees of freedom. In space a rigid body may perform **6 independent motions**:

- **three displacements along three mutually perpendicular axes**
- **three rotations about three mutually perpendicular axes**

Blocking any of those motions at certain support requires a reaction (a force or a moment) appropriate for the direction of motion. One might define in total 63 different supports for spatial case, respective for 63 combinations of blocked motions. Among them only few are of our intereset:

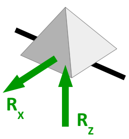
- **Roller support in XY plane**

u_X	u_Y	u_Z	φ_X	φ_Y	φ_Z
✓	✓	✗	✓	✓	✓



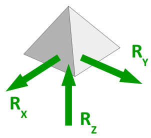
- **Roller support along X axis**

u_X	u_Y	u_Z	φ_X	φ_Y	φ_Z
✓	✗	✗	✓	✓	✓



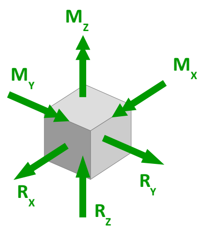
- **Pinned support**

u_X	u_Y	u_Z	φ_X	φ_Y	φ_Z
✗	✗	✗	✓	✓	✓



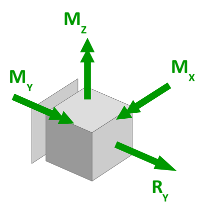
- **Fixed support**

u_X	u_Y	u_Z	φ_X	φ_Y	φ_Z
✗	✗	✗	✗	✗	✗



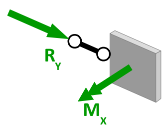
- **Fixed support with displacement in XZ plane**

u_X	u_Y	u_Z	φ_X	φ_Y	φ_Z
✓	✗	✓	✗	✗	✗



- **Constraint on rotation about X axis and on displacement along Y axis**

u_X	u_Y	u_Z	φ_X	φ_Y	φ_Z
✓	✗	✓	✗	✓	✓



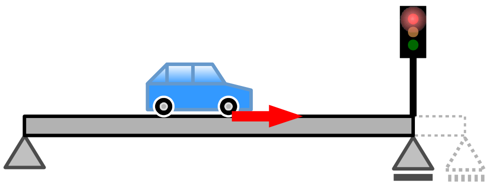
Other types of supports are defined in an analogous way, however, there is no single way of their symbollic representation which would be universally accepted and applied.

TECHNICAL METHODS OF CONSTRUCTING A SUPPORT

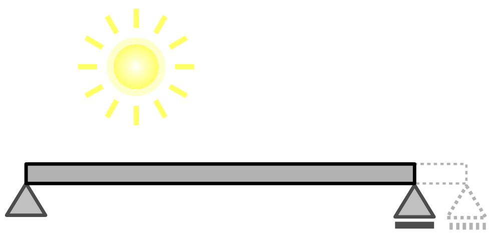
Roller support

Such a support is almost always used in situations in which it is necessary to allow for horizontal constraint while the vertical one is constrained. Other directions of permissible displacement are seldom met in typical technical problems. Permission of displacement is usually required as it is necessary to provide freedom of deformation of the structure (e.g. elongation or contraction) in certain direction. For example:

- Deformation of a bridge span along its axis, due to longitudinal (horizontal) forces due to vehicle breaking.



- Deformation of structural elements due to thermal expansion effect – it is the situation when the element is subject to higher or lower temperature than the one which was present during its installation.



- Deformation of a reinforced-cocnrete element due to its shrinkage during drying.

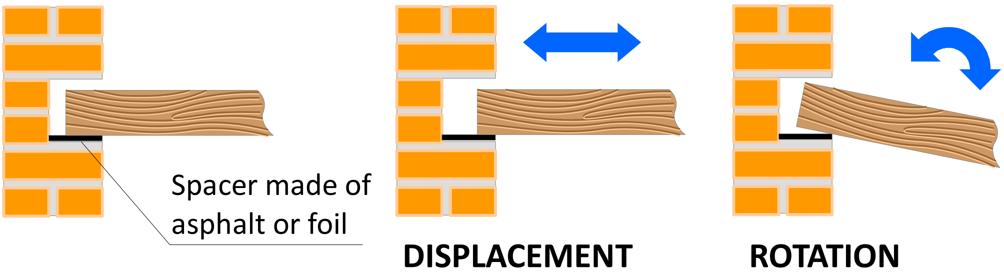
The reason for providing freedom in displacement is that distribution of internal forces depends on the arrangement of supports. An element which may deform freely won't be subject to any force. If the deformation was constrained then certain reaction forces would occur. In extreme cases such forces may be so great that they could result in excessive deformation or even in destruction of the element. Since in certain situation such a “forced deformation” cannot be avoided (e.g. external structural elements subject to sun heating) and possible reactions due to constraining such a displacement may be dangerously large, it is sometimes necessary to provide free displacement in certain direction.

The easiest way of construction of a roller support is simply to place the structural element on an immovable support in such a way that displacement and rotation is possible.

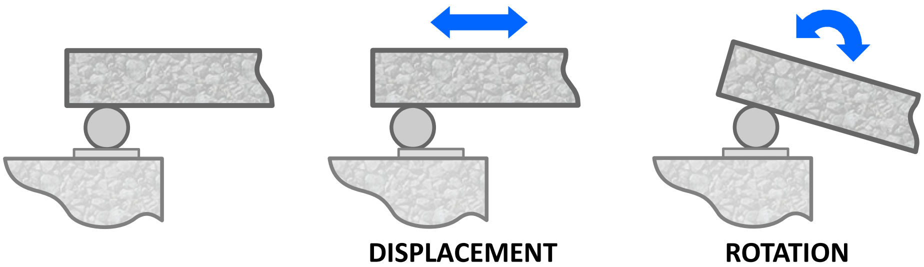
- Possibility of displacement may be provided by minimizing the friction betewwn the supported element and the support. It may be done with the use of spacers of very low friction coefficient, e.g. aluminum foil, asphalt spacers, polished steel plates etc.

- Rotation is possible only when the supporting is not too “deep”, namely when width of the support is small enough that rotation of the supported element is possible at minimum deformation of the support itself.

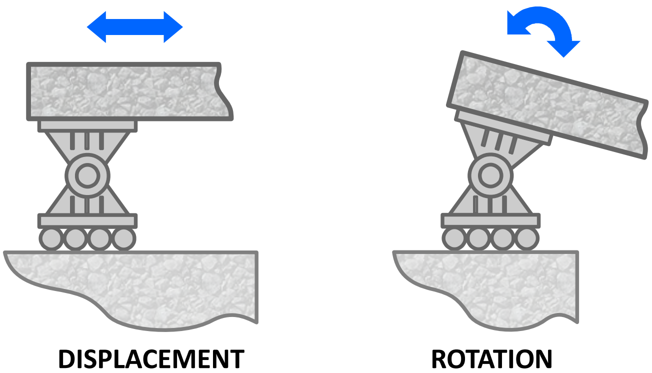
Simple support of a timber beam in a masonry wall may be an example:



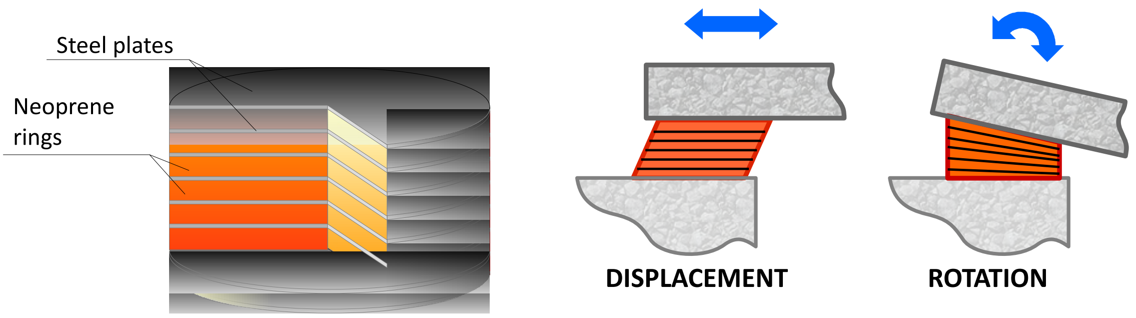
In the situation presented above the presence of asphalt or foil spacer is rather due to protecting the timber against moisture in area of contact with wall, however, another effect is that horizontal displacement of the beam is made easy. It is clear that the above solution is a greately imperfect method of constructing a roller support. In reality friction between beam and spacer will be large enough that a considerably large horizontal reaction (friction force) will occur. Also rotation is not entirely free. In structures of greater importance it is necessary to use more sophisticated solutions that will provide more correct character of supports behaviour. An example of technical execution of roller support is a **rolling bearing**. Smooth steel cylinder is placed in a properly shaped bearing – it allows both for free horizontal displacement (rolling of a cylinder) and for rotation:



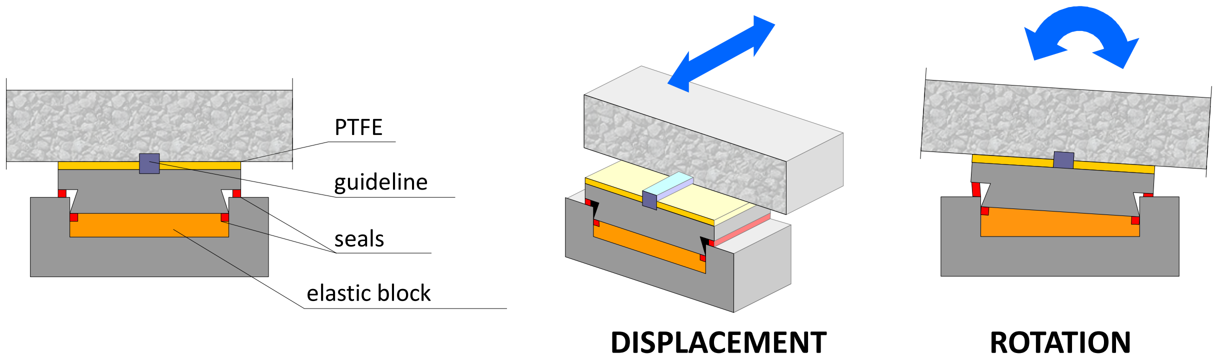
Another method of execution of roller support is construction of appropriate **joint machnery** placed on a system of cylinders allowing for horizontal displacement of the whole machinery:



A relatively new solution (ca. Middle of 20th century) are the supports in the form of a "steel-rubber hamburger" - it is a system of easily deformable rubber-like rings interlaid with stiffening steel plates:



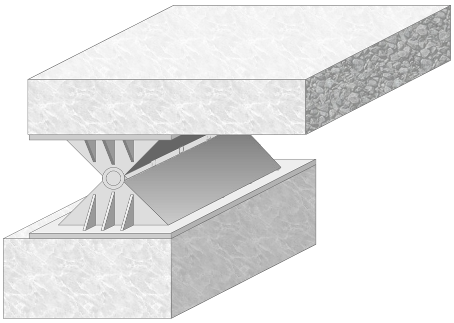
So called **pot bearings** are to some extent similar. Elastic block placed in rigid "pot" allow for rotation of supported element. At the top element of bearing there is a guideline rail allowing for a displacement in given direction on a surface covered with PTFE, which minimizes friction.

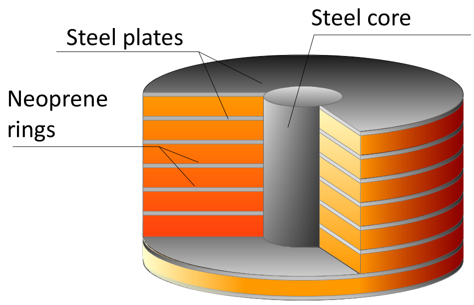


Pinned support

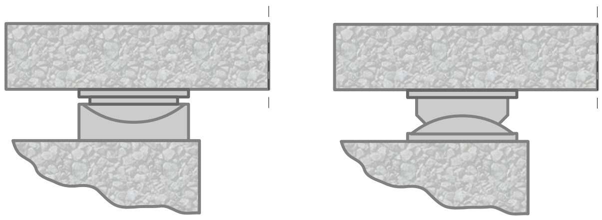
Pinned supports are constructed in a similar way as roller supports – the difference is that displacement has also to be blocked:

- in case of joint machinery we do not use the rolleing cylinders. Both parts of the machinery are anchored to the abutment and to the span structure respectively.
- in case of neoprene bearing it is equipped additionally with a rigid steel core in its axis, which disables horizontal displacement.
- In case of pot bearing we do not use a guideline rail and a PTFE sliding surface that enable displacement.

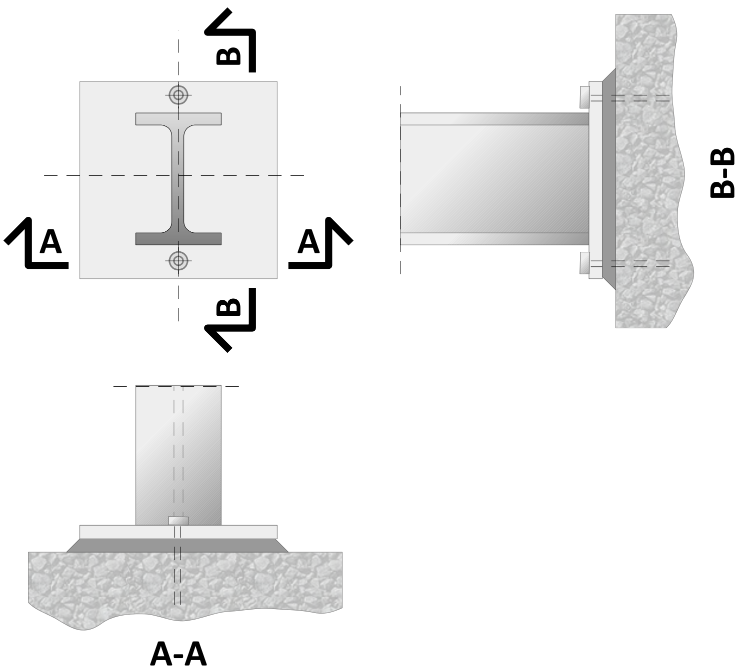




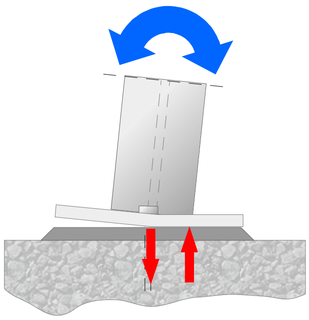
A common solution is a also a **lens bearing**. They will work properly only if the friction between elements of bearing is minimized – the friction may be reduced by application of proper sliding layers made of e.g. PTFE.



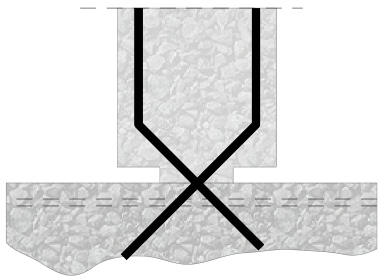
Also other solutions are used – they must block any displacement while still preserving relative freedom in rotation. "Relative freedom" means that rotation of structure results in negligibly small reactions which may be reduced to a couple of forces. What we understand as "negligibly small" depends on chosen criteria (e.g. a certain fraction of permissible moment bending the element). An example of such an approximate solution may be anchoring a steel section to a reinforced-concrete foundation in such a way, that the anchors worked in the plane of bending dedicated to the section.



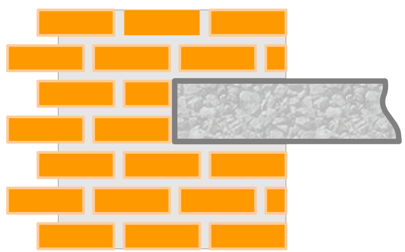
It is clear that the plate to which the section is welded and by which it is anchored to foundation will contact either the grout or concrete foundation – contact stresses occuring there together with forces in achors will give us a couple of forces that provides a moment reaction at the support. Due to small flexular stiffness of thin plate the moment is not large. However, if the bending was performed in a perpendicular plane, then the rotation would be resisted by forces pulling one anchor out and pushing the other one in – as they are in a certain distance one to another, so the “arm” of such a couple of forces may result in a large value of moment. Resistance to the rotation might be so large that presented solution may be closer to fixed support than to the pinned one.



In a similar way a pinned connection between reinforced-cocrete element may be performed. In such a case bent rebars are used – in the area of joint they are very close one to another. Also in this case compressive contact stresses may occur, that – together with tensile forces in rebars – may produce a small moment. Influence of such stresses may by minimized by proper ending a the element.

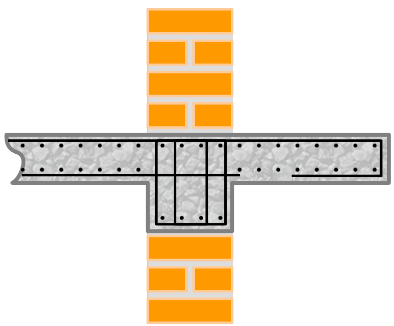


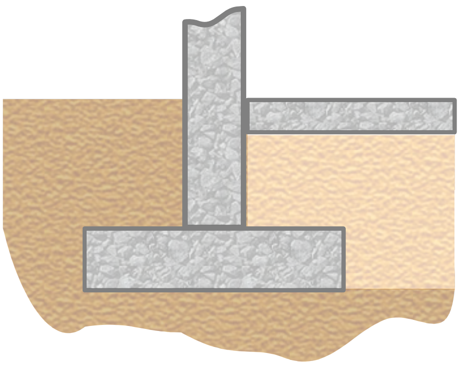
Fixed support



Fixed support is any support that constrains any motion od the element – its displacements or rotations. Typical example of fixed support are all kinds of cantilevers. The self-weigth and stiffeness of the masonry wall, in which such a cantilever is fixed, disables its displacement and rotation.

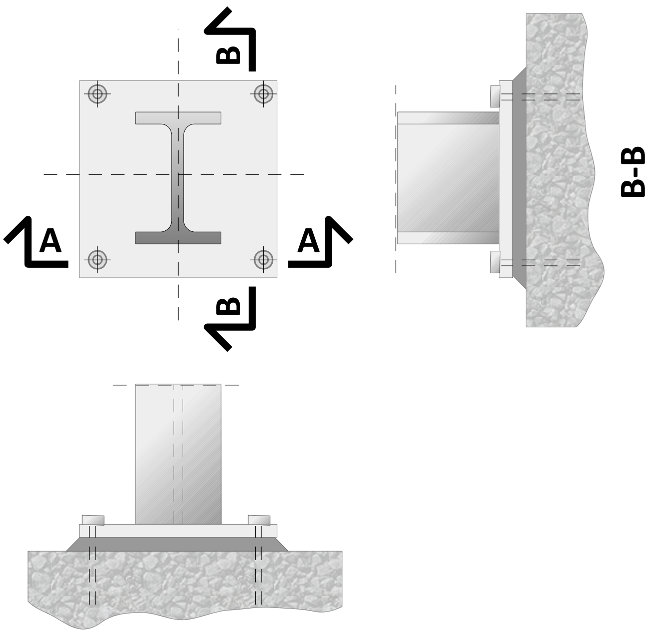
A balcony slab is a specific example. The wall to which a balcony is installed is usually not thick enough to disable rotation completely. The balcony slab is however continued as a floor slab – in this case it is the self-weigth and load applied to the floor slab, that makes the rotation of balcony about the wall minimized.

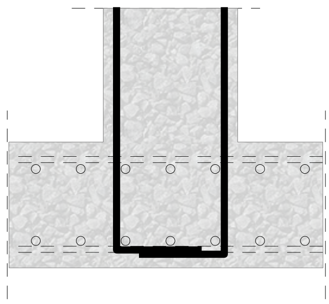




Fixed support is also the most common solution concerning modelling foundation. In case of a column or wall fixed in a footing or in a strip foundation, the medium constraining the motion is the surrounding soil. Because of that structural element supported in very shallow foundations (close to the ground level) are not necessarily to be modelled as fixed elements.

In case when we connect two elements of a structure, one of which should support the other one, constraining its displacement and rotation, it is necessary that such a connection is provided with possibility of bearing forces and moments. Already during discussion on pinned supports we've noticed that widely separated anchors are able to constrain rotation. Similarly, in case of reinforced-concrete elements, it is enough to anchor the reinforcement of one element in the second one in a sufficiently strong way.





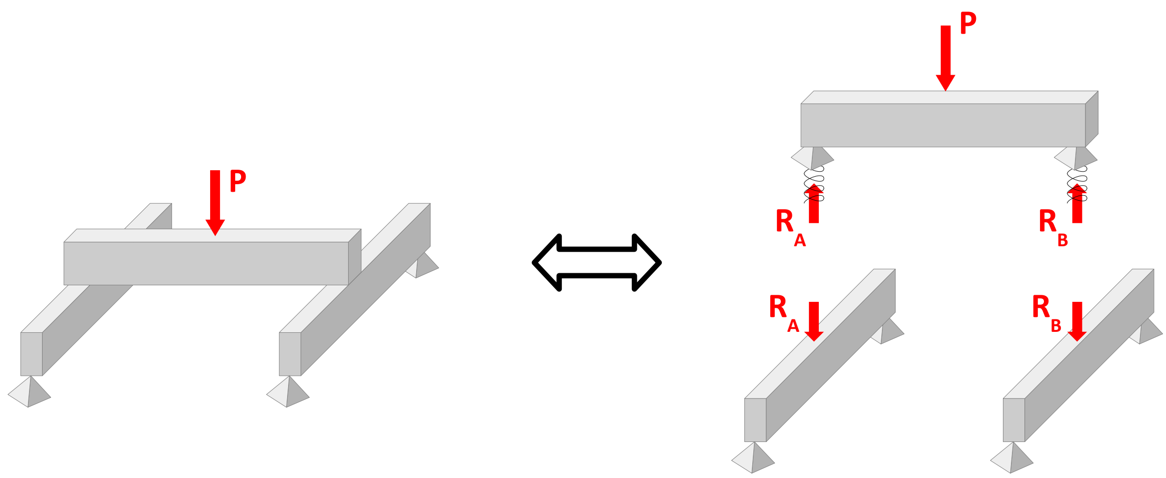
As it can be seen in the above examples, the solutions used in practice are not "pure" solutions in this sense, that permissible displacements are not entirely free – e.g. in case of a roller support, simple support results in presence of a horizontal friction reaction, which should not occur at such a support. It is clear also that in rubber bearings the material's stiffness alone results in presence of both horizontal force and bending moment. On the other hand, neither of the solutions presented above provides a "perfectly rigid" support along required direction. The supports themselves (or their elements) deform due to applied forces, so – for example – vertical displacement at pined suport is never equal exactly zero. It may be even more clearly seen in case of any fixed supports – minimal rotation occurs always. Any methods of construction of fixed support base on constraining the rotation either by stiffness of surrounding material (wall raround the balcony, soil around foundation) or by the use of apropriate anchors. In any case the moment is produced as a couple of reactions, as a response for small but already existing rotation.

Elastic supports

The above examples show us that in practice it is impossible to realize the support that we use in computations in a perfect way – a perfect support should totally disable any displacement, while at every true support the reaction is a response for a certain occurred (very small, however) deformation.

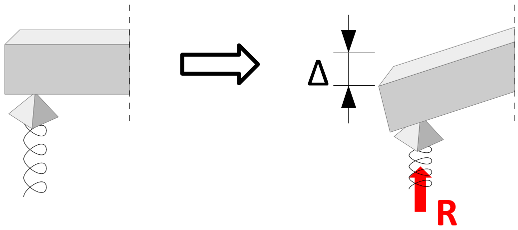
Such a deformation is usually so small that it does not influence the statics of the system – distribution of reactions and internal forces – in any significant way. There are, however, situations, in which deformation of the support is so large, that it influences the distribution of forces in the system. It concerns especially all those situations in which the support is made of a material of small stiffness (a compliant material), such as loose soil, elastomere or polystyrene spacers as well as such situations in which the support itself was constructed as an elastic one, e.g. with the use of springs. Such supports are used (together with appropriate damping elements) as supports for vibrating devices, e.g. turbo generators or engines in compressors, transformers and other ventilation or electric devices, which are always present in buildings. Also the bearing structure itself may vibrate, especially when it is slender – an open-work structure or the one made of elements of small transverse dimensions or of small stiffness – e.g. girders of light platforms.

A specific situation, in which accounting for a deformation of suport is important, is the case, in which we consider the structure as a system of element that interact between each other by forces, not as a single complex system. An example may be a beam supported in the middle of spans of two other perpendicular beams. It is possible to analyze such a three-dimensional beam grid, however it is often easier to analyze the beams separately. It is clear for us that if the bottom beams are thin, then their deformation due to load with a top beam would be so great, that the top beam should be modelled as a beam on elastic supports – stiffness of such an elastic support depends on stiffness of bottom beams.



In **elastic supports**, reactions at those supports are the greater, the larger is the deformation of support. This dependency is most often assumed to be linear, namely, that the reaction is proportional to the displacement at the support. Constant proportionality coefficient k of units N/m is termed **support stiffness**, while its reciprocal – **support compliance**. Elastic supports are usually denoted in an analogous way as the rigid supports with an added spring symbol for supports blocking displacement or two springs (or spiral spring) for those blocking rotation.

$$R = k \cdot \Delta$$



$$R = k \cdot \Delta$$