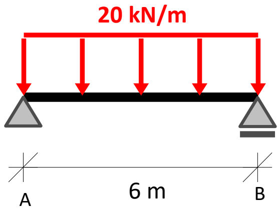
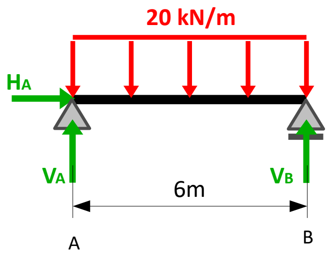
EXERCISE 1

Find the reaction forces with the use of equilibrium equations. Check if your calculations are correct.



SOLUTION:

Each exercise in which we have to find reaction forces should be started with marking proper force markings (arrows) on appropriate supports:



Orientation of reaction may be chosen in any way – it has no influence on final results. If it happens that chosen orientation is wrong (it means that the force will in fact act in an opposite direction than we have already marked with an arrow) then we will simply obtain negative value of reaction. Sometimes we are able to predict orientation of certain reaction so that we can instantly draw an arrow correctly – it is, however, not always so easy to be predicted so we can arbitrary assume orientation of reaction at the beginning of solution.

Reaction may be determined with the use of **equilibrium equations**. There are three such equations for plane systems:

$$\begin{cases} \Sigma X = 0 & \text{Sum of forces along X-axis is equal 0} \\ \Sigma Y = 0 & \text{Sum of forces along Y-axis is equal 0} \\ \Sigma M_P = 0 & \text{Sum of moments about any point P is equal 0} \end{cases}$$

or

$$\begin{cases} \Sigma M_A = 0 \\ \Sigma M_B = 0 \\ \Sigma M_C = 0 \end{cases}$$, points A, B, C may not lie on a single straight line.

It is most easy to use those equations in such a way that we avoid solving a system of equations and only **solve independent (uncoupled) equations with a single unknown**. In our case such a situation occurs when we chose the equation for sum of forces along X-axis:

$$\Sigma X = 0: \quad H_A + 0 = 0 \quad \Rightarrow \quad \boxed{H_A = 0 \text{ kN}}$$

WE can see that sum of forces along Y-axis contains two unknons – reactions V_A and V_B . It won't help us in determining any of them without using any other equation. The only equation we are left with is equating the sum of moments to 0, yet **point about which the moments are calculated may be chosen in any way**. It is the best way to chose such a point that only a single reaction gives a non-zero moment about it. Therefore it must lie on (intersection of) lines of action of those reactions that we want to omit in our equation. Obtained result will be independent of values of those ignored reactions also in the sense that if we made a mistake in determining them, then this error won't influence further results. In our case we could calculated sum of moments about A (omitting influence of reaction V_A) or about B (omitting influence of V_B). The choice is up to us – let it be point A:

$$\Sigma M_A=0:\quad +V_B\cdot 6-20\cdot 6\cdot 3=0 \qquad \Rightarrow \qquad \boxed{V_B=60\ \text{kN}}$$

What's only left is to determine the final reaction – in order to do it, we will make use of sum of vertical forces:

$$\Sigma Y=0:\quad V_A+V_B-20\cdot 6=0 \qquad \Rightarrow \qquad \boxed{V_A=60\ \text{kN}}$$

We could as well use different equation – e.g. Sum of moments about B. The result must be of course the same:

$$\Sigma M_B=0:\quad -V_A\cdot 6+20\cdot 6\cdot 3=0 \qquad \Rightarrow \qquad V_A=60\ \text{kN}$$

Checking if the calculations are correct consists in writing down any equilibrium equation – if all reactions are determined correctly, then such an equation will be automaticly satisfied. Following criteria may be accounted for when looking for an equation for checking our results:

- **We chose an equation which has not been already used –** if we solved it correctly and we made a mistake in a different equation such a check won't inform us about it. If in our solution both sums along axes are used, then check equation will always be a sum of moments.

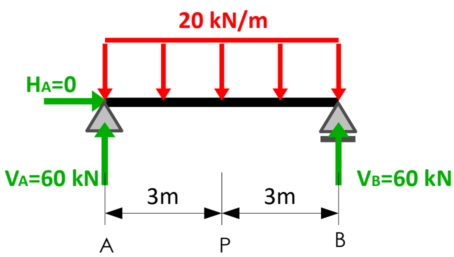
- **A point about which a sum of moment is to be calculated should be chosen in such a way that possibly many (at best: all) determined reactions give a non-zero moment about it** (the point lies outside their line of action) – if any reaction makes a zero moment about a chosen point and we made a mistake in determining this reaction, then such a check won't inform us about it.

- **A point about which a sum of moment is to be calculated should be chosen in such a way that possibly many external loads give zero moment about it** – such an equation is easier to be written down and thus it is more reliable.

In our case a good point for check is the middle of beam's span. Let's denote it with P. Then:

$$\Sigma M_P=-V_A\cdot 3+V_B\cdot 3=-60\cdot 3+60\cdot 3=-180+180=0$$

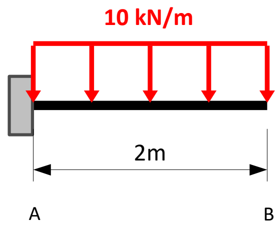
The fact that the sum of moments due to calculated reactions is zero confirms that they were determined correctly.



Since point P lies on line of action of H_A the above equation will be satisfied always, even if this reaction is determined wrongly. On the other hand, this reaction was determined with the use of a very simple equation, the solution of which we are sure of.

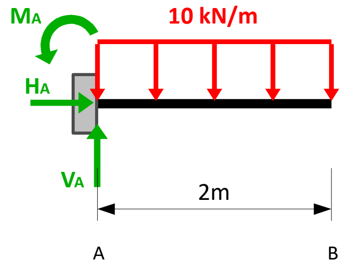
EXERCISE 2

Find the reaction forces with the use of equilibrium equations. Check if your calculations are correct.



SOLUTION:

We assign proper forces to supports:



We use the equilibrium equations:

$$\Sigma X = 0: \quad H_A + 0 = 0 \quad \Rightarrow \quad \boxed{H_A = 0 \text{ kN}}$$

$$\Sigma Y = 0: \quad V_A - 10 \cdot 2 = 0 \quad \Rightarrow \quad \boxed{V_A = 20 \text{ kN}}$$

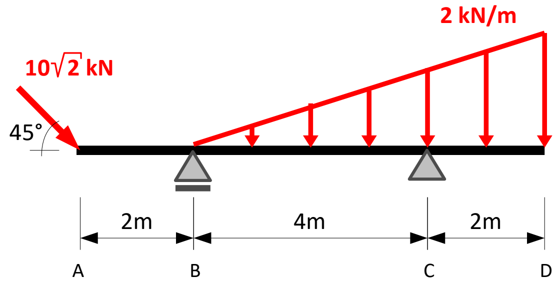
$$\Sigma M_A = 0: \quad +M_A - 10 \cdot 2 \cdot 1 = 0 \quad \Rightarrow \quad \boxed{M_A = 20 \text{ kNm}}$$

Check:

$$\Sigma M_B = 0: \quad +M_A + 10 \cdot 2 \cdot 1 - V_A \cdot 2 = 20 - 20 + 20 \cdot 2 \; = 0$$ **OK!**

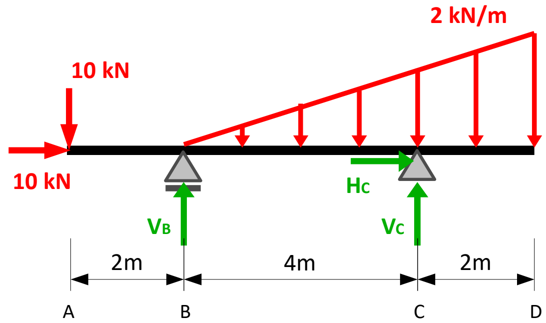
EXERCISE 3

Find the reaction forces with the use of equilibrium equations. Check if your calculations are correct.



SOLUTION:

Let's assign forces to supports. We may instantly decompose an oblique load into hotizontal and vertical component



We use the equilibrium equation:

$$\Sigma X = 0: \quad H_C + 10 = 0 \quad \Rightarrow \quad \boxed{H_C = -10 \text{ kN}}$$

In case when **obtained value is negative** it means that the orientation assumed by us at the beginning is wrong. A common procedure in such a situation is to change the orientation of an arrow in the picture and to change the value of reaction to a positive one. Such a solution requires an attention – it is necessary to clearly mark in the picture (i.e. in all pictures if there are more than one) which orientation is correct and to cross out a negative value obtained in calculations. Neglecting these rules often leads to errors – it concerns especially situations when a single force is present in few pictures, in which it is oriented in different way (these are the cases of "cutting" the structre – what will be discussed later – or equilibrating nodes of truss)

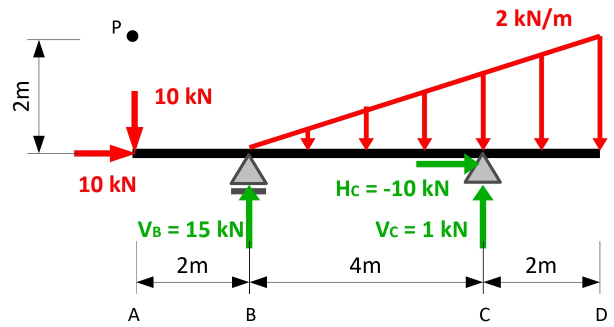
In my opinion it is much better to leave the originally assumed orientation of arrow and to stay with negative value of reaction. Further equations may be written at first symbolically (H_C) and later proper value may be substituted. Such a procedure has an additional advantege – equation written down symbolically is easier to be checked and if it is written correctly then its correctness does not depend on values of reactions determined earlier. Possible corrections are done simply by substituting certain symbols with apropriately modified values.

We determine the rest of reactions:

$$\Sigma M_C = 0: \quad -V_B \cdot 4 + 10 \cdot 6 = 0 \quad \Rightarrow \quad \boxed{V_B = 15 \text{ kN}}$$

$$\Sigma Y = 0: \quad -10 + V_B + V_C - \frac{1}{2} \cdot 6 \cdot 2 = 0 \quad \Rightarrow \quad \boxed{V_C = 1 \text{ kN}}$$

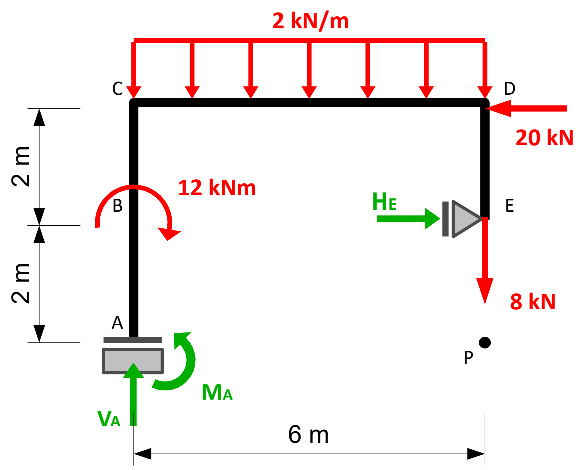
If we want to check our results with a sum of moments, then choice of proper point is not limited to the points belonging to the structure – it may be any point one the plane. Let's chose such a point that does not lie on line of action of negative reaction H_C in order to show how an equation is written first symbolically and then how proper values are substituted:



$$\Sigma M_C = 0: \quad 10 \cdot 2 + V_B \cdot 2 + V_C \cdot 6 + H_C \cdot 2 - \left(\frac{1}{2} \cdot 6 \cdot 2\right) \cdot \left(2 + \frac{2}{3} \cdot 6\right) =$$
$$= 10 \cdot 2 + 15 \cdot 2 + 1 \cdot 6 + (-10) \cdot 2 - \left(\frac{1}{2} \cdot 6 \cdot 2\right) \cdot \left(2 + \frac{2}{3} \cdot 6\right) =$$
$$= 20 + 30 + 6 - 20 - 6 \cdot 6 = 36 - 36 = 0$$

EXERCISE 4

Find the reaction forces with the use of equilibrium equations. Check if your calculations are correct.



SOLUTION:

$$\Sigma X = 0: \quad H_E - 20 = 0 \quad \Rightarrow \quad \boxed{H_E = 20 \text{ kN}}$$

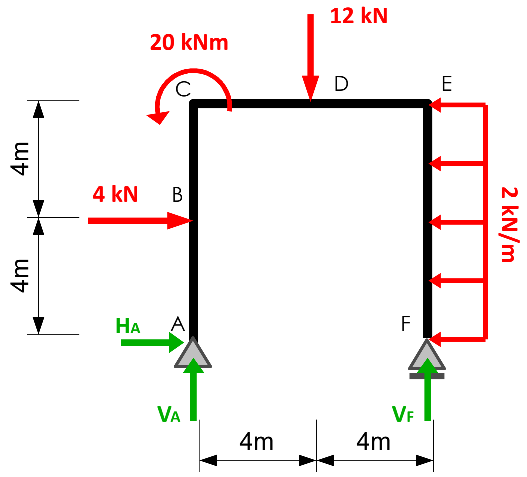
$$\Sigma Y = 0: \quad V_A - 2 \cdot 6 - 8 = 0 \quad \Rightarrow \quad \boxed{V_A = 20 \text{ kN}}$$

$$\Sigma M_E = 0: \quad +M_A - V_A \cdot 6 - 12 + 2 \cdot 6 \cdot 3 + 20 \cdot 2 = 0 \quad \Rightarrow \quad \boxed{M_A = 56 \text{ kNm}}$$

Sprawdzenie:

$$\Sigma M_P = 0: \quad M_A - V_A \cdot 6 - 12 + 2 \cdot 6 \cdot 3 + 20 \cdot 4 - H_E \cdot 2 =$$
$$= 56 - 20 \cdot 6 - 12 + 36 + 80 - 20 \cdot 2 =$$
$$= 56 - 120 - 12 + 36 + 80 - 40 = 0$$

EXERCISE 5
Find the reaction forces with the use of equilibrium equations.



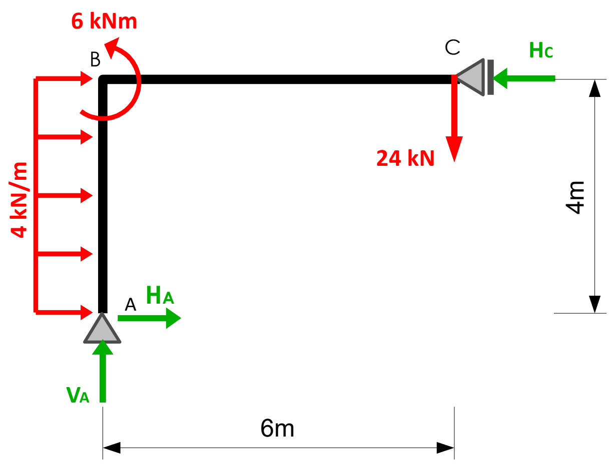
SOLUTION:

$$\Sigma X = 0: \quad H_A + 4 - 2\cdot 8 = 0 \quad \Rightarrow \quad \boxed{H_A = 12 \text{ kN}}$$

$$\Sigma M_A = 0: \quad -4\cdot 4 + 20 - 12\cdot 4 + 2\cdot 8\cdot 4 + V_F\cdot 8 = 0 \quad \Rightarrow \quad \boxed{V_F = -2{,}5 \text{ kN}}$$

$$\Sigma Y = 0: \quad V_A - 12 + V_F = 0 \quad \Rightarrow \quad \boxed{V_A = 14{,}5 \text{ kN}}$$

EXERCISE 6
Find the reaction forces with the use of equilibrium equations.



SOLUTION:

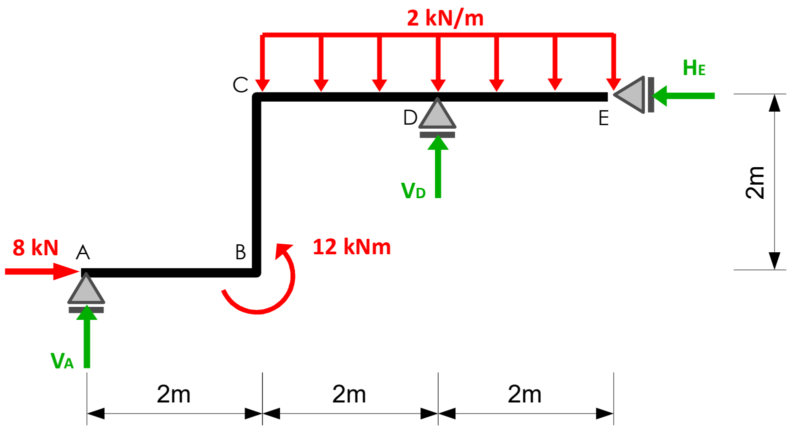
$$\Sigma Y = 0: \quad V_A - 24 = 0 \quad \Rightarrow \quad \boxed{V_A = 24 \text{ kN}}$$

$$\Sigma M_A = 0: \quad -4\cdot 4\cdot 2 + 6 - 24\cdot 6 + H_C\cdot 4 = 0 \quad \Rightarrow \quad \boxed{H_C = 42{,}5 \text{ kN}}$$

$$\Sigma X = 0: \quad H_A + 4\cdot 4 - H_C = 0 \quad \Rightarrow \quad \boxed{H_A = 26{,}5 \text{ kN}}$$

EXERCISE 7

Find the reaction forces with the use of equilibrium equations.



SOLUTION:

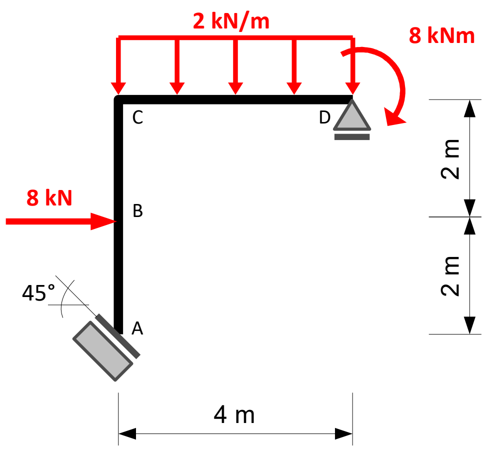
$$\Sigma X = 0: \quad 8 - H_E = 0 \quad \Rightarrow \quad \boxed{H_E = 8 \text{ kN}}$$

$$\Sigma M_D = 0: \quad +8 \cdot 2 - V_A \cdot 4 + 12 = 0 \quad \Rightarrow \quad \boxed{V_A = 7 \text{ kN}}$$

$$\Sigma Y = 0: \quad V_A - 2 \cdot 4 + V_D = 0 \quad \Rightarrow \quad \boxed{V_D = 1 \text{ kN}}$$

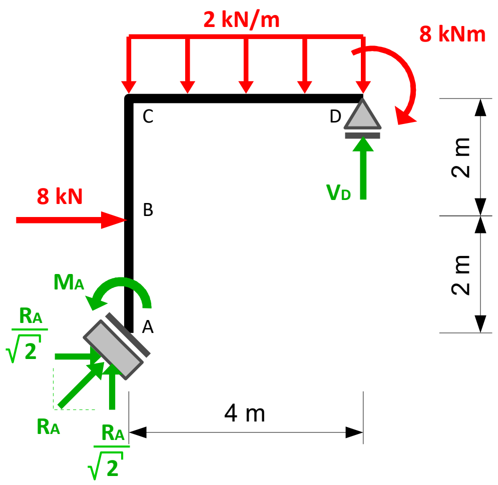
EXERCISE 8

Find the reaction forces with the use of equilibrium equations.



SOLUTION:

In case when there is a support with oblique displacement permissed reaction at such a support is always perpendicular to the direction of permissed displacement. It will always be a single unknown, however we will decompose it into its vertical and horizontal component.



$$\Sigma X=0: \quad \frac{R_A}{\sqrt{2}}+8=0 \quad \Rightarrow \quad \boxed{R_A=-8\sqrt{2}\ \mathrm{kN}}$$

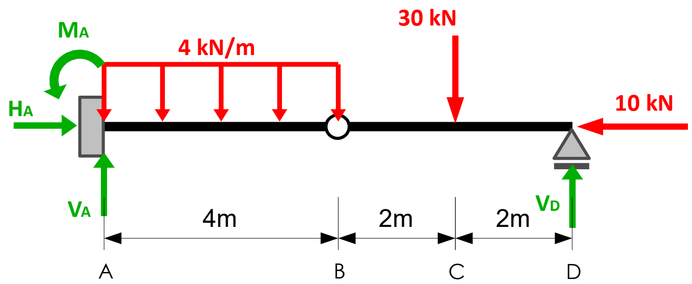
$$\Sigma Y=0: \quad \frac{R_A}{\sqrt{2}}-2\cdot 4+V_D=0 \quad \Rightarrow \quad \boxed{V_D=16\ \mathrm{kN}}$$

When chosing a point about which we would like to calculate sum of moment in order to find M_A we may notice that point D lies at the line of action of both force V_D and reaction R_A - despite fact that D does not lie on line of action of neither component of R_A, yet sum of those two components gives us obviously a zero moment about D:

$$\Sigma M_D=0: \quad +M_A+8\cdot 2+2\cdot 4\cdot 2-8=0 \quad \Rightarrow \quad \boxed{M_A=-24\ \mathrm{kNm}}$$

EXERCISE 9

Find the reaction forces with the use of equilibrium equations. Check if your calculations are correct.



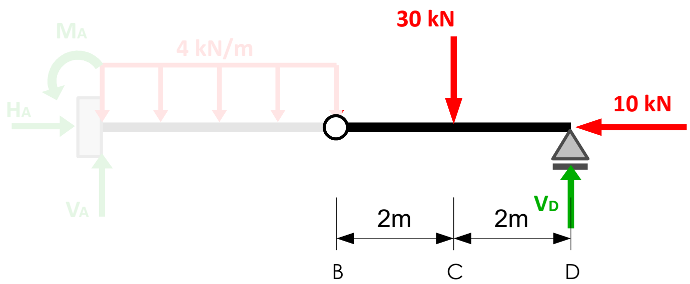
SOLUTION:

In systems with joints there are additionally two more equations for every joint, except three basic equilibrium equations: **sum of moments about the joint due to forces applied to either one or the second part of the structure which meet in the joint**. Those two equations are not independent of the three basic equilibrium equations, so each joint gives us a single additional independent equation which enables us to find unknown reaction.

At the beginning we will find horizontal reaction at fixed suport:

$$\Sigma X = 0: \quad H_A - 10 = 0 \quad \Rightarrow \quad \boxed{H_A = 10\ \text{kN}}$$

Please note, that sum of forces along Y-axis as well as any sum of moments about a point always contain two or more unknowns. Anyway we would be forced to use an equation for **sum of moments about a joint due to forces applied to one or to another side od joint**. If we write down such an equation for forces at right side of the joint, we will obtain an equation depending on only one unknown. When writing down this equation, we totally neglect the forces applied to the second part of the structure.



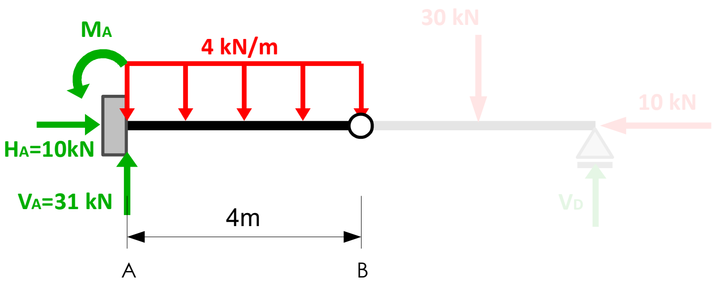
$$\Sigma M_B^{\rightarrow} = 0: \quad -30 \cdot 2 + V_D \cdot 4 = 0 \quad \Rightarrow \quad \boxed{V_D = 15\ \text{kN}}$$

REMARK:
One must remember that sum of moment from one side of a point may be written down **ONLY FOR A JOINT**.

The second vertical reaction may be found from equation of sum of forces along Y-axis:

$$\Sigma Y = 0: \quad V_A - 4 \cdot 4 - 30 + V_D = 0 \quad \Rightarrow \quad \boxed{V_A = 31\ \text{kN}}$$

Moment at fixed support may be determined with the use of the second equation that is delivered to use by the presence of the joint:



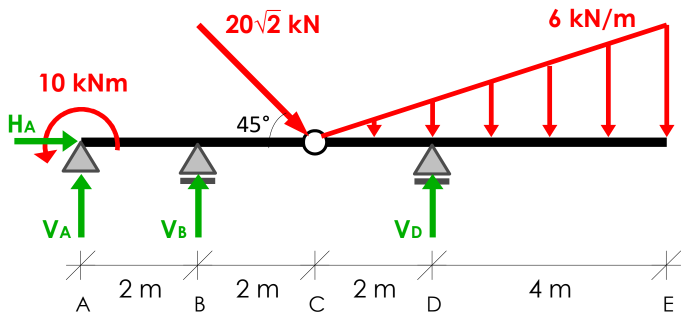
$$\Sigma M_B^{\leftarrow} = 0: \quad +M_A - V_A \cdot 4 + 4 \cdot 4 \cdot 2 = 0 \quad \Rightarrow \quad \boxed{M_A = 92\ \text{kNm}}$$

In order to check our calculations we must calculate the sum of moments (due to total load) about any point. It would be erroneus to chose pont B, since we've already used equations equating sum of moments from both sides of joint to zero. It implies from those two equations that sum of moments due to total load about B will surely be 0 (that's what means that each joint gives us two equartions but only one is independent of the rest of equilibrium equations). We may calculate e.g. sum of moment about C:

$$\Sigma M_C = M_A - V_A \cdot 6 + 4 \cdot 4 \cdot 4 + V_D \cdot 2 = 92 - 31 \cdot 6 + 64 + 15 \cdot 2 = 0$$

EXERCISE 10

Find the reaction forces with the use of equilibrium equations. Check if your calculations are correct.



SOLUTION:

$$\Sigma X = 0: \quad H_A + 20 = 0 \quad \Rightarrow \quad \boxed{H_A = -20 \text{ kN}}$$

$$\Sigma M_C^{\rightarrow} = 0: \quad +V_D \cdot 2 - \left(\frac{1}{2} \cdot 6 \cdot 6\right) \cdot 4 = 0 \quad \Rightarrow \quad \boxed{V_D = 36 \text{ kN}}$$

$$\Sigma M_A = 0: \quad +10 + V_B \cdot 2 - 20 \cdot 4 + V_D \cdot 6 - \left(\frac{1}{2} \cdot 6 \cdot 6\right) \cdot 8 = 0 \quad \Rightarrow \quad \boxed{V_B = -1 \text{ kN}}$$

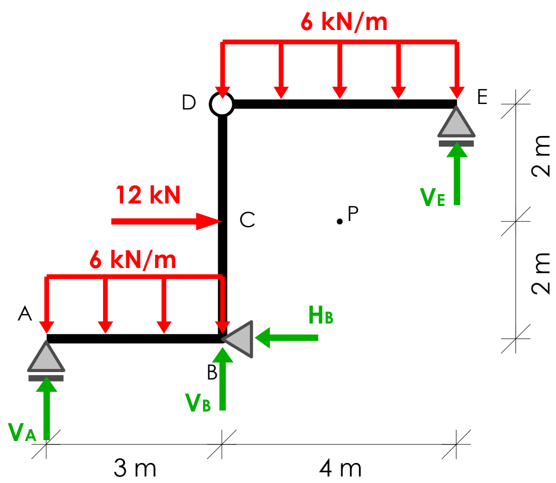
$$\Sigma M_C^{\leftarrow} = : \quad +10 - V_A \cdot 4 - V_B \cdot 2 = 0 \quad \Rightarrow \quad \boxed{V_A = 3 \text{ kN}}$$

Check:

$$\Sigma Y = V_A + V_B + V_D - 20 - \frac{1}{2} \cdot 6 \cdot 6 = 3 - 1 + 36 - 20 - 18 = 0$$ **OK!**

EXERCISE 11

Find the reaction forces and check if your calculations are correct.



SOLUTION:

$$\Sigma M_D^{\rightarrow} = 0: \quad V_E \cdot 4 - 6 \cdot 4 \cdot 2 = 0 \quad \Rightarrow \quad \boxed{V_E = 12 \text{ kN}}$$

$$\Sigma X = 0: \quad 12 - H_B = 0 \quad \Rightarrow \quad \boxed{H_B = 12 \text{ kN}}$$

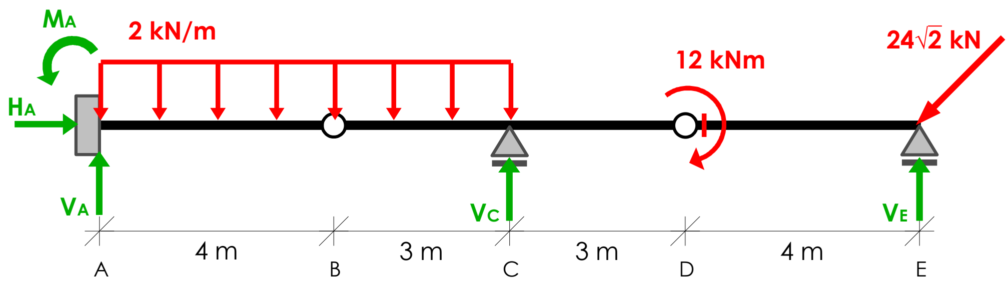
$$\Sigma M_D^{\downarrow} = 0: \quad 12 \cdot 2 - H_B \cdot 4 + 6 \cdot 3 \cdot 1{,}5 - V_A \cdot 3 = 0 \quad \Rightarrow \quad \boxed{V_A = 1 \text{ kN}}$$

$$\Sigma Y = 0: \quad V_A + V_B + V_E - 6 \cdot 3 - 6 \cdot 4 = 0 \quad \Rightarrow \quad \boxed{V_B = 29 \text{ kN}}$$

Check:

$$\Sigma M_P = -V_A \cdot 5 - V_B \cdot 2 - H_B \cdot 2 + V_E \cdot 2 + 6 \cdot 3 \cdot 3{,}5 = -5 - 58 - 24 + 24 + 63 = 0$$ **OK!**

EXERCISE 12
Find the reaction forces.



SOLUTION:

$$\Sigma X = 0: \quad H_A - 24 = 0 \quad \Rightarrow \quad \boxed{H_A = 24\ \text{kN}}$$

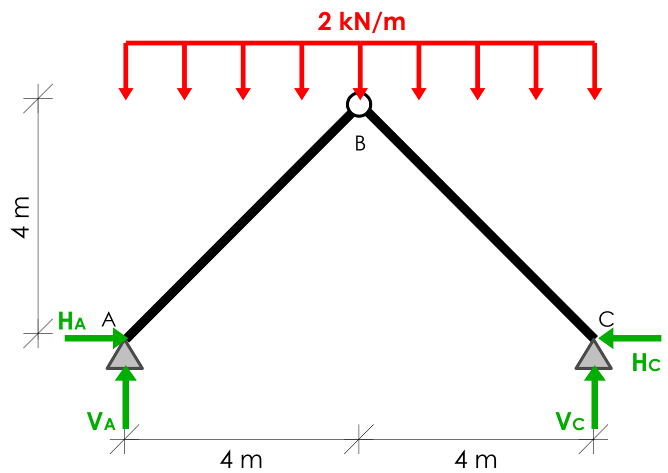
$$\Sigma M_D^{\rightarrow} = 0: \quad -12 - 24\cdot 4 + V_E\cdot 4 = 0 \quad \Rightarrow \quad \boxed{V_E = 27\ \text{kN}}$$

$$\Sigma M_B^{\rightarrow} = 0: \quad -2\cdot 3\cdot 1{,}5 + V_c\cdot 3 - 12 - 24\cdot 10 + V_E\cdot 10 = 0 \quad \Rightarrow \quad \boxed{V_C = -3\ \text{kN}}$$

$$\Sigma Y = : \quad V_A + V_C + V_E - 2\cdot 7 - 24 = 0 \quad \Rightarrow \quad \boxed{V_A = 14\ \text{kN}}$$

$$\Sigma M_B^{\leftarrow} = 0: \quad M_A - V_A\cdot 4 + 2\cdot 4\cdot 2 = 0 \quad \Rightarrow \quad \boxed{M_A = 40\ \text{kNm}}$$

EXERCISE 13
Find the reaction forces.



SOLUTION:

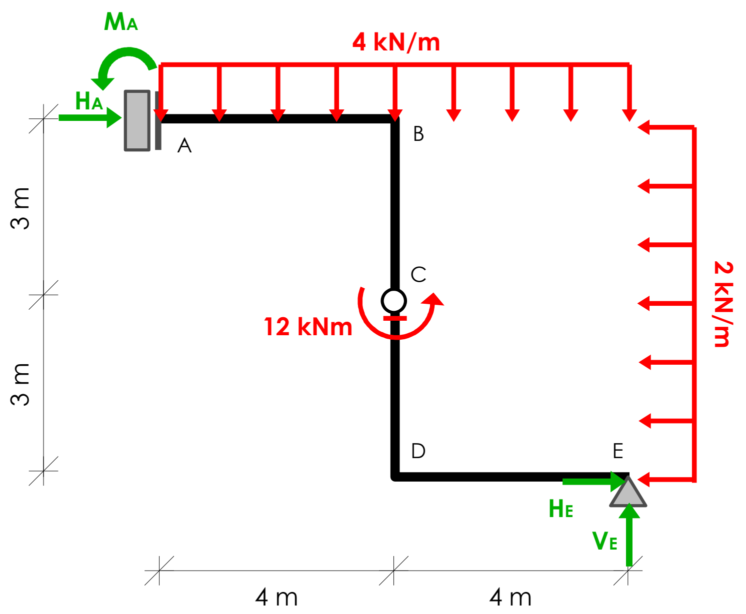
$$\Sigma M_C = 0: \quad -V_A\cdot 8 + 2\cdot 8\cdot 4 = 0 \quad \Rightarrow \quad \boxed{V_A = 8\ \text{kN}}$$

$$\Sigma M_B^{\leftarrow} = 0: \quad -V_A\cdot 4 + H_A\cdot 4 + 2\cdot 4\cdot 2 = 0 \quad \Rightarrow \quad \boxed{H_A = 4\ \text{kN}}$$

$$\Sigma X = 0: \quad H_A - H_C = 0 \quad \Rightarrow \quad \boxed{H_C = 4\ \text{kN}}$$

$$\Sigma Y = : \quad V_A + V_C - 2\cdot 8 = 0 \quad \Rightarrow \quad \boxed{V_C = 8\ \text{kN}}$$

EXERCISE 14
Find the reaction forces.



SOLUTION:

$$\Sigma Y = 0: \quad -4 \cdot 8 + V_E = 0 \quad \Rightarrow \quad \boxed{V_E = 32 \text{ kN}}$$

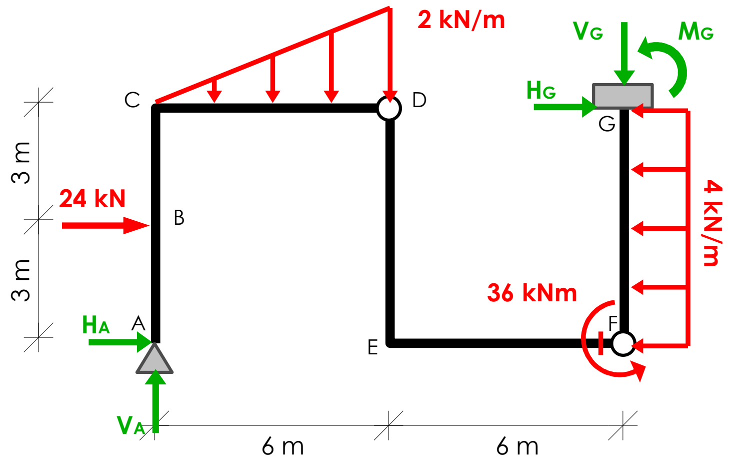
$$\Sigma M_C^{\downarrow} = 0: \quad 12 - 4 \cdot 4 \cdot 2 - 2 \cdot 3 \cdot 1{,}5 + H_E \cdot 3 + V_E \cdot 4 = 0 \quad \Rightarrow \quad \boxed{H_E = -33 \text{ kN}}$$

$$\Sigma X = 0: \quad H_A - 2 \cdot 6 + H_E = 0 \quad \Rightarrow \quad \boxed{H_A = 45 \text{ kN}}$$

$$\Sigma Y = : \quad +M_A - H_A \cdot 3 + 4 \cdot 4 \cdot 2 + 2 \cdot 3 \cdot 1{,}5 = 0 \quad \Rightarrow \quad \boxed{M_A = 94 \text{ kNm}}$$

EXERCISE 15

Find the reaction forces.



SOLUTION:

$$\Sigma M_F^{\leftarrow} = 0: \quad -V_A \cdot 12 - 24 \cdot 3 + \frac{1}{2} \cdot 6 \cdot 2 \cdot 8 + 36 = 0 \quad \Rightarrow \quad \boxed{V_A = 1 \text{ kN}}$$

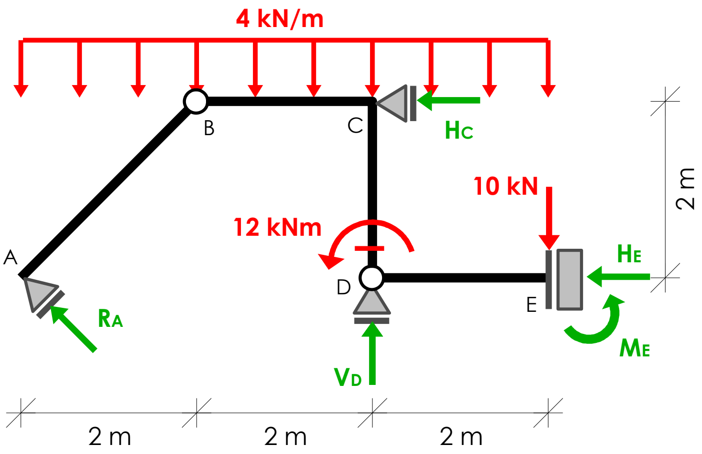
$$\Sigma M_D^{\leftarrow} = 0: \quad -V_A \cdot 6 + H_A \cdot 6 + 24 \cdot 3 + \frac{1}{2} \cdot 6 \cdot 2 \cdot 2 = 0 \quad \Rightarrow \quad \boxed{H_A = -13 \text{ kN}}$$

$$\Sigma X = 0: \quad H_A + 24 - 4 \cdot 6 + H_G = 0 \quad \Rightarrow \quad \boxed{V_C = 13 \text{ kN}}$$

$$\Sigma Y = : \quad V_A - \frac{1}{2} \cdot 6 \cdot 2 - V_G = 0 \quad \Rightarrow \quad \boxed{V_G = -5 \text{ kN}}$$

$$\Sigma M_F^{\uparrow} = 0: \quad +M_G - H_G \cdot 6 + 4 \cdot 6 \cdot 3 = 0 \quad \Rightarrow \quad \boxed{M_G = 6 \text{ kNm}}$$

EXERCISE 16
Find the reaction forces.



SOLUTION:

$$\Sigma M_D^{\rightarrow} = 0: \quad -10\cdot 2 - 4\cdot 2\cdot 1 + M_E = 0 \quad \Rightarrow \quad \boxed{M_E = 28 \text{ kNm}}$$

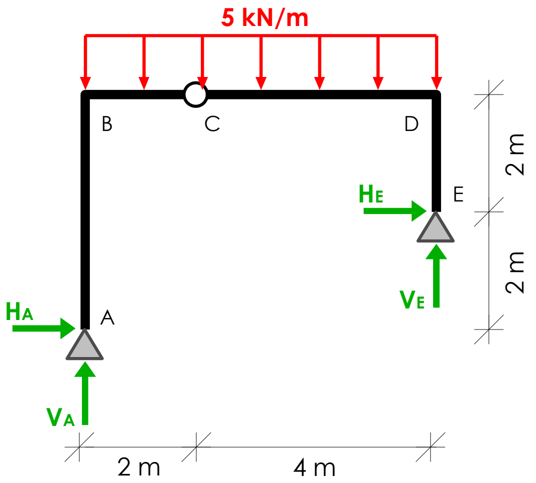
$$\Sigma M_B^{\leftarrow} = 0: \quad -\frac{R_A}{\sqrt{2}}\cdot 2 - \frac{R_A}{\sqrt{2}}\cdot 2 + 4\cdot 2\cdot 1 = 0 \quad \Rightarrow \quad \boxed{R_A = 2\sqrt{2} \text{ kN}}$$

$$\Sigma Y = 0: \quad \frac{R_A}{\sqrt{2}} + V_D - 4\cdot 6 - 10 = 0 \quad \Rightarrow \quad \boxed{V_D = 32 \text{ kN}}$$

$$\Sigma M_D^{\uparrow} =: \quad +12 + H_C\cdot 2 - \frac{R_A}{\sqrt{2}}\cdot 4 + 4\cdot 4\cdot 2 = 0 \quad \Rightarrow \quad \boxed{H_C = -18 \text{ kN}}$$

$$\Sigma X = 0: \quad -\frac{R_A}{\sqrt{2}} - H_C - H_E = 0 \quad \Rightarrow \quad \boxed{H_E = 16 \text{ kN}}$$

EXERCISE 17
Find the reaction forces.



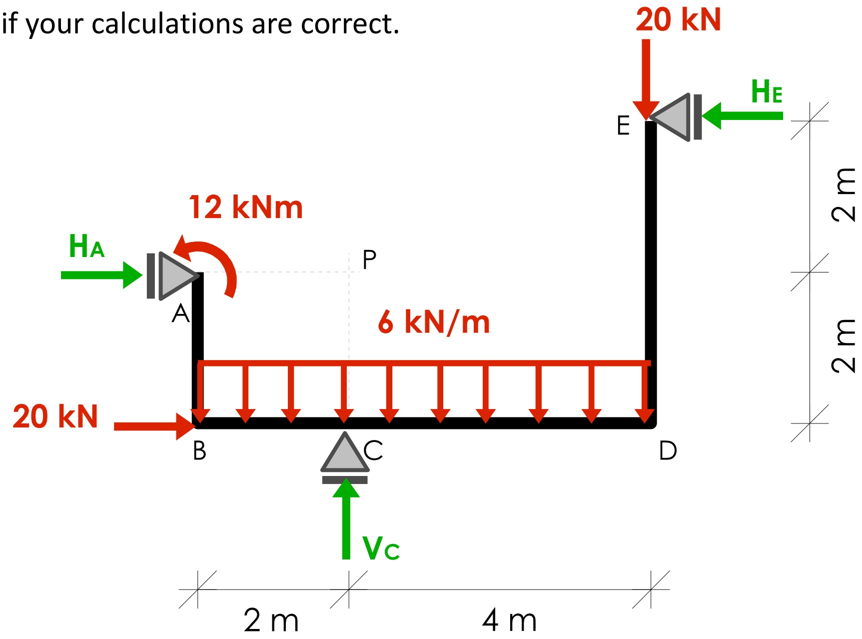
SOLUTION:

$$\begin{cases} \Sigma M_E = 0 \\ \Sigma M_C^{\leftarrow} = 0 \end{cases} \Rightarrow \begin{cases} +H_A \cdot 2 - V_A \cdot 6 + 5 \cdot 6 \cdot 3 = 0 \\ +H_A \cdot 4 - V_A \cdot 2 + 5 \cdot 2 \cdot 1 = 0 \end{cases} \Rightarrow \boxed{\begin{cases} H_A = 6 \text{ kN} \\ V_A = 17 \text{ kN} \end{cases}}$$

$$\Sigma X = 0: \quad H_A + H_E = 0 \quad \Rightarrow \quad \boxed{H_E = -6 \text{ kN}}$$

$$\Sigma Y = 0: \quad V_A + V_E - 5 \cdot 6 = 0 \quad \Rightarrow \quad \boxed{V_E = 13 \text{ kN}}$$

EXERCISE 18
Find reactions and check if your calculations are correct.



$$\Sigma Y=0: \quad V_C - 6\cdot 6 - 20 = 0 \quad \Rightarrow \quad \boxed{V_C=56}$$

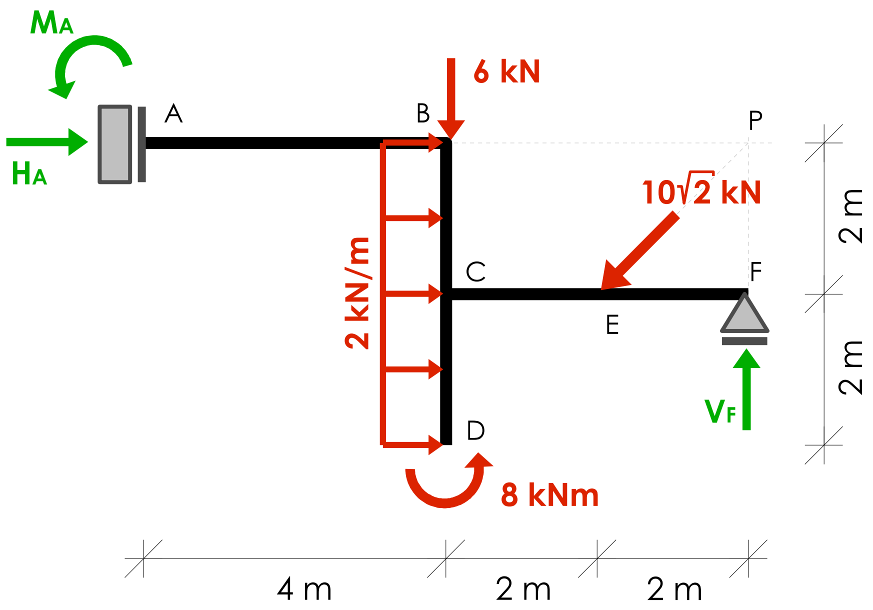
$$\Sigma M_P=0: \quad +20\cdot 2 - 6\cdot 6\cdot 1 + 12 - 20\cdot 4 + H_E\cdot 2 = 0 \quad \Rightarrow \quad \boxed{H_E = 32}$$

$$\Sigma X=0: \quad H_A + 20 - H_E = 0 \quad \Rightarrow \quad \boxed{H_A = 12}$$

Check:

$$\Sigma M_D = -H_A\cdot 2 + 12 - V_C\cdot 4 + 6\cdot 6\cdot 3 + H_E\cdot 4 =$$
$$-12\cdot 2 + 12 - 56\cdot 4 + 6\cdot 6\cdot 3 + 32\cdot 4 = 0$$

EXERCISE 19
Find reactions and check if your calculations are correct.



$$\Sigma X = 0: \quad H_A + 2 \cdot 4 - 10 = 0 \quad \Rightarrow \quad \boxed{H_A = 2}$$

$$\Sigma Y = 0: \quad V_F - 6 - 10 = 0 \quad \Rightarrow \quad \boxed{V_F = 16}$$

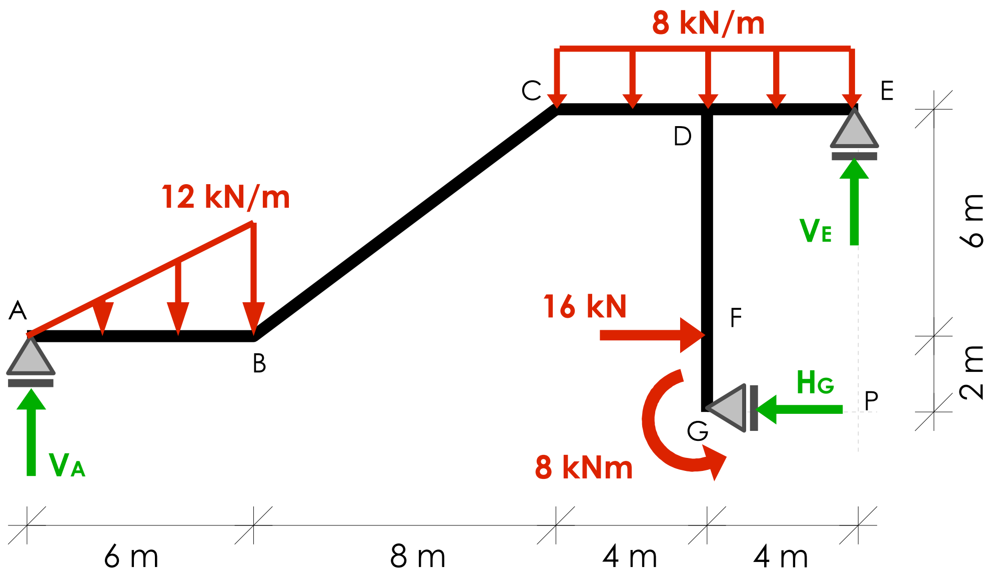
$$\Sigma M_P = 0: \quad M_A + 6 \cdot 4 + 2 \cdot 4 \cdot 2 + 8 = 0 \quad \Rightarrow \quad \boxed{M_A = -48}$$

Check:

$$\Sigma M_E = +M_A - H_A \cdot 2 + 6 \cdot 2 + 8 + V_F \cdot 2 =$$
$$-48 - 2 \cdot 2 + 6 \cdot 2 + 8 + 16 \cdot 2 = 0$$

EXERCISE 20

Find reactions at supports.



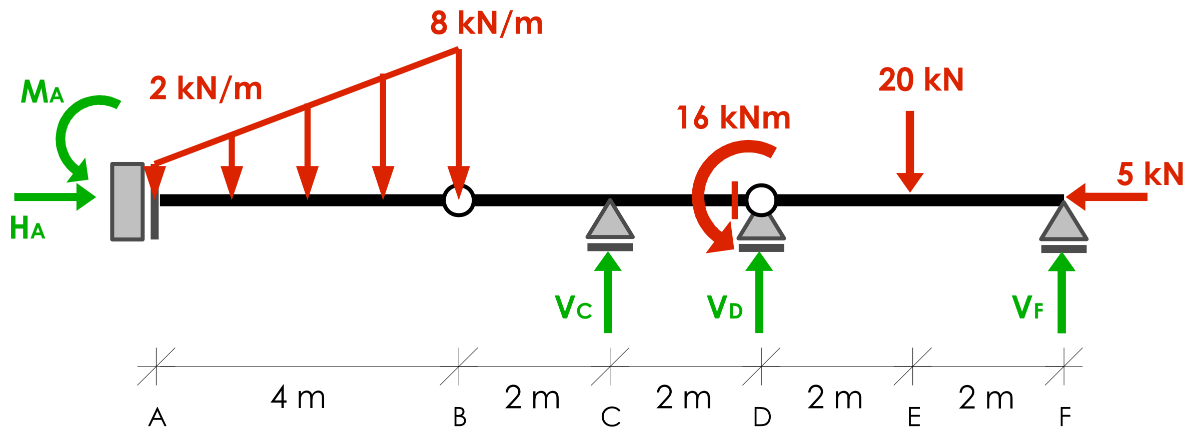
$$\Sigma X = 0: \quad 16 - H_G = 0 \quad \Rightarrow \quad \boxed{H_G = 16}$$

$$\Sigma M_P = 0: \quad -V_A \cdot 22 + \left[\frac{1}{2} \cdot 6 \cdot 12\right] \cdot \left[16 + \frac{1}{3} \cdot 6\right] + 8 \cdot 8 \cdot 4 - 16 \cdot 2 + 8 = 0 \quad \Rightarrow \quad \boxed{V_A = 40}$$

$$\Sigma Y = 0: \quad V_A - \frac{1}{2} \cdot 6 \cdot 12 - 8 \cdot 8 + V_E = 0 \quad \Rightarrow \quad \boxed{V_E = 60}$$

EXERCISE 21

Find reactions and check if your calculations are correct.



$$\Sigma X = 0: \quad H_A - 5 = 0 \quad \Rightarrow \quad \boxed{H_A = 5}$$

$$\Sigma M_B^{lewo} = 0: \quad +M_A + [2\cdot 4]\cdot\left[\frac{1}{2}\cdot 4\right] + \left[\frac{1}{2}\cdot 4\cdot(8-2)\right]\cdot\left[\frac{1}{3}\cdot 4\right] = 0 \quad \Rightarrow \quad \boxed{M_A = -32}$$

$$\Sigma M_D^{prawo} = 0: \quad -20\cdot 2 + V_F\cdot 4 = 0 \quad \Rightarrow \quad \boxed{V_F = 10}$$

$$\Sigma M_D^{lewo} = 0: \quad +M_A + [2\cdot 4]\cdot\left[4+\frac{1}{2}\cdot 4\right] + \left[\frac{1}{2}\cdot 4\cdot(8-2)\right]\cdot\left[4+\frac{1}{3}\cdot 4\right] + 16 - V_C\cdot 2 = 0 \quad \Rightarrow \quad \boxed{V_C = 48}$$

$$\Sigma Y = 0: \quad -2\cdot 4 - \frac{1}{2}\cdot 4\cdot(8-2) + V_C + V_D - 20 + V_F = 0 \quad \Rightarrow \quad \boxed{V_D = -18}$$

Check:

$$\Sigma M_E = +M_A + [2\cdot 4]\cdot\left[6+\frac{1}{2}\cdot 4\right] + \left[\frac{1}{2}\cdot 4\cdot(8-2)\right]\cdot\left[6+\frac{1}{3}\cdot 4\right] + 16 - V_C\cdot 4 - V_D\cdot 2 + V_F\cdot 2 =$$

$$-32 + 8\cdot 8 + 12\cdot\frac{22}{3} + 16 - 48\cdot 4 - (-18)\cdot 2 + 10\cdot 2 = 0$$

EXERCISE 22

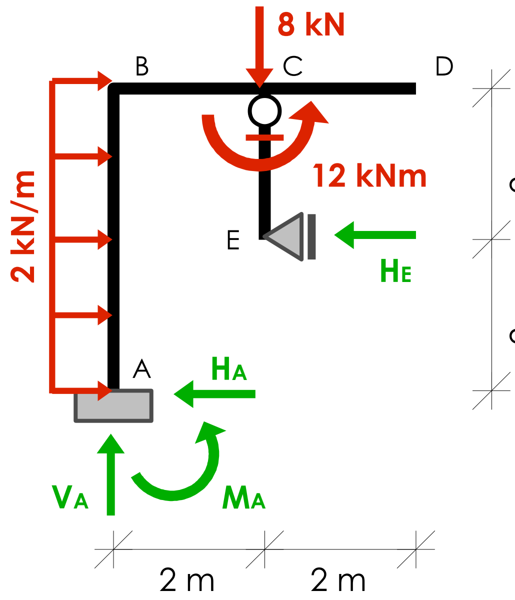
Find reactions at supports.

$$\Sigma M_C^{dół}=0:\quad +12 - H_E \cdot 2 = 0 \quad \Rightarrow \quad \boxed{H_E = 6}$$

$$\Sigma X=0:\quad +2\cdot 4 - H_A - H_E = 0 \quad \Rightarrow \quad \boxed{H_A = 2}$$

$$\Sigma Y=0:\quad V_A - 8 = 0 \quad \Rightarrow \quad \boxed{V_A = 8}$$

$$\Sigma M_E = 0:\quad M_A - H_A \cdot 2 - V_A \cdot 2 + 12 = 0 \quad \Rightarrow$$
$$\Rightarrow \quad \boxed{M_A = 8}$$



EXERCISE 23

Find reactions at supports.

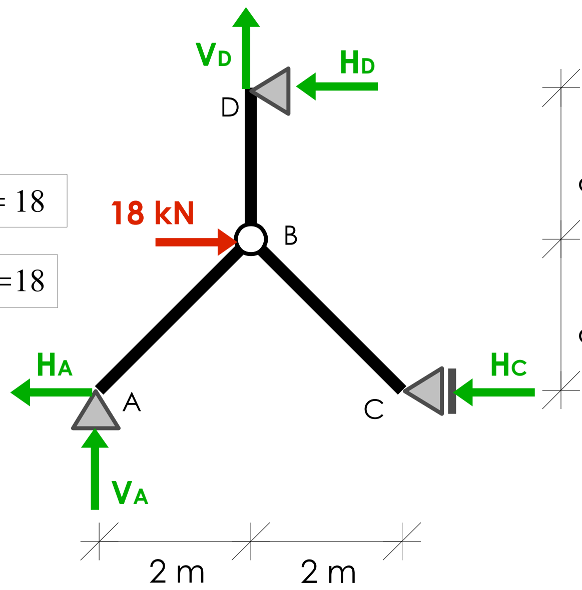
$$\Sigma M_B^{prawo-dół}=0:\quad -H_C \cdot 2 = 0 \quad \Rightarrow \quad \boxed{H_C = 0}$$

$$\Sigma M_B^{góra}=0:\quad H_D \cdot 2 = 0 \quad \Rightarrow \quad \boxed{H_D = 0}$$

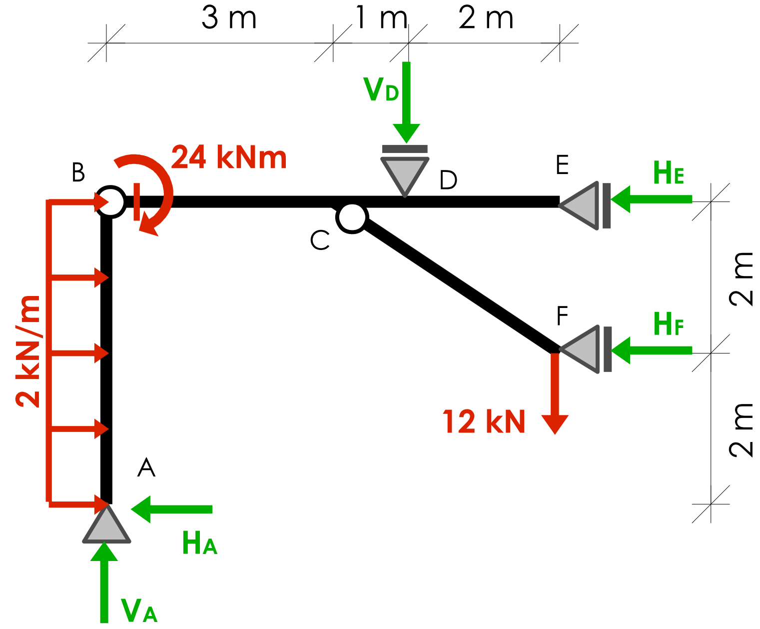
$$\Sigma X=0:\quad -H_A + 18 - H_D - H_C = 0 \quad \Rightarrow \quad \boxed{H_A = 18}$$

$$\Sigma M_A=0:\quad -18\cdot 2 + V_D \cdot 2 + H_D \cdot 4 = 0 \quad \Rightarrow \quad \boxed{V_D = 18}$$

$$\Sigma Y=0:\quad V_A + V_D = 0 \quad \Rightarrow \quad \boxed{V_A = -18}$$



EXERCISE 24
Find reactions at supports.



$$\Sigma M_B^{dół}=0: \quad +2\cdot4\cdot2 - H_A\cdot4 = 0 \quad \Rightarrow \quad \boxed{H_A = 4}$$

$$\Sigma M_C^{dół}=0: \quad -12\cdot3 - H_F\cdot2 = 0 \quad \Rightarrow \quad \boxed{H_F = -18}$$

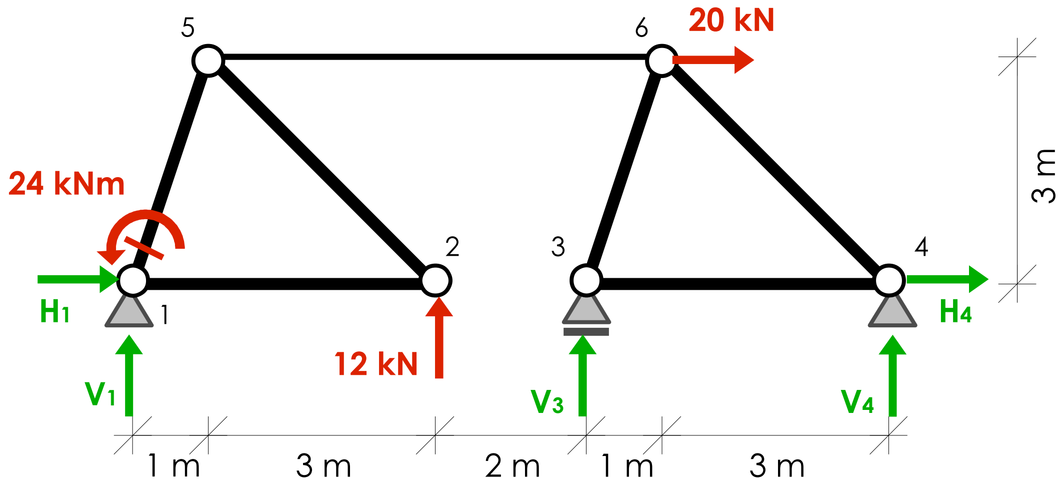
$$\Sigma X=0: \quad -H_A + 2\cdot4 - H_E - H_F = 0: \quad \Rightarrow \quad \boxed{H_E = 22}$$

$$\Sigma M_B^{prawo}=0: \quad -24 - V_D\cdot4 - 12\cdot6 - H_F\cdot2 = 0 \quad \Rightarrow \quad \boxed{V_D = -15}$$

$$\Sigma Y=0: \quad V_A - V_D - 12 = 0 \quad \Rightarrow \quad \boxed{V_A = -3}$$

EXERCISE 25

Find reactions at supports and a force in bowstring 5-6.



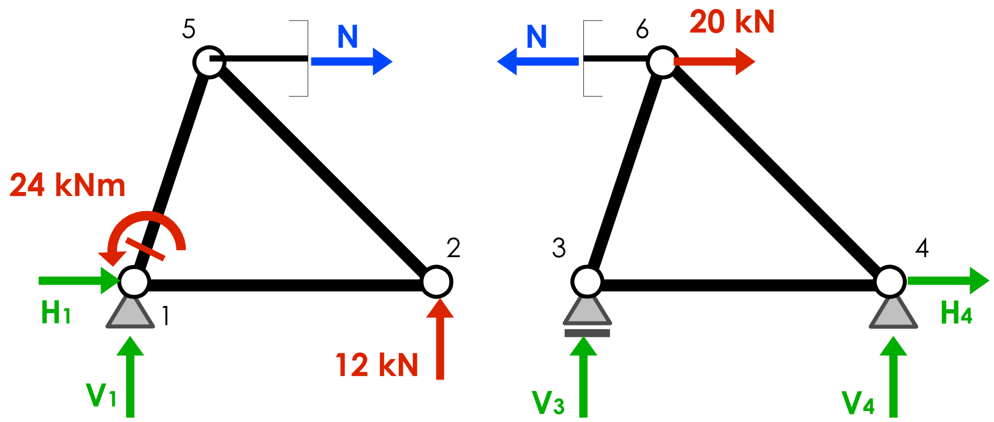
$$\begin{cases} \Sigma M_5^{dól}=0 \\ \Sigma M_6^{lewo}=0 \end{cases} \Rightarrow \begin{cases} +24 + H_1\cdot 3 - V_1\cdot 1 + 12\cdot 3=0 \\ +24 + H_1\cdot 3 - V_1\cdot 7 - 12\cdot 3=0 \end{cases} \Rightarrow \boxed{\begin{cases} H_1=-24 \\ V_1=-12 \end{cases}}$$

$$\Sigma X=0: \quad H_1 + 20 + H_4 = 0 \quad \Rightarrow \quad \boxed{H_4 = 4}$$

$$\Sigma M_4=0: \quad +24 - V_1\cdot 10 - 12\cdot 6 - V_3\cdot 4 - 20\cdot 3 = 0 \quad \Rightarrow \quad \boxed{V_3 = 3}$$

$$\Sigma Y = 0: \quad V_1 + 12 + V_3 + V_4 = 0: \quad \Rightarrow \quad \boxed{V_4 = -3}$$

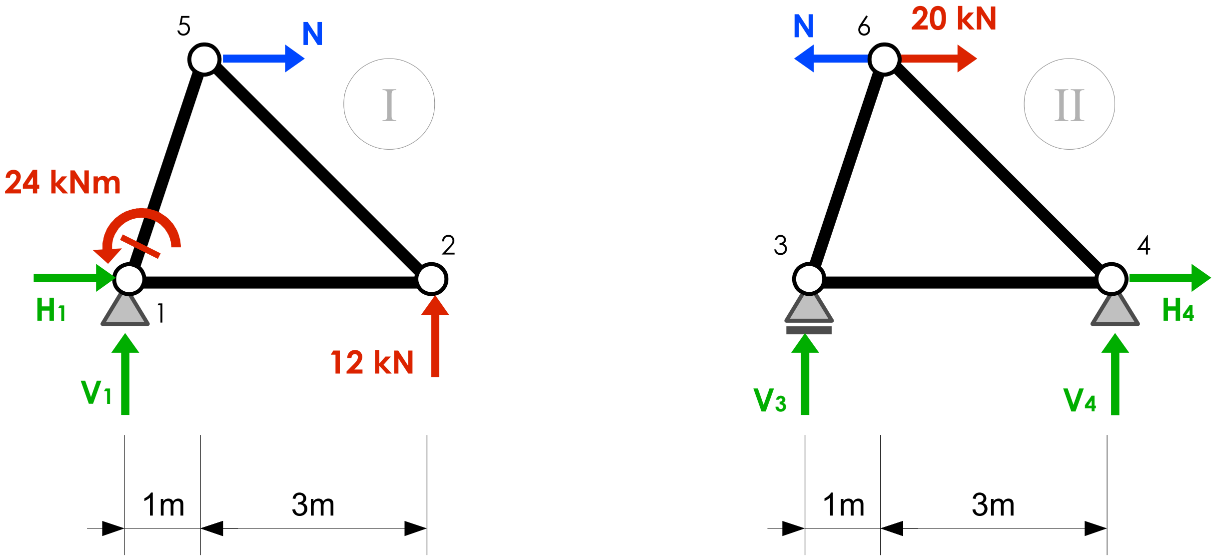
In order to find a force in bowstring we make a cut through the bowstring:



The force is determined from equilibrium of system of forces applied to any part of the structure. For example, for the left part:

$$\Sigma X=0: \quad H_1 + N = 0 \quad \Rightarrow \quad \boxed{N = 24}$$

In order to find reaction forces it was necessary to solve a system of equations. We may avoid solving such a system if we chose a different solution of the EXERCISE. We may cut through bar 5-6 at the very beginning. We get then two separate parts. If a whole mechanical system is in equilibrium then any separated part of it (with additional forces describing mutual interaction between parts) must be in equilibrium - three basic equilibrium equations (and possible additional equations due to joints) may be applied to each of the part separately. In our case deleting bar 5-6 results in that joints in points 5 and 6 to not allow for any rotation (they are included in a single rigid body), so they do not provide any additional equation.



Part I – 3 unknowns and 3 equilibrium equations:

$$\Sigma Y^{I} = 0: \quad V_1 + 12 = 0 \quad \Rightarrow \quad \boxed{V_1 = -12\,\text{kN}}$$

$$\Sigma M_1^{I} = 0: \quad 24 + 12\cdot 4 - N\cdot 3 = 0 \quad \Rightarrow \quad \boxed{N = 24\ \text{kN}}$$

$$\Sigma X^{I} = 0: \quad H_1 + N = 0 \quad \Rightarrow \quad \boxed{H_1 = -24\,\text{kN}}$$

Determined value of axial force in bar 5-6 is applied to part II of the structure:

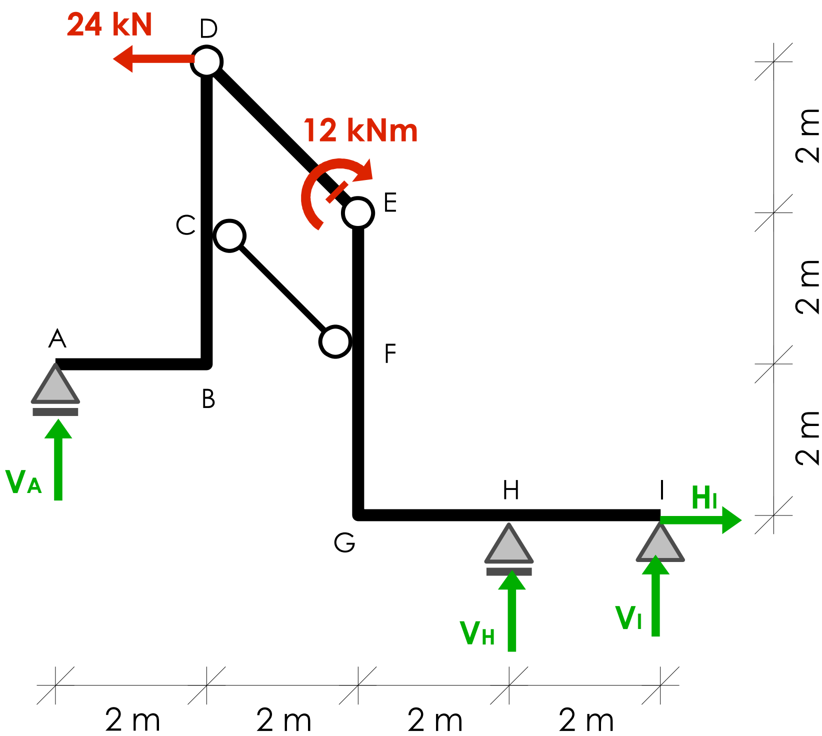
$$\Sigma X^{II} = 0: \quad H_4 + 20 - N = 0 \quad \Rightarrow \quad \boxed{H_4 = 4\,\text{kN}}$$

$$\Sigma M_4^{II} = 0: \quad -V_3\cdot 4 - 20\cdot 3 + N\cdot 3 = 0 \quad \Rightarrow \quad \boxed{V_3 = 3\ \text{kN}}$$

$$\Sigma Y^{II} = 0: \quad V_3 + V_4 = 0 \quad \Rightarrow \quad \boxed{V_4 = -3\ \text{kN}}$$

EXERCISE 26

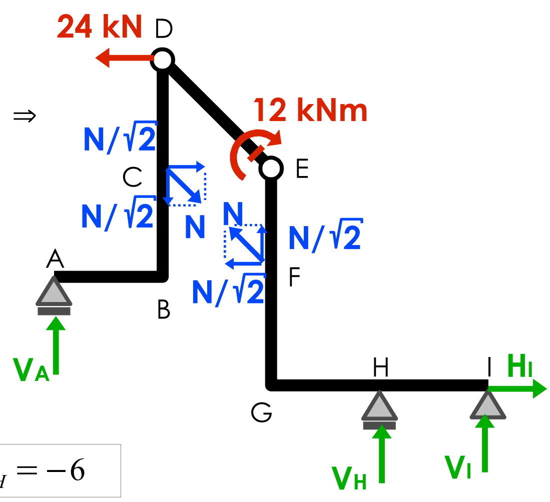
Find reactions at supports and a force in bowstring C-F.



$$\Sigma X=0: \quad -24+H_I=0 \quad \Rightarrow \quad \boxed{H_I=24}$$

In order to get additional equations (sum of moments due to load applied to one side of a joint) we will cut the structure through bowstring C-F:

$$\begin{cases}\Sigma M_D^{lewo}=0\\ \Sigma M_E^{lewo}=0\end{cases} \Rightarrow \begin{cases}+\frac{N}{\sqrt{2}}\cdot 2-V_A\cdot 2=0\\ -12+\frac{N}{\sqrt{2}}\cdot 2+24\cdot 2-V_A\cdot 4=0\end{cases} \Rightarrow$$



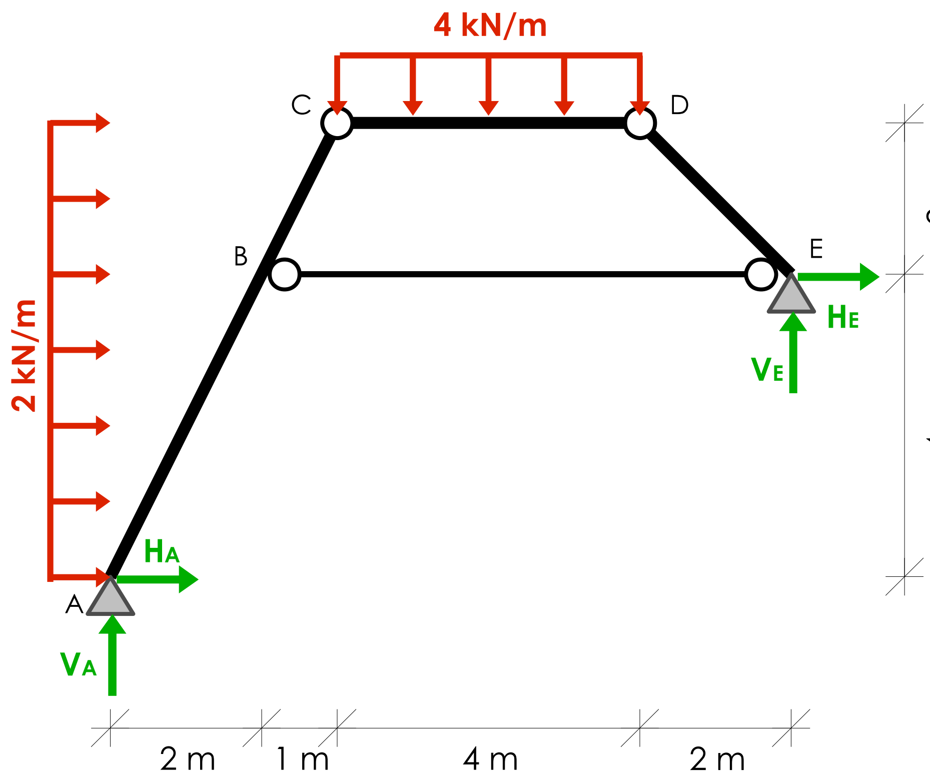
$$\Rightarrow \boxed{\begin{cases}V_A=18\\ N=18\sqrt{2}\end{cases}}$$

$$\Sigma M_I=0: \quad -V_H\cdot 2-12+24\cdot 6-V_A\cdot 8=0 \quad \Rightarrow \quad \boxed{V_H=-6}$$

$$\Sigma Y=0: \quad V_A+V_H+V_I=0 \quad \Rightarrow \quad \boxed{V_I=-12}$$

EXERCISE 27

Find reactions at supports and a force in bowstring B-E.

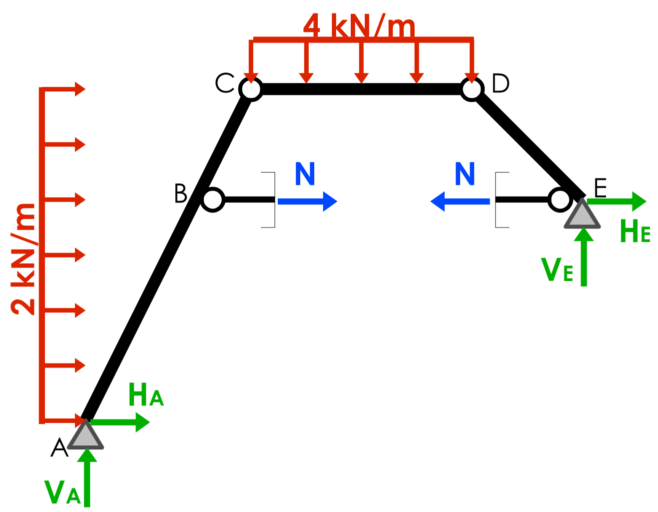


In order to get additional equations (sum of moments due to load applied to one side of a joint) we will cut the structure through bowstring B-E:

$$\begin{cases} \Sigma M_D^{prawo}=0 \\ \Sigma M_C^{prawo}=0 \\ \Sigma M_A=0 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} -N\cdot 2 + H_E\cdot 2 + V_E\cdot 2=0 \\ -N\cdot 2 + H_E\cdot 2 + V_E\cdot 6-4\cdot 4\cdot 2 = 0 \\ -H_E\cdot 4 + V_E\cdot 9 - 4\cdot 4\cdot 5 - 2\cdot 6\cdot 3 = 0 \end{cases}$$

$$\Rightarrow \boxed{\begin{cases} V_E=8 \\ H_E=-11 \\ N=-3 \end{cases}}$$

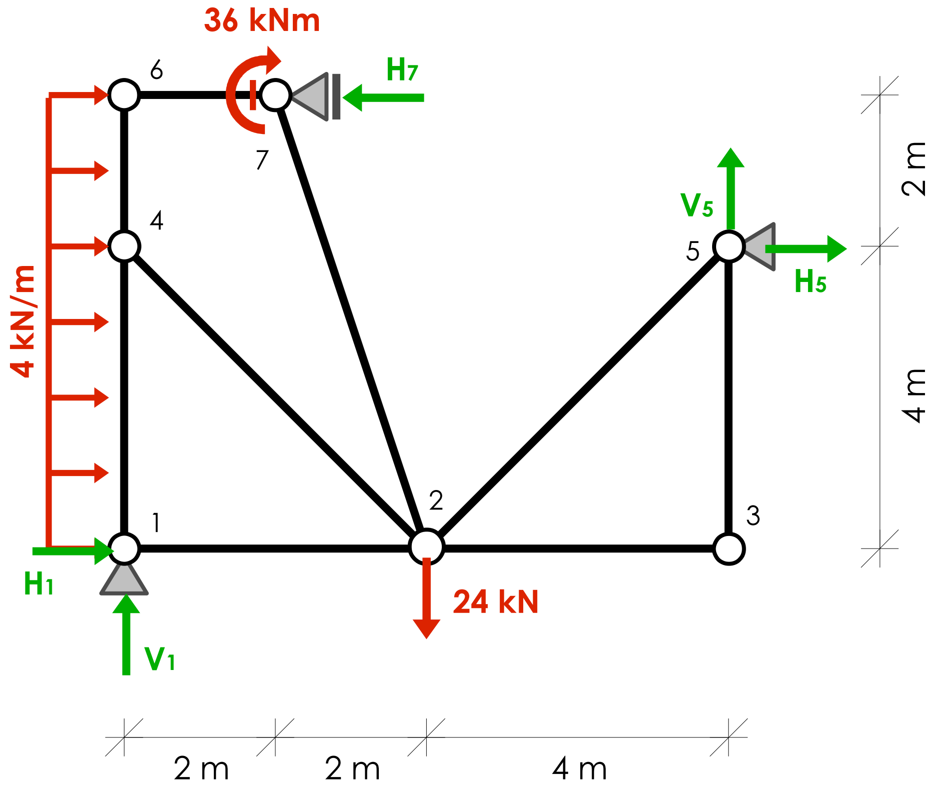


$$\Sigma X=0: \quad H_A + 2\cdot 6 + H_E = 0 \quad \Rightarrow \quad \boxed{H_A=-1}$$

$$\Sigma Y=0: \quad V_A - 4\cdot 4 + V_E = 0 \quad \Rightarrow \quad \boxed{V_A = 8}$$

EXERCISE 28

Find reactions at supports.



In order to get additional equations (sum of moments due to load applied to one side of a joint) we will cut the structure through bowstring 2-7:

$$\Sigma M_6^{prawo}=0:$$

$$-36-\frac{3N}{\sqrt{10}}\cdot 2=0 \quad \Rightarrow \quad \boxed{N=-6\sqrt{10}}$$

$$\Sigma M_4^{góra}=0:$$

$$-4\cdot 2\cdot 1-36-\frac{3N}{\sqrt{10}}\cdot 2-\frac{N}{\sqrt{10}}\cdot 2+H_7\cdot 2=0$$

$$\Rightarrow \boxed{H_7=-2}$$

$$\Sigma M_2^{lewo}=0:$$

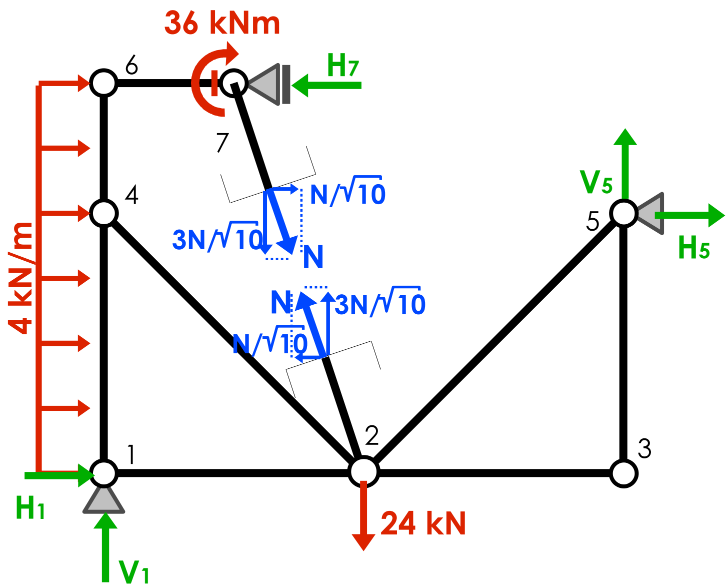
$$-V_1\cdot 4+H_7\cdot 6-36-4\cdot 6\cdot 3=0$$

$$\Rightarrow \boxed{V_1=-30}$$

$$\Sigma Y=0: \quad V_1-24+V_5=0 \quad \Rightarrow \quad \boxed{V_5=54}$$

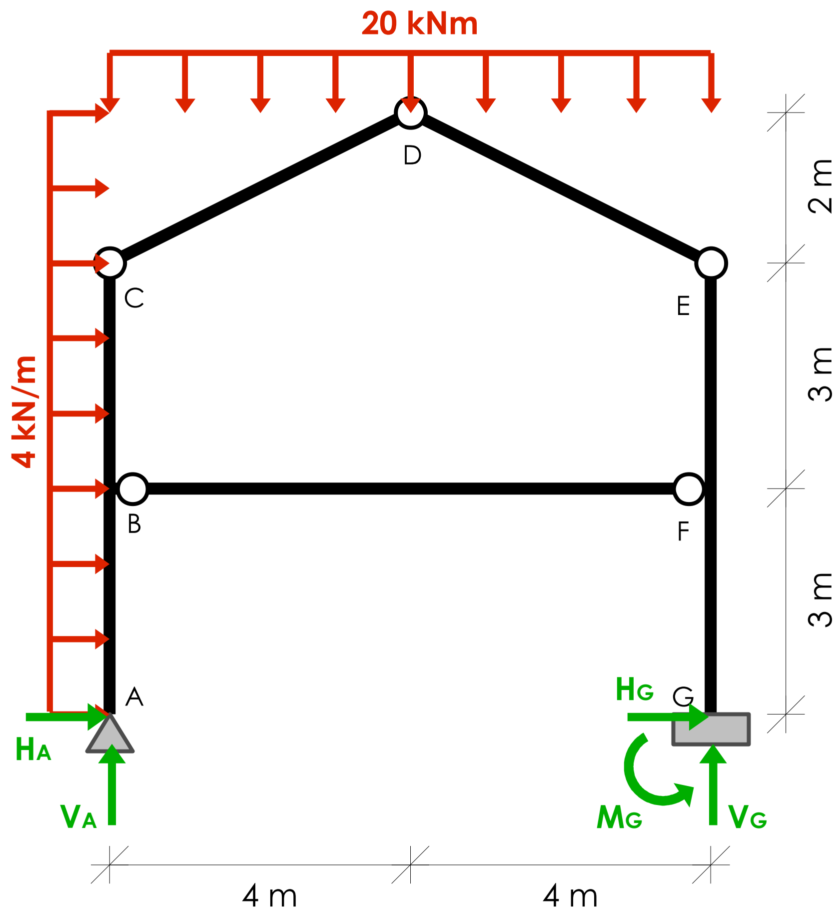
$$\Sigma M_2^{prawo}=0: \quad V_5\cdot 4-H_5\cdot 4=0 \quad \Rightarrow \quad \boxed{H_5=54}$$

$$\Sigma X=0: \quad -H_7+H_1+H_5+4\cdot 6=0 \quad \Rightarrow \quad \boxed{H_1=-80}$$



EXERCISE 29

Find reactions at supports.

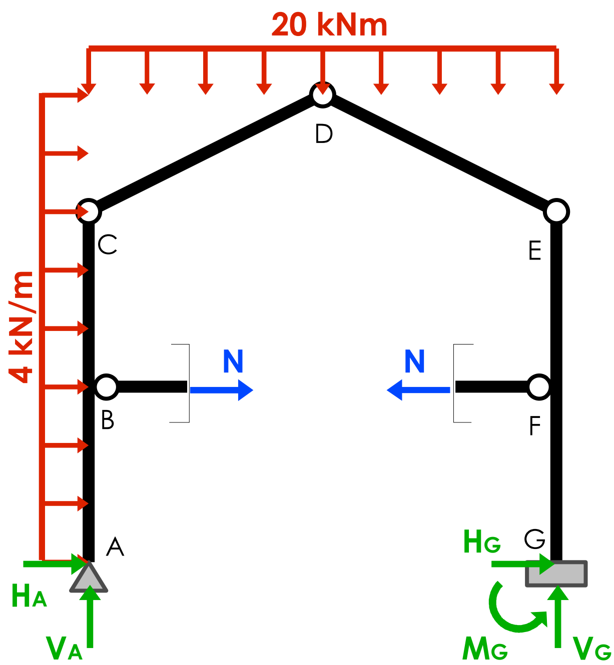


In order to get additional equations (sum of moments due to load applied to one side of a joint) we will cut the structure through bowstring B-F:

$$\begin{cases} \Sigma M_C^{dół}=0 \\ \Sigma M_D^{lewo}=0 \\ \Sigma M_E^{lewo}=0 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} +4\cdot 6\cdot 3+N\cdot 3+H_A\cdot 6=0 \\ +4\cdot 8\cdot 4+N\cdot 5+H_A\cdot 8-V_A\cdot 4+20\cdot 4\cdot 2=0 \\ +4\cdot 8\cdot 2+N\cdot 3+H_A\cdot 6-V_A\cdot 8+20\cdot 8\cdot 4=0 \end{cases}$$

$$\Rightarrow \boxed{\begin{cases} H_A=-74 \\ V_A=79 \\ N=124 \end{cases}}$$



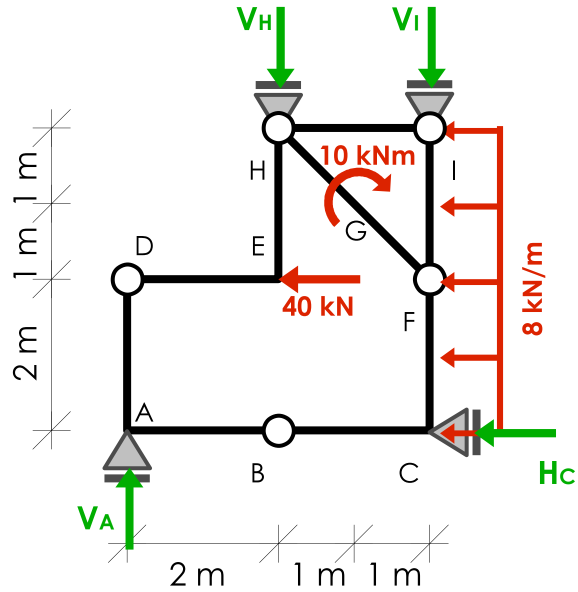
$$\Sigma X=0: \quad H_A+4\cdot 8+H_G=0 \quad \Rightarrow \quad \boxed{H_G=42}$$

$$\Sigma Y=0: \quad V_A-20\cdot 8+V_G=0 \quad \Rightarrow \quad \boxed{V_G=81}$$

$$\Sigma M_E^{dół}=0: \quad -N\cdot 3+H_G\cdot 6+M_G=0 \quad \Rightarrow \quad \boxed{M_G=120}$$

EXERCISE 30

Find reactions at supports.



$$\Sigma X = 0: \quad -40 - 8 \cdot 4 - H_C = 0 \quad \Rightarrow \quad \boxed{H_C = -72}$$

In order to get additional equations (sum of moments due to load applied to one side of a joint) we will cut the structure through joint B:

$$\begin{cases} \Sigma M_D^{dół} = 0 \\ \Sigma M_F^{dół} = 0 \end{cases} \Rightarrow \begin{cases} +H_B \cdot 2 - V_B \cdot 2 = 0 \\ -H_B \cdot 2 - V_B \cdot 2 - H_C \cdot 2 - 8 \cdot 2 \cdot 1 = 0 \end{cases} \Rightarrow$$

$$\Rightarrow \boxed{\begin{cases} H_B = 32 \\ V_B = 32 \end{cases}}$$

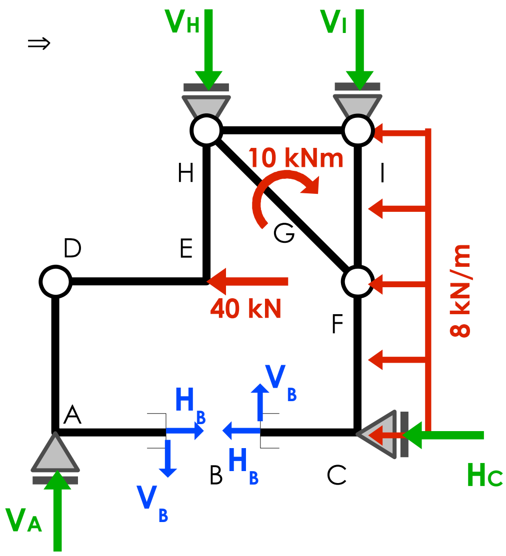
$$\Sigma M_H^{dół} = 0: \quad +N \cdot 4 - V_A \cdot 2 - 40 \cdot 2 = 0 \quad \Rightarrow \quad \boxed{V_A = 24}$$

$$\Sigma M_C = 0:$$

$$-V_A \cdot 4 + 40 \cdot 2 + 8 \cdot 4 \cdot 2 - 10 + V_H \cdot 2 = 0 \quad \Rightarrow$$

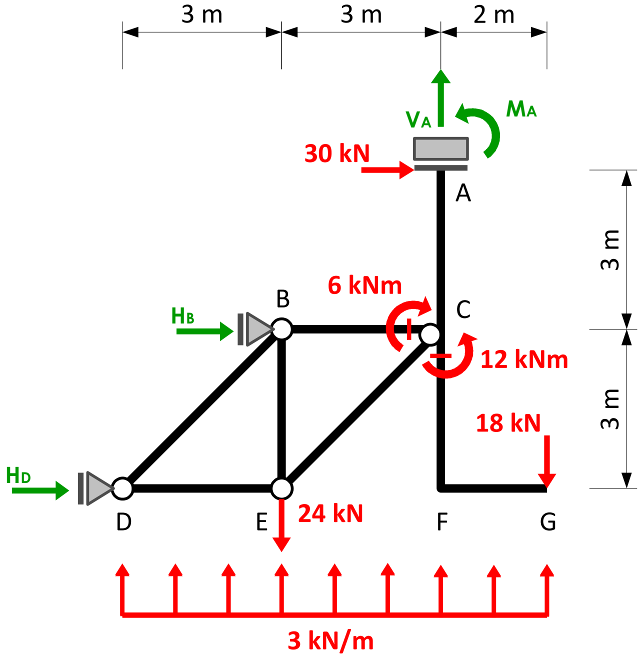
$$\Rightarrow \boxed{V_H = -19}$$

$$\Sigma Y = 0: \quad V_A - V_H - V_I = 0 \quad \Rightarrow \quad \boxed{V_I = 43}$$



EXERCISE 31

Find reactions with the use of equilibrium equations.



SOLUTION:

$$\Sigma M_C^{\rightarrow} = 0: \quad 12 + M_A - 30\cdot 3 - 18\cdot 2 + 3\cdot 2\cdot 1 = 0 \quad \Rightarrow \quad \boxed{M_A = 108 \text{ kNm}}$$

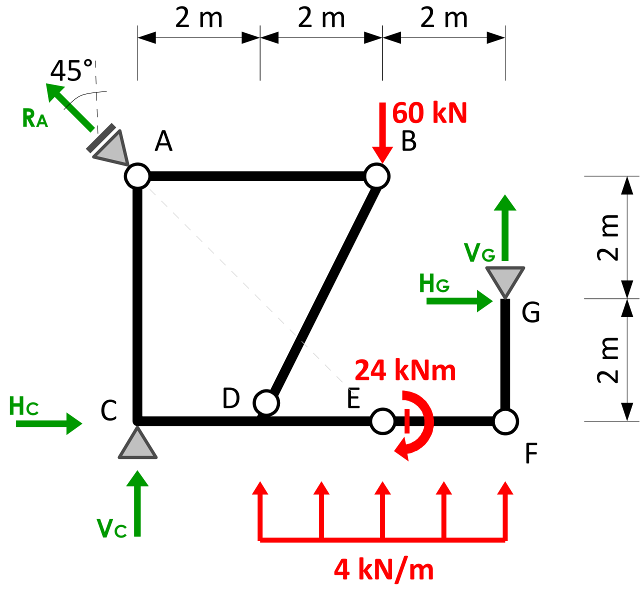
$$\Sigma Y = 0: \quad V_A - 18 - 24 + 3\cdot 8 = 0 \quad \Rightarrow \quad \boxed{V_A = 18 \text{ kN}}$$

$$\Sigma M_C^{\leftarrow} = 0: \quad -6 + 24\cdot 3 + H_D\cdot 3 - 3\cdot 6\cdot 3 = 0 \quad \Rightarrow \quad \boxed{H_D = -4 \text{ kN}}$$

$$\Sigma X = 0: \quad H_D + H_B + 30 = 0 \quad \Rightarrow \quad \boxed{H_B = -26 \text{ kN}}$$

EXERCISE 32

Find reactions with the use of equilibrium equations.



SOLUTION:

$$\Sigma M_F^{\uparrow} = 0: \quad \Rightarrow \quad \boxed{H_G = 0 \text{ kN}}$$

$$\Sigma M_E^{\rightarrow} = 0: \; -24 + 4{\cdot}2{\cdot}1 - H_G{\cdot}2 + V_G{\cdot}2 = 0 \quad \Rightarrow \quad \boxed{V_G = 8 \text{ kN}}$$

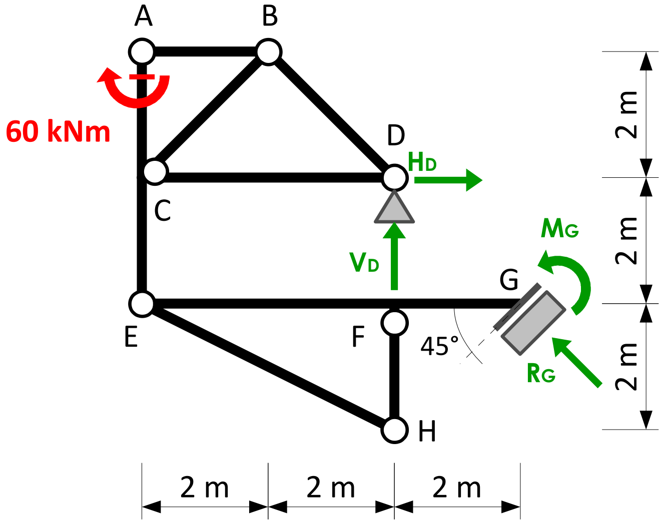
$$\Sigma M_E^{\leftarrow} = 0: \; -4{\cdot}2{\cdot}1 - V_C{\cdot}4 = 0 \quad \Rightarrow \quad \boxed{V_C = -2 \text{ kN}}$$

$$\Sigma Y = 0: \; V_C + V_G - 60 + 4{\cdot}4 + \frac{R_A}{\sqrt{2}} = 0 \quad \Rightarrow \quad \boxed{R_A = 38\sqrt{2} \text{ kN}}$$

$$\Sigma X = 0: \; H_C + H_G - \frac{R_A}{\sqrt{2}} = 0 \quad \Rightarrow \quad \boxed{H_C = 38 \text{ kN}}$$

EXERCISE 33

Find reactions with the use of equilibrium equations.



SOLUTION:

$$\Sigma M_D = 0: \; -60 + M_G = 0 \quad \Rightarrow \quad \boxed{M_G = 60 \text{ kNm}}$$

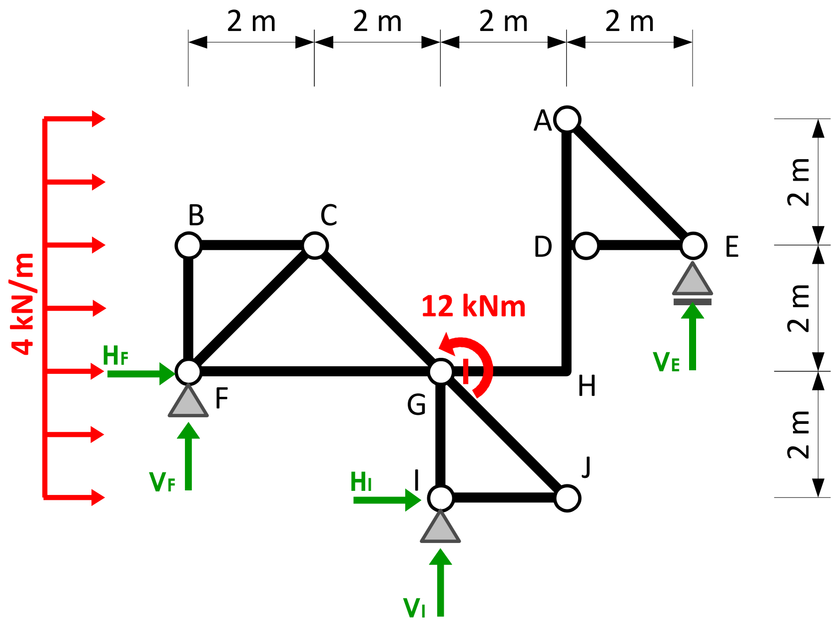
$$\Sigma M_E^{\rightarrow} = 0: \; M_G + \frac{R_G}{\sqrt{2}} \cdot 6 = 0 \quad \Rightarrow \quad \boxed{R_G = -10\sqrt{2} \text{ kN}}$$

$$\Sigma Y = 0: \; V_D + \frac{R_G}{\sqrt{2}} = 0 \quad \Rightarrow \quad \boxed{V_D = 10 \text{ kN}}$$

$$\Sigma X = 0: \; H_D - \frac{R_G}{\sqrt{2}} = 0 \quad \Rightarrow \quad \boxed{H_D = -10 \text{ kN}}$$

EXERCISE 34

Find reactions with the use of equilibrium equations.



SOLUTION:

$\Sigma M_G^{\downarrow} = 0: \ H_I \cdot 2 + 4 \cdot 2 \cdot 1 = 0 \quad \Rightarrow \quad \boxed{H_I = -4 \text{ kN}}$

$\Sigma X = 0: \ 4 \cdot 6 + H_F + H_I = 0 \quad \Rightarrow \quad \boxed{H_F = -20 \text{ kN}}$

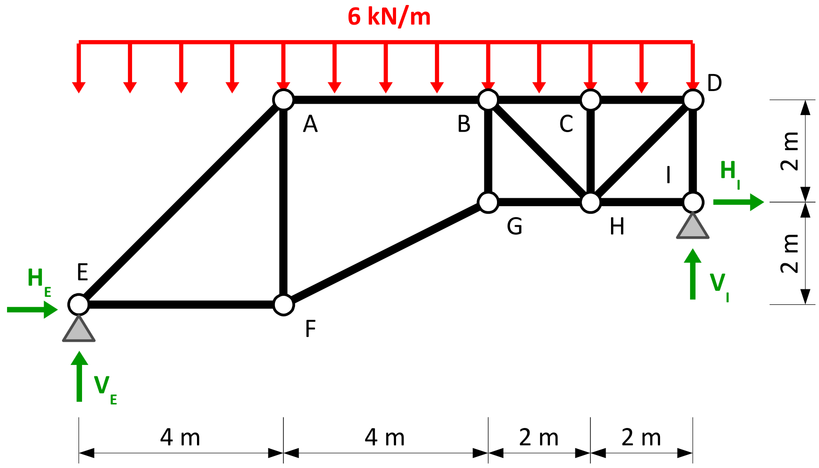
$\Sigma M^{\leftarrow} = 0: \ -V_F \cdot 4 - 4 \cdot 2 \cdot 1 = 0 \quad \Rightarrow \quad \boxed{V_F = -2 \text{ kN}}$

$\Sigma M^{\rightarrow} = 0: \ 12 + V_E \cdot 4 - 4 \cdot 2 \cdot 3 = 0 \quad \Rightarrow \quad \boxed{V_E = 3 \text{ kN}}$

$\Sigma Y = 0: \ V_F + V_I + V_E = 0 \quad \Rightarrow \quad \boxed{V_I = -1 \text{ kN}}$

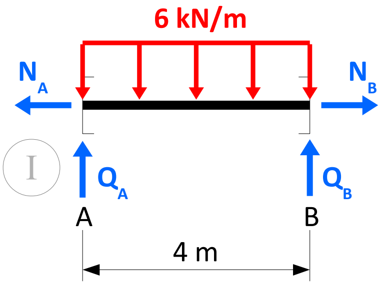
EXCERCISE 35

Find reactions at supports and axial force in bars AB and FG with the use of equilibrium equations.



SOLUTION:

In order to get required number of equations we have to perform a cut through structure. We may cut through joints A, B, F and G. Bar FG is a truss bar – only axial force occurs in it. Bar AB is a bent bar which is loaded transversally, so joints at its end bear also transverse force. We will determine it from equilibrium of bar AB:

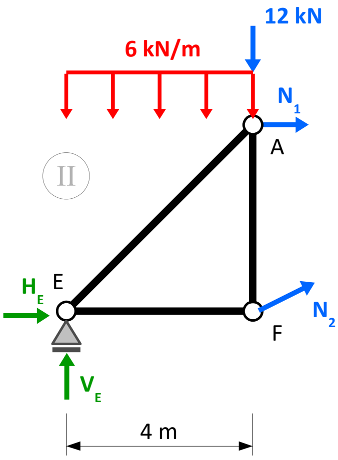


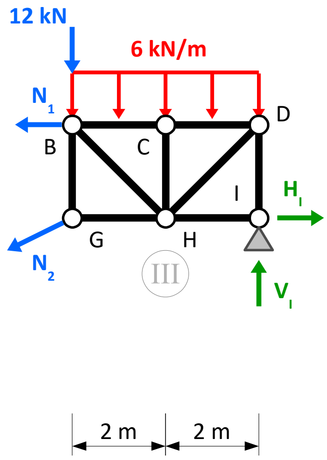
$$\Sigma X^I = 0: \quad \Rightarrow \quad N_A = N_B$$

$$\Sigma M_A^I = 0: \quad Q_B \cdot 4 - 6 \cdot 4 \cdot 2 = 0 \quad \Rightarrow \quad \boxed{Q_B = 12 \text{ kN}}$$

$$\Sigma M_B^I = 0: \quad -Q_A \cdot 4 + 6 \cdot 4 \cdot 2 = 0 \quad \Rightarrow \quad \boxed{Q_A = 12 \text{ kN}}$$

We may draw:





Sum of moment about D for the right part of structure gives us:

$$\Sigma M_D^{III} = 0: \quad 12 \cdot 4 + 6 \cdot 4 \cdot 2 + H_I \cdot 2 = 0 \qquad \Rightarrow \qquad \boxed{H_I = -48 \text{ kN}}$$

The rest of equations may be written down for the whole (not cutted) system:

$$\Sigma X = 0: \quad H_E + H_I = 0 \qquad \Rightarrow \qquad \boxed{H_E = 48 \text{ kN}}$$

$$\Sigma M_E = 0: \quad -6 \cdot 12 \cdot 6 + V_I \cdot 12 - H_I \cdot 2 = 0 \qquad \Rightarrow \qquad \boxed{V_I = 28 \text{ kN}}$$

$$\Sigma Y = 0: \quad V_E + V_I - 6 \cdot 12 = 0 \qquad \Rightarrow \qquad \boxed{V_E = 44 \text{ kN}}$$

Unknown axial forces may be found e.g. from equilibrium of the left part of structure:

$$\Sigma Y^{II}: \quad V_E + \frac{N_2}{\sqrt{5}} - 12 - 6 \cdot 4 = 0 \qquad \Rightarrow \qquad \boxed{N_2 = -8\sqrt{5} \text{ kN}}$$

$$\Sigma X^{II}: \quad H_E + \frac{2}{\sqrt{5}} N_2 + N_1 = 0 \qquad \Rightarrow \qquad \boxed{N_1 = -32 \text{ kN}}$$

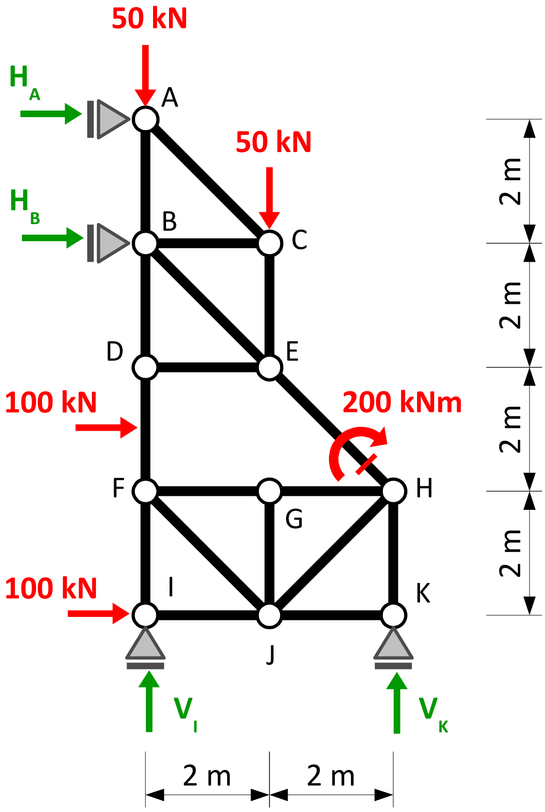
Check may be performed by writing down the sum of forces applied to the right part of structure:

$$\Sigma Y^{III} = -12 - 6 \cdot 4 - \frac{N_2}{\sqrt{5}} + V_I = -12 - 24 + 8 + 28 = 0$$

$$\Sigma X^{III} = -N_1 - \frac{2}{\sqrt{5}} N_2 + H_I = 32 + 16 - 48 = 0$$

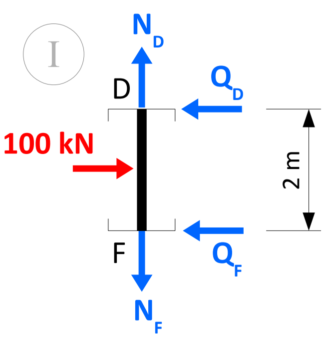
EXCERCISE 36

Find reactions at suports and axial force in bars EF and EH with the use of equilibrium equations.



SOLUTION:

In order to get required number of equations we may perform a cut through joints D, E, F and G. Bars DF and EG are loaded transversally so the joints at their ends bear both longitudinal and transverse forces. Transverse forces may be determined from equilibrium fo those bars.

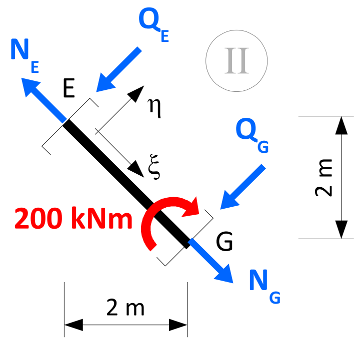


$$\Sigma Y^{I} = 0 \quad \Rightarrow \quad N_F = N_D$$

$$\Sigma M_D^{I} = 0: \ 100 \cdot 1 - Q_F \cdot 2 = 0 \quad \Rightarrow \quad \boxed{Q_F = 50 \text{ kN}}$$

$$\Sigma M_F^{I} = 0: \ -100 \cdot 1 + Q_D \cdot 2 = 0 \quad \Rightarrow \quad \boxed{Q_D = 50 \text{ kN}}$$

For oblique bars it is convenient to introduce a local coordinate system, one axis of which is parallel to the bar's axis.

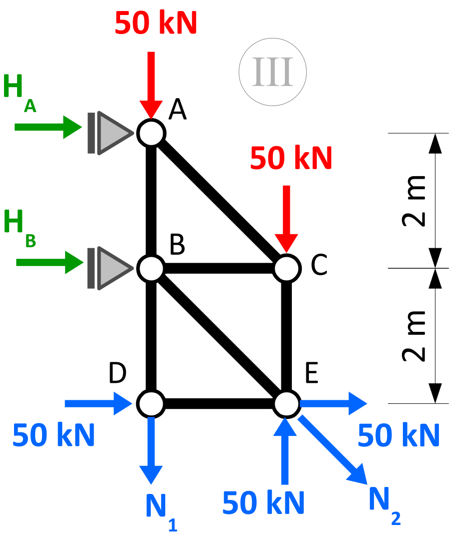


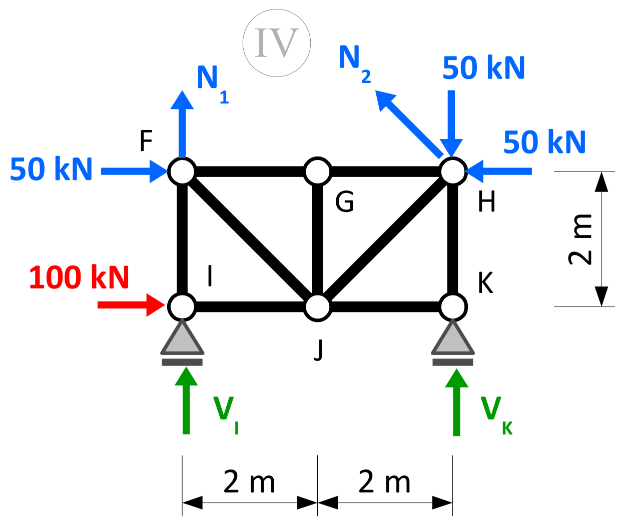
$$\Sigma \xi^{II} = 0 \quad \Rightarrow \quad N_E = N_G$$

$$\Sigma M_E^{II} = 0:\ -200 - Q_G \cdot 2\sqrt{2} = 0 \quad \Rightarrow \quad \boxed{Q_G = -50\sqrt{2}\ \text{kN}}$$

$$\Sigma M_G^{II} = 0:\ -200 + Q_E \cdot 2\sqrt{2} = 0 \quad \Rightarrow \quad \boxed{Q_E = 50\sqrt{2}\ \text{kN}}$$

Oblique foces may be decomposed into horizontal and vertical components:





Sum of moment about B for upper part of the structure:

$$\Sigma M_B^{III} = 0:\ -H_A \cdot 2 - 50 \cdot 2 + 50 \cdot 2 + 50 \cdot 2 + 50 \cdot 2 = 0 \quad \Rightarrow \quad \boxed{H_A = 100\ \text{kN}}$$

Sum of forces along X-axis for the whole structure:

$$\Sigma X = 0:\ H_A + H_B + 100 + 100 = 0 \quad \Rightarrow \quad \boxed{H_B = -300\ \text{kN}}$$

Sum of forces along X-axis for bottom part of structure:

$$\Sigma X^{IV} = 0:\ 100 + 50 - 50 - \frac{N_2}{\sqrt{2}} = 0 \quad \Rightarrow \quad \boxed{N_2 = 100\sqrt{2}\ \text{kN}}$$

Sum of forces along Y-axis for the upper part of the structure:

$$\Sigma Y^{III} = 0:\ -50 - 50 + 50 - N_1 - \frac{N_2}{\sqrt{2}} = 0 \quad \Rightarrow \quad \boxed{N_1 = -150\ \text{kN}}$$

Sum of moments about H for bottom part of the structure:

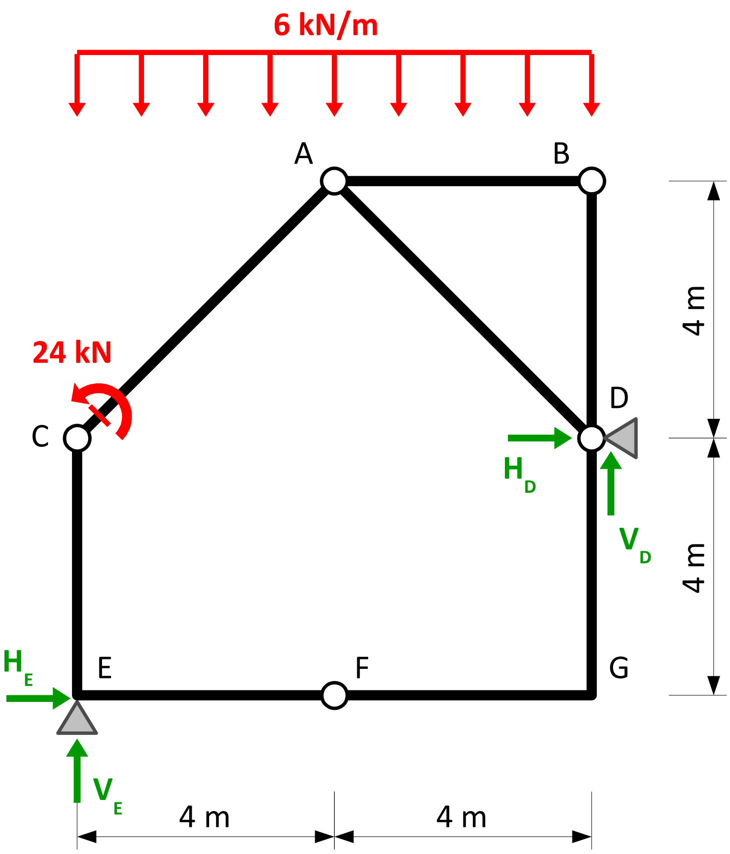
$$\Sigma M_H^{IV} = 0: \; -N_1 \cdot 4 + 100 \cdot 2 - V_I \cdot 4 = 0 \quad \Rightarrow \quad \boxed{V_I = 200 \text{ kN}}$$

Sum of forces along Y-axis for the bottom part of the structure:

$$\Sigma Y^{IV} = 0: \; N_1 + V_I + V_K + \frac{N_2}{\sqrt{2}} - 50 = 0 \quad \Rightarrow \quad \boxed{V_K = -100 \text{ kN}}$$

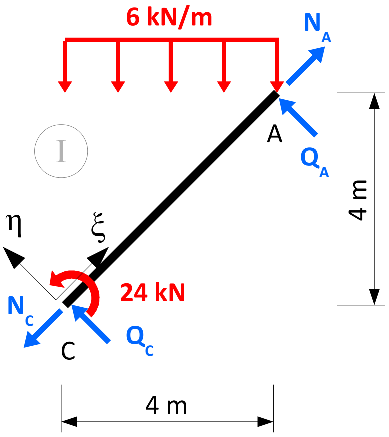
EXCERCISE 37

Find reactions at supports with the use of equilibrium equations.



SOLUTION:

We cut out bar AC:



$$\Sigma M_C^I = 0:$$

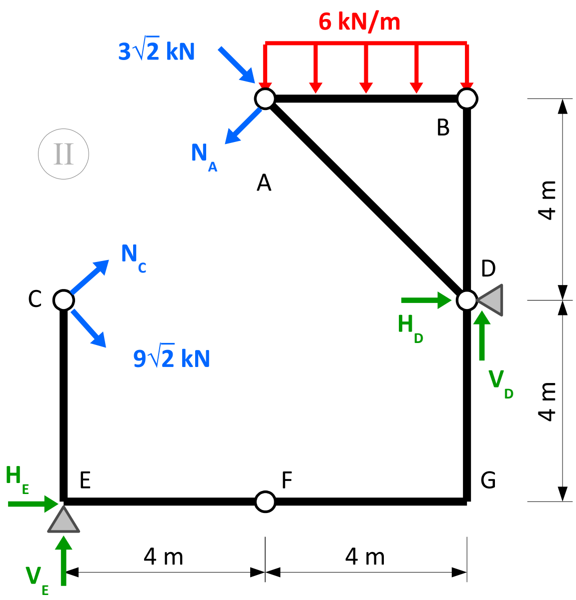
$$24 - 6 \cdot 4 \cdot 2 + Q_A \cdot 4\sqrt{2} = 0 \quad \Rightarrow \quad \boxed{Q_A = 3\sqrt{2}\ \text{kN}}$$

$$\Sigma M_A^I = 0:$$

$$24 + 6 \cdot 4 \cdot 2 - Q_C \cdot 4\sqrt{2} = 0 \quad \Rightarrow \quad \boxed{Q_C = 9\sqrt{2}\ \text{kN}}$$

$$\Sigma \xi^I = 0:$$

$$-N_C + N_A - \frac{6 \cdot 4 \cdot 1}{\sqrt{2}} = 0 \quad N_C = N_A - 12\sqrt{2}$$



$$\Sigma M_D^{II\uparrow} = 0:$$

$$6\cdot 4\cdot 2 + N_A\cdot 4\sqrt{2} = 0 \quad \Rightarrow \quad \boxed{N_A = -6\sqrt{2}\,\text{kN}}$$

$$\boxed{N_C = N_A - 12\sqrt{2} = -18\sqrt{2}\,\text{kN}}$$

$$\Sigma M_F^{II\leftarrow} = 0:$$

$$-V_E\cdot 4 - N_C\cdot 4\sqrt{2} = 0 \quad \Rightarrow \quad \boxed{V_E = 36\ \text{kN}}$$

The rest of equations may be written down for the whole structure:

$$\Sigma Y = 0:$$

$$V_E + V_D - 6\cdot 8 = 0 \quad \Rightarrow \quad \boxed{V_D = 12\ \text{kN}}$$

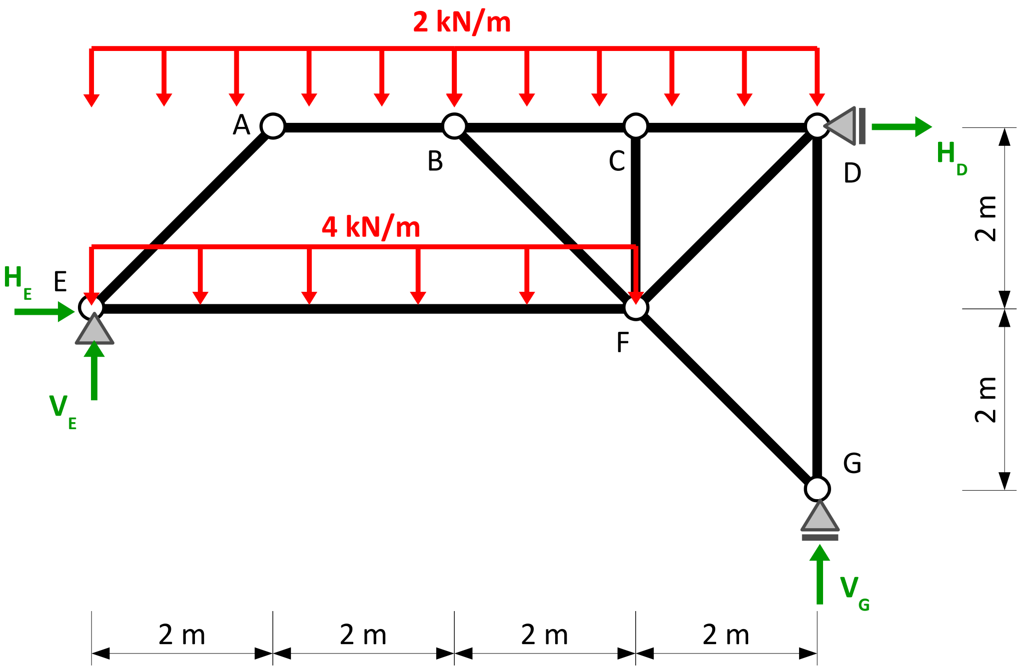
$$\Sigma M_E = 0:$$

$$-H_D\cdot 4 + V_D\cdot 8 + 24 - 6\cdot 8\cdot 4 = 0 \quad \Rightarrow \quad \boxed{H_D = -18\ \text{kN}}$$

$$\Sigma X = 0 \quad H_E + H_D = 0 \quad \Rightarrow \quad \boxed{H_E = 18\ \text{kN}}$$

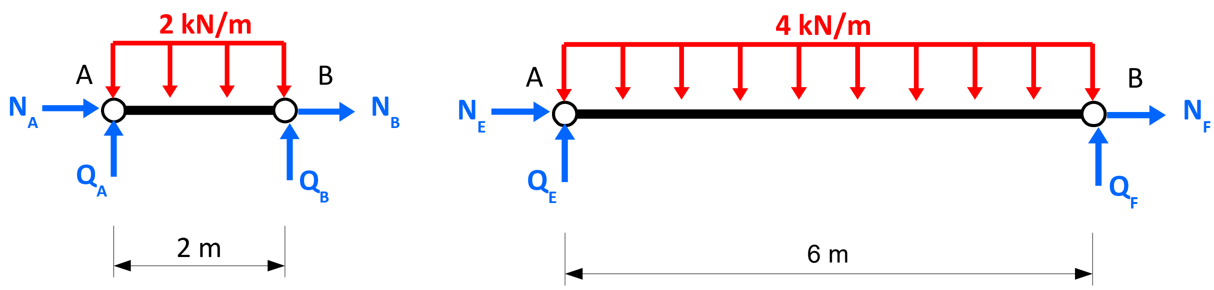
EXCERCISE 38

Find reactions at supports with the use of equilibrium equations.



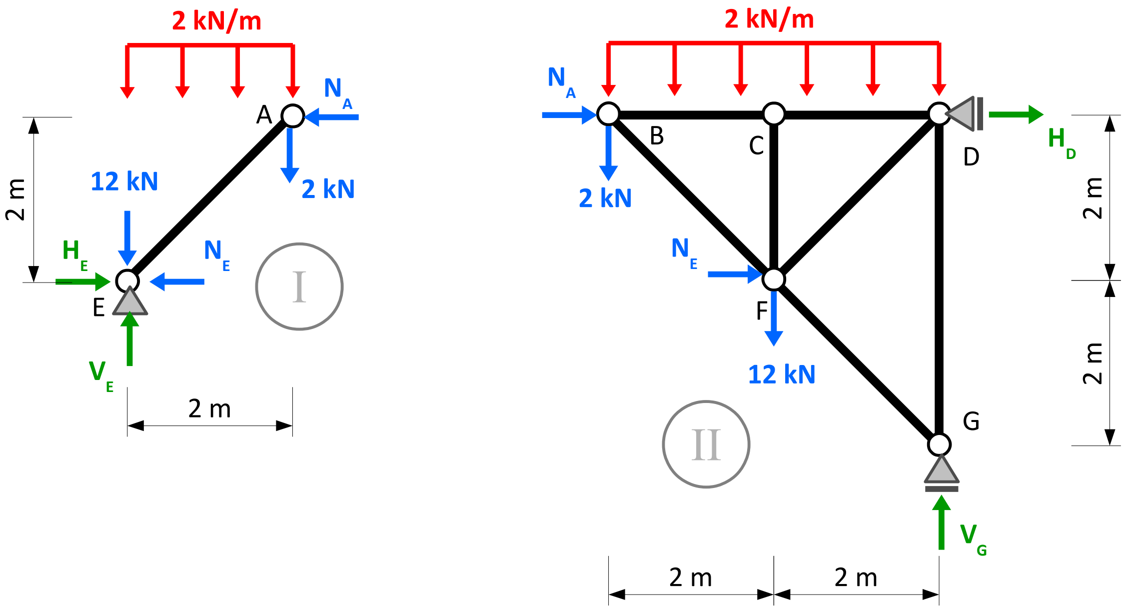
SOLUTION:

We will cut out bars AB and EF. Tansverse forces in joints A, B, E and F may be determined from equilibrium of those bars. In both cases the system of forces is symmetric, co finding unknown values is easy:



$$Q_A = Q_B = \frac{1}{2} \cdot 2 \cdot 2 = 2 \text{ kN}$$
$$N_B = -N_A$$

$$Q_E = Q_F = \frac{1}{2} \cdot 4 \cdot 6 = 12 \text{ kN}$$
$$N_F = -N_E$$



$$\Sigma Y^{I} = V_E - 12 - 2 - 2 \cdot 2 = 0 \quad \Rightarrow \quad \boxed{V_E = 18 \text{ kN}}$$

4 kN/m

$$\Sigma Y^{II} = V_G - 12 - 2 - 2 \cdot 4 = 0 \quad \Rightarrow \quad \boxed{V_G = 22 \text{ kN}}$$

$$\Sigma M_E^{I} = N_A \cdot 2 - 2 \cdot 2 - 2 \cdot 2 \cdot 1 = 0 \quad \Rightarrow \quad N_A = 4 \text{ kN}$$

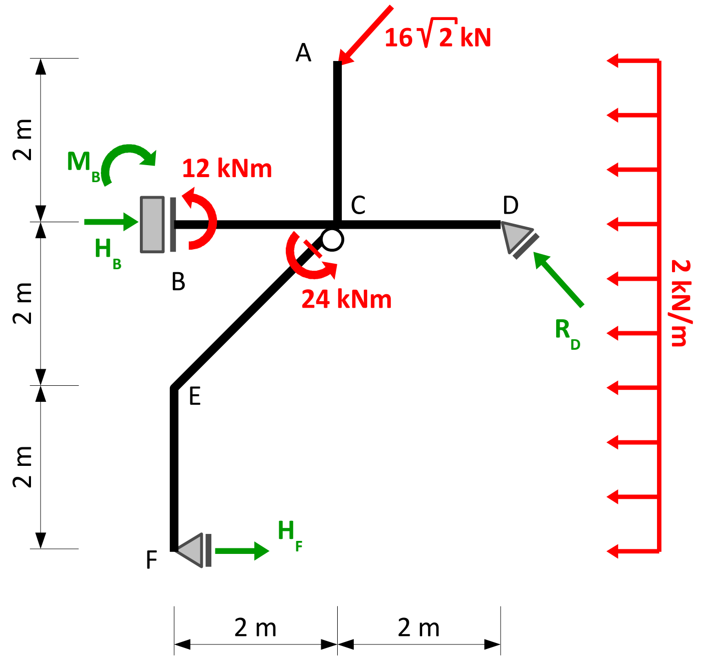
$$\Sigma M_D^{II} = N_E \cdot 2 + 2 \cdot 4 + 12 \cdot 2 + 2 \cdot 4 \cdot 2 = 0 \quad \Rightarrow \quad N_E = -24 \text{ kN}$$

$$\Sigma X^{I} = H_E - N_E - N_A = 0 \quad \Rightarrow \quad \boxed{H_E = -20 \text{ kN}}$$

$$\Sigma X^{II} = N_A + N_E + H_D = 0 \quad \Rightarrow \quad \boxed{H_D = 20 \text{ kN}}$$

EXCERCISE 39

Find reactions at supports by dividing the system into simple frames.

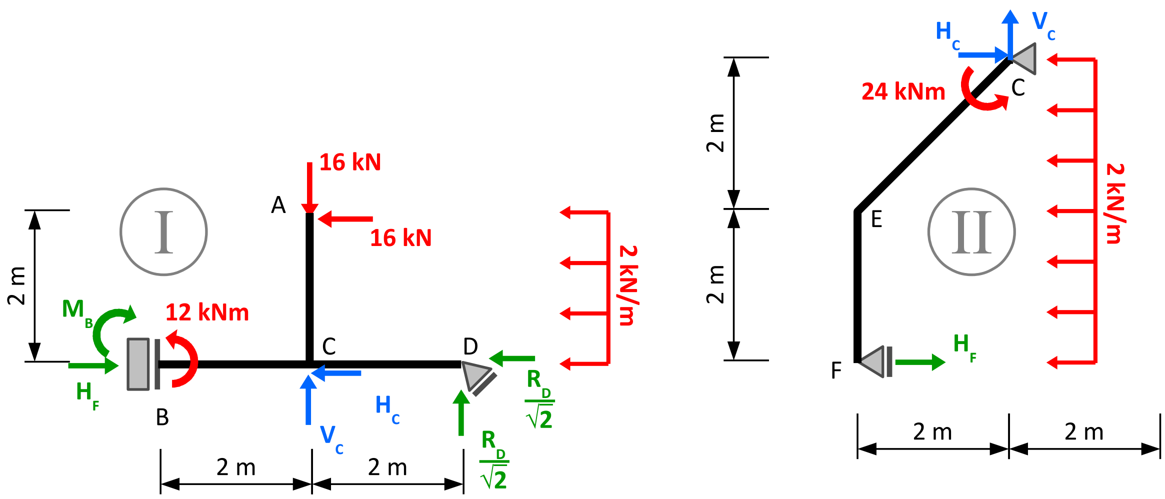


SOLUTION:

We may distinguish two simple frames in the system above:

- Frame ABCD of a shape of inverted „T"
- Frame FEC of a shape of bent bar

Frame ABCD has enough supports to remain in equilibrium. For this reason it constitutes a support for frame FEC – both frames are connected in a joint, which bears horizontal and vertical force. Those forces may be determined as reaction forces at a fictious support of frame FEC and they are treated as external load for frame ABCD.



Frame II:

$$\Sigma Y^{II} = V_C = 0 \quad \Rightarrow \quad V_C = 0 \text{ kN}$$

$$\Sigma M_C = 24 - 2{\cdot}4{\cdot}2 + H_F{\cdot}4 = 0 \quad \Rightarrow \quad H_F = -2 \text{ kN}$$

$$\Sigma X^{II} = H_F + H_C - 2{\cdot}4 = 0 \quad \Rightarrow \quad H_C = 10 \text{ kN}$$

Frame I:

$$\Sigma M_D = -V_C{\cdot}2 + 16{\cdot}2 + 16{\cdot}2 + 2{\cdot}2{\cdot}1 + 12 - M_B = 0 \quad \Rightarrow \quad M_B = 80 \text{ kNm}$$

$$\Sigma Y^{I} = -16 + V_C + \frac{R_D}{\sqrt{2}} = 0 \quad \Rightarrow \quad R_D = 16\sqrt{2} \text{ kN}$$

$$\Sigma X^{I} = H_F - 16 - H_C - \frac{R_D}{\sqrt{2}} - 2{\cdot}2 = 0 \quad \Rightarrow \quad H_F = 46 \text{ kN}$$