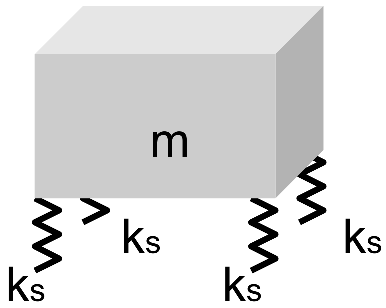
EXERCISE 1

A complex scientific apparatus which is meant to be launched into the space has mass equal 1000 kg and rests on four identical springs placed one by another. Stiffness of springs was examined in an terrestrial laboratory in Houston – dead load of this body resulted in 2cm displacement. A complex scientific apparatus was then placed in a satellite which was then raised to Low Earth Orbit (LEO) with the use of Ariane 4 rocket. Being in the state of weightlessness the apparatus was deflected from the equilibrium position with 5cm. Find the natural angular frequency, frequency and period, find and solve the equation of motion (free vibration) of this syste



SOLUTION:

- In order to determine the natural angluar frequency, stiffness of the system must be found.
- According to the Hooke's Law, stiffness k is defined as a proportionality coefficient between the acting force and resultant displacement

$$F = k\,x \quad \Rightarrow \quad k = \frac{F}{x} = const. \quad [k] = \frac{\mathrm{N}}{\mathrm{m}}$$

- In the dynamic analysis we use only global stiffness which can be found even without accounting for the fact that the springs work in a parallel manner:

Acting test force: $F = mg \approx 10000\ \mathrm{N}$
Test displacement: $x = 0{,}02\ \mathrm{m}$
Stiffness: $k = F/x = 500000\ \mathrm{N/m}$

- **Dynamical characteristics of the system**

Natural angular frequency: $\omega = \sqrt{k/m} \approx 22{,}36\ \mathrm{rad/s}$
Natural frequency: $\nu = \dfrac{\omega}{2\pi} \approx 3{,}58\ \mathrm{Hz}$
Natural period: $T = \dfrac{1}{\nu} \approx 0{,}28\ \mathrm{s}$

- **Equation of motion and its solution** – its right hand side is equal 0 since the body is in LEO and no gravity acts on it.

Equation: $m\ddot{x} + k\,x = 0 \quad \Rightarrow \quad 1000\,\ddot{x} + 500000\,x = 0$
Initial conditions: $x(t=0) = x_0 = 0{,}05\ \mathrm{m}$ - initial displacement
$\dot{x}(t=0) = v_0 = 0\ \mathrm{m/s}$ - initial velocity

General solution: $x = A\sin(\omega t) + B\cos(\omega t)$

Constants of integration are found with the use of initial conditions:

$$x = A\sin(\omega t) + B\cos(\omega t) \quad \Rightarrow \quad x(0) = B \quad \Rightarrow \quad B = 0{,}05$$
$$\dot{x} = A\omega\cos(\omega t) - B\omega\sin(\omega t) \quad \Rightarrow \quad \dot{x}(0) = A \quad \Rightarrow \quad A = 0$$

Particular solution: $x = 0{,}05 \cdot \sin(22{,}36\,t)$

EXERCISE 2

On a spring of stiffness 2000 N/m there is a body of mass 1 kg, which may perform horizontal vibration. A harmonic driving force was applied to the body. Its amplitude is 100 N and frequency is 7 Hz.

- What is the maximum amplitude of vibration of the body?
- How the natural angular frequency and maximum amplitude would change is damping was accounted for? Assume that logarithmic damping decrement is equal $\Delta = 0{,}1$.

SOLUTION:
Maximum amplitude is found with the use of the dynamic coefficient η . The coeficient determines how many times the maximum amplitude of steady vibration driven by harmonic force is greater than displacement due to static application of maximum value of the force (according to Hooke's Law):

$$\eta = \frac{x_{max}}{x_{st}} = \frac{\omega_0^2}{\sqrt{(\omega_0^2 - \lambda^2)^2 + 4\gamma^2\omega_0^2\lambda^2}}$$

Static displacement: $$x_{st} = \frac{P_0}{k} = \frac{100}{2000} = 0{,}05\,\mathrm{m}$$

Natural angular frequency: $$\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{2000}{1}} \approx 44{,}72\ \mathrm{rad/s}$$

UNDAMPED VIBRATION

- In case of undamped vibration we assume $\gamma = 0$: $\lim\limits_{\gamma \to 0} \eta = \dfrac{\omega_0^2}{|\omega_0^2 - \lambda^2|}$
- In the formulae above the frequency of driving force must be substituted in rad/s:

$$\lambda = 2\pi \cdot 7\ \mathrm{Hz} \approx 43{,}98\ \mathrm{rad/s}$$

Dynamic coefficient: $$\eta = \frac{\omega_0^2}{|\omega_0^2 - \lambda^2|} \approx \frac{44{,}72^2}{\sqrt{(44{,}72^2 - 43{,}98^2)^2}} \approx 30{,}46$$

Maximum amplitude: $$x_{max} = \eta x_{st} \approx 30{,}46 \cdot 0{,}05 = 1{,}523\,\mathrm{m}$$

NOTE: Frequency of the driving force is close to the eigenfrequency of the system, so the phenomenon of resonance occurs – obtained value of the dynamic coefficient is very high.

DAMPED VIBRATION

First, we have to find the dimensionless damping coefficient (**damping ratio**):

$$\Delta = \frac{2\pi\gamma}{\sqrt{1-\gamma^2}} \quad \Rightarrow \quad \gamma = \frac{\Delta}{\sqrt{\Delta^2 + 4\pi^2}} \approx 0{,}0159$$

Natural angular frequency of subcritically damped vibration (for which $\gamma < 1$ - only for such a situtation logarithmic decrement is defined) is equal:

$$\omega_1 = \omega_0\sqrt{1-\gamma^2} \approx 44{,}72\sqrt{1 - 0{,}0159^2} \approx 44{,}71\ \mathrm{rad/s}$$

Dynamic coefficient: $$\eta = \frac{\omega_0^2}{\sqrt{(\omega_0^2 - \lambda^2)^2 + 4\gamma^2\omega_0^2\lambda^2}} \approx \frac{44{,}72^2}{\sqrt{(44{,}72^2 - 43{,}98^2)^2 + 4\,0{,}0159^2 \cdot 44{,}72^2 \cdot 43{,}98^2}} \approx 22{,}06$$

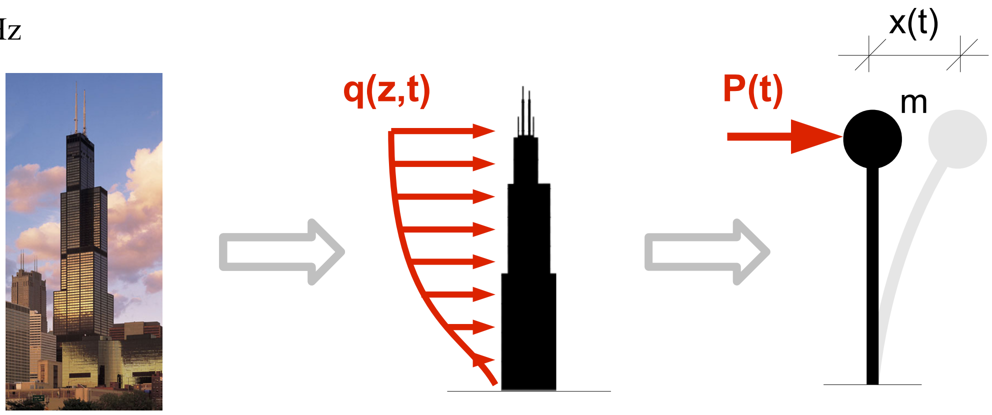
Maximum amplitude: $$x_{max} = \eta x_{st} \approx 22{,}06 \cdot 0{,}05 = 1{,}103\,\mathrm{m}$$

EXERCISE 3

Willis Tower in Chicago (formerly known as the Sears Tower) is a skyscraper of height (up to the top floor) 412,7m – it is the 13th highest freestanding structure in the world. Its total mass is equal ca. 200000 tonnes. Simplifying, the building may be modeled as a point mass at the top of an elastic cantilever. Let's assume that the wind load is replaced with a resultant applied to the point mass and that it is described with a function:

$$P(t) = P_0 \sin(\lambda t)$$

gdzie: $P_0 = 90000\,\mathrm{kN}$
$\lambda = 0{,}2\ \mathrm{Hz}$



Neglecting the damping and assuming that at maximum wind blows the building deflects with 15cm find the buildings stiffness and its natural angular frequency.

SOLUTION:
Stiffness of the building will be found by dividing the value of applied force with respective displacement. We know that for an elastic system of one degree of freedom with harmonic driving force the displacement is described with a function:

$$x(t) = A_{max} \sin(\lambda t)$$

Maximum amplitude is equal: $A_{max} = \eta x_{st} = \dfrac{\omega_0^2}{|\omega_0^2 - \lambda^2|} \cdot \dfrac{P_0}{k}$

Remembering that $\omega_0 = \sqrt{k/m}$ we may compare it with the maximum recorded value:

$$\frac{P_0}{m\left(\dfrac{k}{m} - \lambda^2\right)} = A_{max} = 0{,}15\ \mathrm{m} \qquad \Rightarrow \qquad k = \frac{P_0}{A_{max}} + \lambda^2 m$$

Hence:

Stiffness: $$k = \frac{P_0}{A_{max}} + \lambda^2 m = \frac{90000000}{0{,}15} + 1{,}257^2 \cdot 200000000 = 916009800\ \mathrm{N/m}$$

Natural angular frequency: $$\omega_0 = \sqrt{\frac{k}{m}} \approx 2{,}14\ \mathrm{rad/s} \qquad \Rightarrow \qquad \nu_0 = \frac{\omega_0}{2\pi} \approx 0{,}341\ \mathrm{Hz}$$

EXERCISE 4

A body of mass 5 kg is attached to a spring of stiffness 180 N/m and it may perform horizontal vibration. What is the amplitude of free vibration if in the initial time the body is deflected with 10cm from the equilibrium position and it was given velocity 3 m/s in a direction onwards from this position?

SOLUTION:

Natural angular frequency: $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{180}{5}} = 6 \text{ rad/s}$

General solution of free undamped vibration equation:

displacement: $x = A_1 \sin(\omega t) + A_2 \cos(\omega t)$

velocity: $\dot{x} = A_1 \omega \cos(\omega t) - A_2 \omega \sin(\omega t)$

Constants of integration are found with the use of initial conditions:

initial displacement: $x(0) = A_2 = x_0 = 0{,}1 \text{ m}$

initial velocity: $\dot{x}(0) = A_1 \omega = v_0 \quad \Rightarrow \quad A_1 = \frac{v_0}{\omega} = \frac{3}{6} = 0{,}5 \text{ m}$

Amplitude of vibration: $A = \sqrt{A_1^2 + A_2^2} \approx 0{,}51 \text{ m}$

EXERCISE 5

On a steel substructure in form of a simply supported beam of span length 4m, in the middle of the span there is a motor of total mass 1250 kg working at speed 3000 rpm. A mass rotating at eccentricity 5cm is equal 5% of total mass of the motor.

Total mass: $m = 1250\,\text{kg}$

Rotating mass: $m_w = 62{,}5\,\text{kg}$

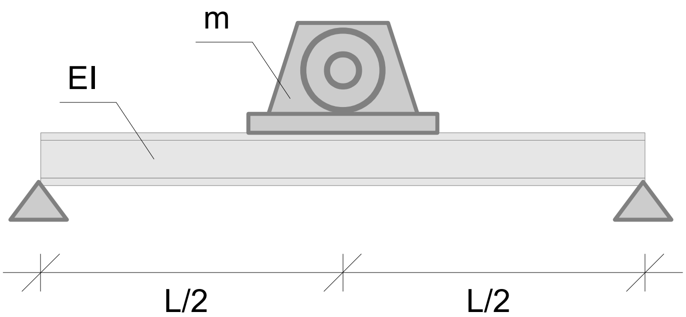
Rotation speed: $n = 3000\,\text{obr./min}$

Eccentricity: $R = 5\ \text{cm}$

Stiffness: $k = \dfrac{48\,EI}{L^3}$

Moment of inertia of beam section: $I = 3880\,\text{cm}^4$

Young modulus of steel: $E = 210\ \text{GPa}$



Estimate:

- Static deflection of substructure due to its self weight
- eigenfrequency of system
- Amplitude of steady vibration driven by motor

SOLUTION:

- **Static deflection**

Stiffness: $$k = \frac{48 \cdot 200 \cdot 10^9 \cdot 3880 \cdot 10^{-8}}{4^3} \approx 6111000\ \text{N/m}$$

Dead load: $$P_g = mg \approx 12500\ \text{N}$$

Static deflection due to dead load: $$A_g = \frac{P_{st}}{k} \approx 0{,}00205\ \text{m}$$

- **Eigenfrequency**

Natural angular frequncy: $$\omega = \sqrt{k/m} \approx 69{,}91\ \text{rad/s}$$

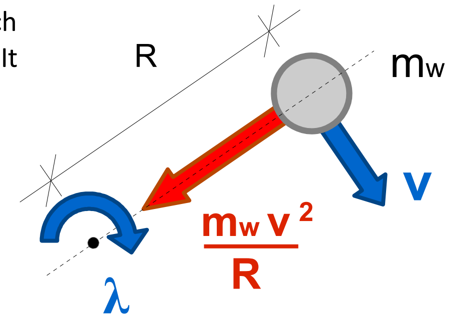
Eigenfrequency: $$\nu = \frac{\omega}{2\pi} \approx 11{,}13\ \text{Hz}$$

- ### Amplitude of steady driven vibration

Natural frequency of driving force: $$\lambda = \frac{n \cdot 2\pi}{60\,\mathrm{s}} = 314{,}16\ \frac{\mathrm{rad}}{\mathrm{s}}$$

Driving force transmited to the beam is a reaction on shaft's axis which holds the rotating body at constant distance from the center of rotation. It is equal the centripetal force:

$$P_0 = \frac{m_w v^2}{R}$$



Linear velocity v is determined assumping that the body rotates with constant angular velocity λ . Hence:

$$v = \lambda R \quad \Rightarrow \quad P_0 = m_w R \lambda^2$$

Driving force: $$P(t) = P_0 \cdot \sin(\lambda t)$$

Amplitude of driving force: $$P_0 = m_w R \lambda^2 = 62{,}5 \cdot 0{,}05 \cdot (314{,}16)^2 \approx 308425\ \mathrm{N}$$

Amplitude of driven vibration

$$A_{max} = \eta \cdot x_{st} = \frac{P_0}{\left|m(\omega^2 - \lambda^2)\right|} \approx \left|\frac{308425}{1250 \cdot \left[(69{,}91)^2 - (314{,}16)^2\right]}\right| = 0{,}00263\ \mathrm{m}$$

Amplitude of driven vibration is equal ca. 2,6 mm.

NOTE: Natural angular frequency is smaller the the angular frequency of driving force – during the starting phase of motor's work, when rotational speed increases, the system in fact works for a short time in resonance, until the motor speed raises sufficiently high. This requires a separate analysis and protection against possible damage.

EXERCISE 6

A machine of total mass 40kg rests on a rubber-like spacer of thickness h=10cm and dimensions 1 x 1m. Young modulus of the spacer is 10 kPa. The motor works at speed 600 rpm. Eccentric mass is equal 2 kg and it is with respect to rotation axis with 10 cm. Find the amplitude of driven vibration in two cases:

- neglecting the damping
- assuming the damping coefficient c = 800 Ns/m.

Spacer stiffness: k= EA/h. What is the critical value of damping coefficient?

SOLUTION:

Total vibrating mass: $m = 40 \text{ kg}$

Stiffness: $k = \frac{EA}{h} = \frac{10 \cdot 10^3 \cdot 1}{0{,}1} = 100000 \text{ N/m}$

Natural angular frequency: $\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{100000}{40}} = 50 \text{ rad/s}$

Angular frequency of driving force: $\lambda = \frac{n \cdot 2\pi}{60} = \frac{600 \cdot 2\pi}{60} \approx 62{,}83 \text{ rad/s}$

Amplitude of driving force: $P_0 = m_w \cdot r \cdot \lambda^2 = 2 \cdot 0{,}1 \cdot 62{,}83^2 \approx 790 \text{ N}$

Static deflection: $x_{st} = \frac{P_0}{k} = \frac{790}{100000} = 0{,}0079 \text{ m}$

1) NO DAMPING

Dynamic coefficient: $\eta = \frac{\omega_0^2}{\sqrt{(\omega_0^2 - \lambda^2)^2 + 4\gamma^2\omega_0^2\lambda^2}} = \frac{\omega_0^2}{|\omega_0^2 - \lambda^2|} = 1{,}727$

Amplitude of vibration: $x_{max} = \eta x_{st} = 0{,}0136 \text{ m}$

2) DAMPING

Damping coefficient: $c = 800 \text{ Ns/m}$

Critical value of Damping coefficient: $c_{kr} = 2\sqrt{k\,m} = 2\sqrt{100000 \cdot 40} = 4000 \text{ Ns/m}$

Damping ratio: $\gamma = \frac{c}{c_{kr}} = \frac{800}{4000} = 0{,}2$

Dynamic coefficient: $\eta = \frac{\omega_0^2}{\sqrt{(\omega_0^2 - \lambda^2)^2 + 4\gamma^2\omega_0^2\lambda^2}} = 1{,}30$

Amplitude of vibration: $x_{max} = \eta x_{st} = 0{,}0103 \text{ m}$

EXERCISE 7

In the middle of a room there is an industrial washing machine. The room is rectangular in plan and its dimensions are L x B = 6,4 x 4 m. Its floor is a reinforced-concrete slab 20 cm thick, clamped at edges. In order to reduce the vibration transmitted to the floor, a XPS polystyrene spacer of dimensions 1,5 m x 1,5 m was placed under the washing machine. The machine of total mass 350 kg, during spin-drying works at steady state at speed 1000 rpm. Eccentric mass is equal 30 kg and it is displaced with 10 cm.

- What is the natural angular frequency if the polystyrene spacer is 2cm thick?
- What is the amplitude of vibration in steady state spin-drying?
- What should be the thickness of the spacer so that the machine worked in a resonance when spin-drying?

Flexural stiffness of the RC floor: $k_1 = \dfrac{E_c t^3}{0{,}17\,B^2}$

Young modulus of concrete: $E_c = 32\ \text{GPa}$

Poisson ratio of concrete: $\nu_c = 0{,}2$

Longitudinal stiffness of spacer: $k_2 = \dfrac{E_s A}{h_s}$

Young modulus of polystyrene: $E_c = 10\ \text{kPa}$

Logarithmic damping decrement of polystyrene spacer: $\Delta = 0{,}1$

SOLUTION:

Natural frequency of driving force: $\lambda = \dfrac{n \cdot 2\pi}{60\text{s}} \approx 105\ \text{rad/s}$

Amplitude of driving force: $P_0 = m_w r \lambda^2 = 33075\ \text{N}$

- **Natural angular frequency:**

Stiffness of the slab: $k_1 = \dfrac{E_c t^3}{0{,}17\,B^2} = \dfrac{32\cdot 10^9 \cdot 0{,}2^3}{0{,}17\cdot 4^2} = 94117647\ \text{N/m}$

Stiffness of spacer: $k_2 = \dfrac{E_s A}{h_s} = \dfrac{10\cdot 10^3 \cdot (1{,}5)^2}{0{,}02} = 1125000\ \text{N/m}$

The spacer rests on the slab so it is a serial connection of springs. Resultant stiffness:

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} \quad \Rightarrow \quad k = \frac{k_1 k_2}{k_1 + k_2} \approx 1111712\ \text{N/m}$$

Damping ratio : $\gamma = \dfrac{\Delta}{\sqrt{\Delta^2 + 4\pi^2}} \approx 0{,}0159$

Natural angular frequency of undamped vibration: $\omega_0 = \sqrt{\dfrac{k}{m}} \approx 56{,}243\ \text{rad/s}$

Natural angular frequency of damped vibration: $\omega_1 = \omega_0 \sqrt{1-\gamma^2} \approx 56{,}236\ \text{rad/s}$

- **Amplitude**

Static deflection: $$x_{st} = \frac{P_0}{k} = \frac{33075}{1111712} = 0{,}03\ \mathrm{m}$$

Dynamic coefficient: $$\eta = \frac{\omega_0^2}{\sqrt{(\omega_0^2 - \lambda^2)^2 + 4\gamma^2\omega_0^2\lambda^2}} \approx 0{,}402$$

Amplitude: $$x_{max} = \eta x_{st} = 0{,}012\ \mathrm{m}$$

- **Resonance state:**

Dynamic coefficient reaches maximum value when the natural angular frequency is close to the angular frequency of driving force – precisely, when $\omega_0\sqrt{1-2\gamma^2} = \lambda$. Hence:

$$\sqrt{\frac{k}{m}} = \frac{\lambda}{\sqrt{1-2\gamma^2}} \quad \Rightarrow \quad k = \frac{\lambda^2 m}{\sqrt{1-2\gamma^2}} \quad \Rightarrow \quad \frac{k_1 k_2}{k_1 + k_2} = \frac{\lambda^2 m}{\sqrt{1-2\gamma^2}} \quad \Rightarrow \quad k_2 = \frac{\lambda^2 m k_1}{(k_1 - \lambda^2 m)\sqrt{1-2\gamma^2}}$$

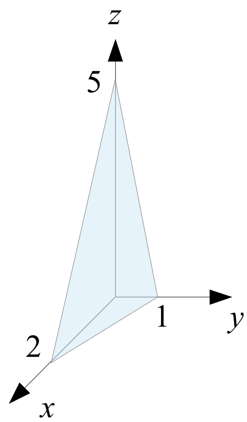
Finally:

$$h_s = \frac{E_s A (k_1 - \lambda^2 m)\sqrt{1-2\gamma^2}}{\lambda^2 m k_1} = \frac{10\cdot 10^3\cdot(1{,}5)^2\cdot(94117647 - 105^2\cdot 350)\sqrt{1-2\cdot 0{,}0159^2}}{105^2\cdot 350\cdot 94117647} \approx 0{,}0055\,\mathrm{m}$$

The washing machine will work in a resonance if the spacer is 5,5 mm thick.

EXERCISE 8

Material point of mass m may perform a motion on a plane shown in the figure. It is subject of uniform field of gravitational force. Find a trajectory of this point knowing that in the initial moment it is located in point (4,-2,5) and it has zero velocity.



SOLUTION:

Plane equation: $f: \ \dfrac{x}{2} + \dfrac{y}{1} + \dfrac{z}{5} = 1$

Canonical form of equation: $f: \ 5x + 10y + 2z - 10 = 0$

Reaction force that holds the point on the plane is always oriented perpendicularly to that plane. Reaction force vector is thus proportional to the gradient of the function describing that plane.

Reaction force: $\mathbf{R} = \alpha \cdot \mathrm{grad}\, f = \alpha \left[\dfrac{\partial f}{\partial x}, \dfrac{\partial f}{\partial y}, \dfrac{\partial f}{\partial z} \right] = [5\alpha, 10\alpha, 2\alpha]$

Gravity force: $\mathbf{G} = [0, 0, -mg]$

Total force: $\mathbf{F} = \mathbf{R} + \mathbf{G}$

Equations of motion are determined with the use of Newtons 2nd principle of motion:

$$\frac{\mathrm{d}}{\mathrm{d}t}(m\dot{\mathbf{r}}) = \mathbf{F} \quad \Rightarrow \quad \begin{cases} m\ddot{x} = 5\alpha \\ m\ddot{y} = 10\alpha \\ m\ddot{z} = 2\alpha - mg \end{cases}$$

Unknown value of parameter α will be found by double differentiation of the equation of plane with respect to time and by expressing the obtained accelerations with the use of the above relations:

$$\frac{\mathrm{d}f}{\mathrm{d}t}: \ 5\dot{x} + 10\dot{y} + 2\dot{z} = 0$$

$$\frac{\mathrm{d}^2 f}{\mathrm{d}t^2}: \ 5\ddot{x} + 10\ddot{y} + 2\ddot{z} = 0 \quad \Rightarrow \quad 25\alpha + 100\alpha + 4\alpha - 2mg = 0 \quad \Rightarrow \quad \alpha = \frac{2mg}{129}$$

Obtained relation is substituted into the equations of motion which are then integrated:

$$\begin{cases} \ddot{x} = \dfrac{10g}{129} \\ \ddot{y} = \dfrac{20g}{129} \\ \ddot{z} = \dfrac{4g}{129} - g = -\dfrac{125}{129}g \end{cases} \Rightarrow \begin{cases} \dot{x} = \dfrac{10}{129}gt + A_1 \\ \dot{y} = \dfrac{20}{129}gt + B_1 \\ \dot{z} = -\dfrac{125}{129}gt + C_1 \end{cases} \Rightarrow \begin{cases} x = \dfrac{5}{129}gt^2 + A_1 t + A_2 \\ y = \dfrac{10}{129}gt^2 + B_1 t + B_2 \\ z = -\dfrac{125}{258}gt^2 + C_1 t + C_2 \end{cases}$$

Constants of integration are determined with the use of initial conditions:

$$\mathbf{r}(t=0) = [4, -2, 5] \quad \Rightarrow \quad A_2 = 4, \ B_2 = -2, \ C_2 = 5$$

$$\dot{\mathbf{r}}(t=0) = [0, 0, 0] \quad \Rightarrow \quad A_1 = 0, \ B_1 = 0, \ C_1 = 0$$

Finally:

$$\begin{cases} x = \dfrac{5}{129}gt^2 + 4 \\ y = \dfrac{10}{129}gt^2 - 2 \\ z = -\dfrac{125}{258}gt^2 + 5 \end{cases}$$

EXERCISE 9

A point of mass *m* is placed in the uniform field of gravitational forces. At the initial moment it is placed at height H_1 and it moves downwards with velocity v_1. Find the velocity of the point when it is at height $H_2 < H_1$. With the use of law of conservation of energy.

SOLUTION:

Total mechanical energy at initial moment: $E = m g H_1 + \frac{m v_1^2}{2}$

Total mechanical energy at terminal moment: $E = m g H_2 + \frac{m v_2^2}{2}$

Conservation of energy:

$$m g H_1 + \frac{m v_1^2}{2} = m g H_2 + \frac{m v_2^2}{2}$$

$$2 g H_1 + v_1^2 = 2 g H_2 + v_2^2$$

$$v_2 = \sqrt{v_1^2 + 2 g (H_1 - H_2)}$$

EXERCISE 10

A point of mass *m* moves in an uniform gravitational field along a curve given by euqation

a) $y=4x^3+5$

b) $y=2x^3-3x^2-36x$

Find the location of equilibrium points for those trajectories and determine their character

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SOLUTION:

a) **Generalized coordinate**: $q = x$

Potential energy: $E_P = mgy = mg(4q^3+5)$

Minimum of potential energy: $\frac{\mathrm{d}E_p}{\mathrm{d}q} = mg(12q^2) = 0 \quad \Rightarrow \quad q_0 = 0$

Character of equilibrium point: $\left.\frac{\mathrm{d}^2E_p}{\mathrm{d}q^2}\right|_{q_0} = mg(24q)|_{q_0} = 0 \quad \Rightarrow$ **neutral equilibrium**

b) **Generalized coordinate**: $q = x$

Potential energy: $E_P = mgy = mg(2q^3-3q^2-36q)$

Minimum of potential energy: $\frac{\mathrm{d}E_p}{\mathrm{d}q} = 6mg(q^2-q-6) = 0 \quad \Rightarrow \quad q_0 = -2 \ \vee \ q_0 = 3$

Character of equilibrium point:

$$\left.\frac{\mathrm{d}^2E_p}{\mathrm{d}q^2}\right|_{q=-2} = 6mg(2q-1)|_{q=-2} = -30\,mg < 0 \quad \Rightarrow$$ **unstable equilibrium**

$$\left.\frac{\mathrm{d}^2E_p}{\mathrm{d}q^2}\right|_{q=3} = 6mg(2q-1)|_{q=3} = 30\,mg > 0 \quad \Rightarrow$$ **stable equilibrium**