

DEFORMATION EQUATIONS

$$x_1 := 50 \cdot X_1 + 55 \cdot X_2$$

$$x_2 := (-25) \cdot X_1 + 10 \cdot X_2$$

DEFORMATION GRADIENT

$$F := \begin{bmatrix} \frac{d}{d X_1} x_1 & \frac{d}{d X_2} x_1 \\ \frac{d}{d X_1} x_2 & \frac{d}{d X_2} x_2 \end{bmatrix} = \begin{bmatrix} 50 & 55 \\ -25 & 10 \end{bmatrix} \quad |F| = 1875$$

DEFORMATION TENSOR

$$C := F^T \cdot F = \begin{bmatrix} 3125 & 2500 \\ 2500 & 3125 \end{bmatrix}$$

EIGENVALUES OF DEFORMATION TENSOR

$$c_1 := \frac{C_{11} + C_{22}}{2} + \sqrt{\left(\frac{C_{11} - C_{22}}{2}\right)^2 + (C_{12})^2} = 5625 \quad c_2 := \frac{C_{11} + C_{22}}{2} - \sqrt{\left(\frac{C_{11} - C_{22}}{2}\right)^2 + (C_{12})^2} = 625$$

EIGENVECTORS

$$\varphi := \arctg\left(\frac{C_{12}}{c_1 - C_{22}}\right) = 45 \text{ deg}$$

$$v_1 := [\cos(\varphi) \quad \sin(\varphi)] = [0,7071 \quad 0,7071]$$

$$v_2 := [-\sin(\varphi) \quad \cos(\varphi)] = [-0,7071 \quad 0,7071]$$

TRANSFORMATION MATRIX

$$A := \begin{bmatrix} v_1 & v_2 \\ v_2 & v_1 \end{bmatrix} = \begin{bmatrix} 0,7071 & 0,7071 \\ -0,7071 & 0,7071 \end{bmatrix}$$

**DEFORMATION TENSOR
IN EIGENAXES**

$$\begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix} = \begin{bmatrix} 5625 & 0 \\ 0 & 625 \end{bmatrix}$$

**STRETCH TENSOR
IN EIGENAXES**

$$U := \begin{bmatrix} \sqrt{c_1} & 0 \\ 0 & \sqrt{c_2} \end{bmatrix} = \begin{bmatrix} 75 & 0 \\ 0 & 25 \end{bmatrix}$$

**INVERSE OF STRETCH TENSOR
IN EIGENAXES**

$$iU := \begin{bmatrix} \frac{1}{\sqrt{c_1}} & 0 \\ 0 & \frac{1}{\sqrt{c_2}} \end{bmatrix} = \begin{bmatrix} 0,0133 & 0 \\ 0 & 0,04 \end{bmatrix}$$

**STRETCH TENSOR
IN ORIGINAL COORINATE SYSTEM**

$$U := A^T \cdot U \cdot A = \begin{bmatrix} 50 & 25 \\ 25 & 50 \end{bmatrix}$$

**INVERSE OF STRETCH TENSOR
IN ORIGINAL COORINATE SYSTEM**

$$iU := A^T \cdot iU \cdot A = \begin{bmatrix} 0,0267 & -0,0133 \\ -0,0133 & 0,0267 \end{bmatrix}$$

$$\text{ROTATION TENSOR} \quad R := F \cdot iU = \begin{bmatrix} 0,6 & 0,8 \\ -0,8 & 0,6 \end{bmatrix}$$

CHECK

$$|R| = 1 \quad U^T - U = \begin{bmatrix} -1,2506 \cdot 10^{-13} & 0 \\ 0 & -1,2506 \cdot 10^{-13} \end{bmatrix} \quad F - R \cdot U = \begin{bmatrix} 1,8993 \cdot 10^{-13} & 3,0437 \cdot 10^{-13} \\ 2,435 \cdot 10^{-14} & 4,0583 \cdot 10^{-14} \end{bmatrix}$$