Yielding criteria

Basic formulae for the substitute stress

Coulomb-Tresca-Guest hypothesis

$$\sigma_{CTG} = \sqrt{\sigma^2 + 4\tau^2}$$

Huber-Mises-Hencky hypothesis

$$\sigma_{HMH} = \sqrt{\sigma^2 + 3\tau^2}$$

Coulomb-Mohr hypothesis

$$\sigma_{CM} = (1-k)\frac{\sigma}{2} + (1+k)\frac{1}{2}\sqrt{(\sigma)^2 + 4\tau^2}$$

Scheme of solution

1. The cross-sectional characteristics
2. The cross-sectional forces
3. Matrix of stress at the point.
4. The substitute stress formula

Examples

Example 13.1

Design the dimensions of the column cross section, provided that R = 150 MPa, q = 0.5 kN/m, P = 20 kN, l = 2m. Determine the substitute stress at the point K using the hypothesis of Huber-Mises-Hencky.

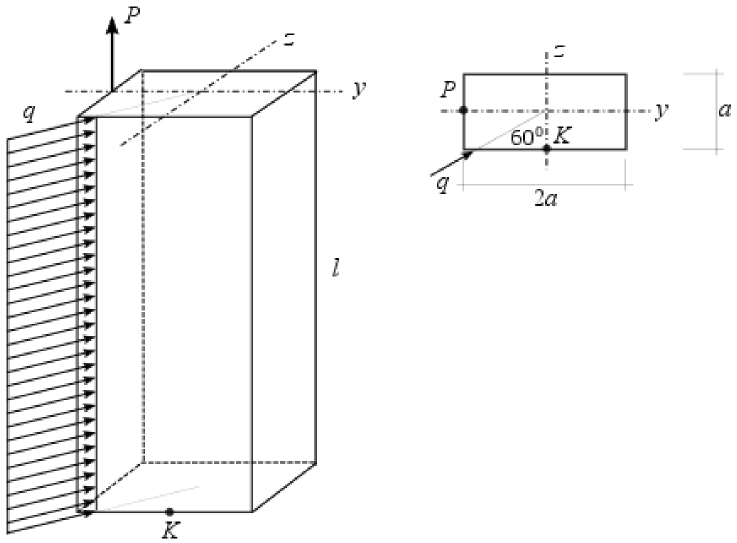

Fig. 13.1 Column with load

Solution

cross-sectional characteristics

area: $2a^2$

cross-sectional factors: $W_y = \frac{2a^3}{6} = \frac{a^3}{3}$, $W_z = \frac{4a^3}{6} = \frac{2a^3}{3}$

cross-sectional forces

$$N = 20000 \text{ N}$$

$$M_y = -500 \text{ Nm}$$

$$M_z = -866 - 20000a$$

extreme value of the normal stress (at the fixed end)

$$max\sigma_x = \frac{N}{A} + \frac{M_y}{W_y} - \frac{M_z}{W_z} \leq R$$

$$\frac{20000}{2a^2} + \frac{3 \cdot 500}{a^3} + 3\frac{866 + 20000a}{2a^3} \leq 150 \cdot 10^6$$

$$a = 0.03 \text{ m} \rightarrow A = 18 \text{ cm}^2, W_y = 9 \text{ cm}^3, W_z = 18 \text{ cm}^3$$

$$\max \sigma_x = \frac{20000}{18 \cdot 10^{-4}} + \frac{500}{9 \cdot 10^{-6}} + \frac{1466}{18 \cdot 10^{-6}} = 148.2 \text{ MPa} < R$$

the stress state at the point *K*:

$$\sigma_x = \frac{N}{A} + \frac{M_y}{W_y} = \frac{20000}{18 \cdot 10^{-6}} + \frac{500}{9 \cdot 10^{-6}} = 66.67 \text{ MPa}$$

$$\tau_{xy} = \frac{3}{2} * \frac{866}{18 \cdot 10^{-4}} = 0.722 \text{ MPa}$$

the substitute stress at the point *K*:

$$\sigma_{HMH} = \sqrt{\sigma^2 + 3\tau^2} = \sqrt{66.67^2 + 3 \cdot 0.722^2} = 66.7 \text{ MPa}$$

Example 13.2

Determine the Coulomb-Tresca-Guest And Huber-Mises-Hencky substitute stress at the point *A* in the fixed end of a bar in Fig. 13.2.

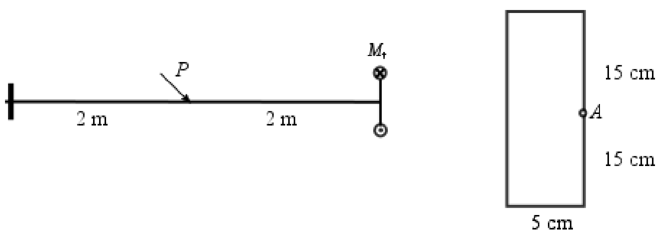

Fig. 13.2 Bar with load

Solution

the cross-sectional forces:

$$N = 10\text{ kN},\qquad Q = 10\text{ kN},\qquad M_y = -20\text{ kNm},\qquad M_t = 10\text{ kNm}$$

the cross-sectional characteristics

$$A = 150\ cm^2,\qquad W_y = 11250\ cm^4,\qquad W_s = \alpha(6)\cdot 30\cdot 5^2 = 0.299\cdot 30\cdot 5^2 = 224.3\ cm^3$$

the stress matrix

tension

$$\sigma_x = \frac{N}{A} = \frac{10\cdot 10^3}{150\cdot 10^{-4}} = 0.667\text{ MPa}$$

bending

$$\sigma_x = \frac{M_y}{I_y} z_A = 0$$

shearing

$$\tau_A = \frac{3}{2}\cdot\frac{10\cdot 10^3}{150\cdot 10^{-4}} = 1.00\text{ MPa}$$

(down)

torsion

$$\tau_A = \frac{M_t}{W_t} = \frac{10\cdot 10^3}{224.3\cdot 10^{-6}} = 44.6\text{ MPa}$$

(down)

finally

$$\sigma_x = 0.667\text{ MPa},\qquad \tau_A = 44.6 + 1.0 = 45.6\text{ MPa}$$

the substitute stress

$$\sigma_{CTG} = \sqrt{\sigma^2 + 4\tau^2} = \sqrt{0.667^2 + 4\cdot 45.6^2} = 91.2\text{ MPa}$$

$$\sigma_{HMH} = \sqrt{\sigma^2 + 43} = \sqrt{0.667^2 + 3\cdot 45.6^2} = 78.98\text{ MPa}$$

Example 13.3

Determine the Huber-Mises-Hencky substitute stress at the points K_1 and K_2 in the section $\alpha - \alpha$ of the bar structure in Fig. 13.3.

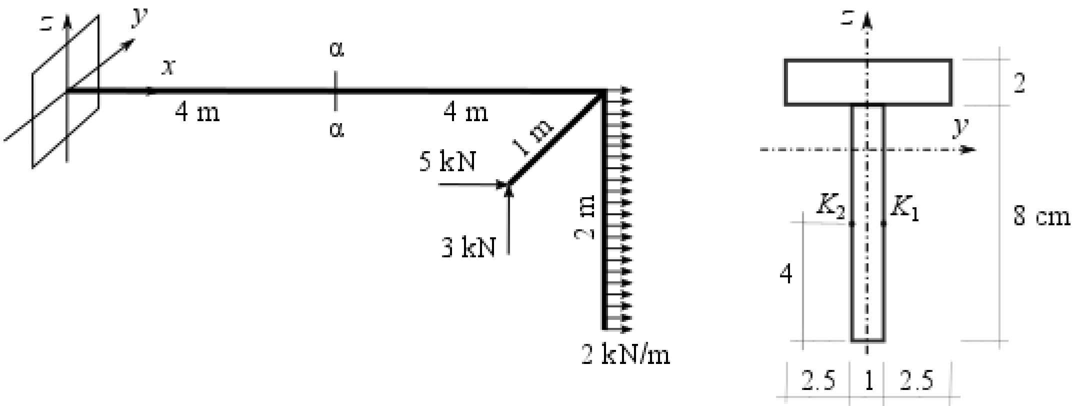

Fig. 13.3 Bar structure with load

Solution

the cross-sectional forces at $\alpha - \alpha$

axial force, $N = 5 + 4 = 9$ kN

shear force, $Q_z = 3$ kN

torque, $M_x = -3$ kNm

bending moment, $M_y = -16$ kNm

bending moment, $M_z = 5$ kNm

the cross-section characteristics

area, $A = 2 \cdot 6 + 8 \cdot 1 = 20$ cm^2

position of centroid, $z_0 = \frac{8 \cdot 1 \cdot 4 + 2 \cdot 6 \cdot 9}{20} = 7$ cm

inertia moment, $I_y = \frac{6 \cdot 8}{12} + 12 \cdot 4 + \frac{1 \cdot 8^3}{12} + 8 \cdot 9 = 166.7$ cm^4

inertia moment, $I_z = \frac{8 \cdot 1}{12} + \frac{2 \cdot 6^3}{12} = 36.67$ cm^4

the stress matrix

normal stress, $\sigma_x = \frac{N}{A} + \frac{M_y}{I_y} z - \frac{M_z}{I_z} y$

$$\sigma_x^{K_1} = \frac{9 \cdot 10^3}{20 \cdot 10^{-4}} + \frac{16 \cdot 10^3}{166.7 \cdot 10^{-8}} 0.03 - \frac{5 \cdot 10^3}{36.67 \cdot 10^{-8}} 0.005 = 224.3 \text{ MPa}$$

$$\sigma_x^{K_2} = \frac{9 \cdot 10^3}{20 \cdot 10^{-4}} + \frac{16 \cdot 10^3}{166.7 \cdot 10^{-8}} 0.03 + \frac{5 \cdot 10^3}{36.67 \cdot 10^{-8}} 0.005 = 360.7 \text{ MPa}$$

from the shear force

$$\tau_{xz} = \frac{3 \cdot 10^3 \cdot 4 \cdot 1 \cdot 5 \cdot 10^{-6}}{166.7 \cdot 10^{-8} \cdot 0.01} = 3.6 \text{ MPa}$$

from the torque

$$I_t = \beta b^3 h, \qquad W_t = \alpha b^2 h,$$

$$\frac{h}{b} = 3 \rightarrow \alpha = 0.267, \beta = 0.263,$$

$$\frac{h}{b} = 8 \rightarrow \alpha = 0.307, \beta = 0.307$$

$$I_{t1} = 1.26 \cdot 10^{-7}\ \mathrm{m}^4,\ \ I_{t2} = 2.456 \cdot 10^{-8}\ \mathrm{m}^4, I_t = 1.51 \cdot 10^{-7}$$

$$M_1 = \frac{I_{t1}}{I_t} M_t = 2503\ \mathrm{Nm}, \qquad M_2 = \frac{I_{t2}}{I_t} M_t = 488\ \mathrm{Nm}$$

$$W_{t2} = 0.307 \cdot 0.01^2 \cdot 0.08 = 2.46 \cdot 10^{-6}\ \mathrm{m}^3$$

$$\tau^{K_1} = -\frac{M_2}{W_{t2}} = -\frac{488}{2.46 \cdot 10^{-6}} = -198.9\ \mathrm{MPa}$$

$$\tau^{K_2} = \frac{M_2}{W_{t2}} = \frac{488}{2.46 \cdot 10^{-6}} = 198.9\ \mathrm{MPa}$$

the shear stress

$$\tau_{xz}^{K_1} = -198.9 + 3.6 = -195.3\ \mathrm{MPa}, \qquad \tau_{xz}^{K_2} = 198.9 + 3.6 = 202.5\ \mathrm{MPa}$$

the Huber-Mises-Hencky substitute stress

$$\tau_{HMH} = \sqrt{\sigma_x^2 + 3\tau_{xz}^2} \rightarrow \sigma_{HMH}^{K_1} = 405.9\ MPa, \ \ \sigma_{HMH}^{K_2} = 503.1\ \mathrm{MPa}$$

Review problems

Problem 13.1

Determine the substitute stress at the point A in the fixed end of the bar in Fig. 13.4. $P = 4\sqrt{2}$ kN, $M = 12\sqrt{2}$ kNm.

Ans.: $\sigma_{CTG} = 7.42$ MPa.

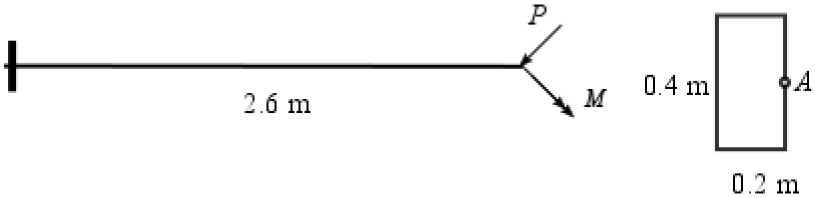

Fig. 13.4 Bar with load

Problem 13.2

Determine the substitute stress according the CTG (HMH or CM) hypothesis at the point *A* (*B* or *C*), given the all necessary data (the dimensions and loads) in the bar shown in Fig. 13.5. The input data: $P_1 = 10$ kN, $P_2 = 40$ kN, $q = 20\,\frac{\text{kN}}{\text{m}}$, $l = 2$ m, $b \times h = 6 \times 12$ cm.

Ans.: $\sigma_{CTG}^{A} = 544.0$ MPa, $\sigma_{CTG}^{B} = 93.17$ MPa.

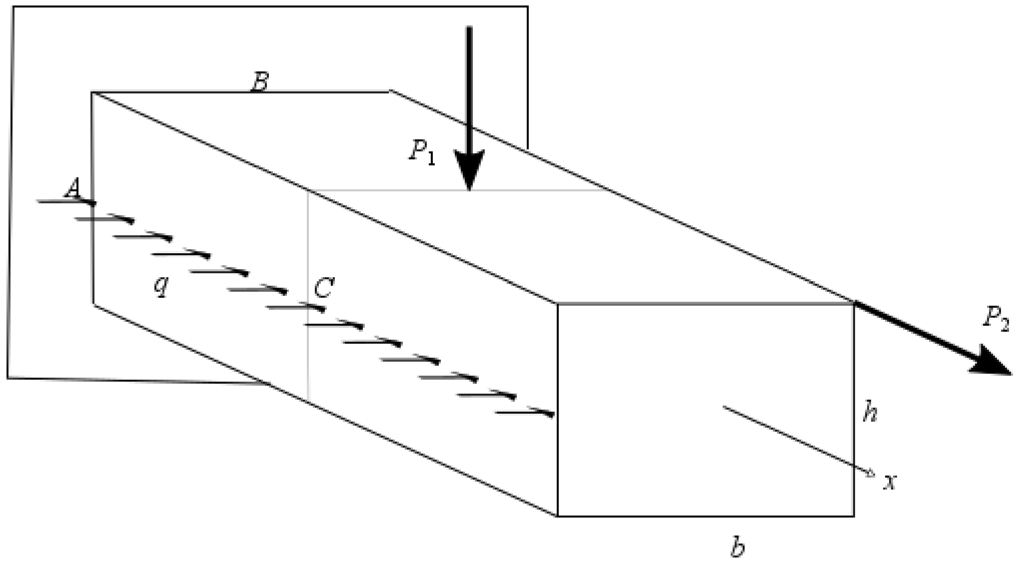

Fig. 13.5 Bar with loads