Limit plastic moment

Basic formulae for the plastic cross-sectional factor

the neutral axis position in simple bending

$$A_1 = A_2 \rightarrow z_0$$

the limit plastic factor, calculated in regard of the central axis

$$\overline{\overline{W}} = 2S_y^{(1)} = 2S_y^{(2)}$$

the limit plastic factor, calculated in regard of the neutral axis

$$\overline{\overline{W}} = S_{y_0}^1 + S_{y_0}^2$$

the limit plastic moment of the cross-section

$$\overline{\overline{M}} = \overline{\overline{W}} R_e$$

(the limit elastic factor of the cross section: $\overline{W} = \frac{I_y}{z_{\max}}$)

the shape coefficient

$$k = \frac{\overline{\overline{M}}}{\overline{M}} = \frac{\overline{\overline{W}}}{\overline{W}}$$

Examples

Example 14.1

Determine the shape coefficient for the cross section in Fig. 14.1.

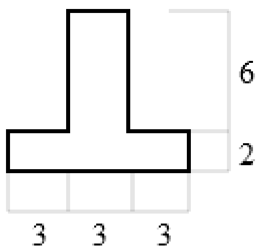

Fig. 14.1 Cross-section

Solution

cross-section area: $A = 9 \cdot 2 + 3 \cdot 6 = 36 \text{ cm}^2$

position of the neutral axis: $z_0 = \frac{36}{2 \cdot 9} = 2 \text{ cm}$

position of the centroid: $z_c = \frac{18 \cdot 1 + 18 \cdot 5}{36} = 3 \text{ cm}$

cross-section inertia moment:

$$I_y = \frac{9 \cdot 2^3}{12} + 18 \cdot 2^2 + \frac{3 \cdot 6^3}{12} + 18 \cdot 2^2 = 204 \text{ cm}^4$$

elastic cross-section factor:

$$\bar{W} = \frac{I_y}{z_{\max}} = \frac{204}{5} = 40.8 \text{ cm}^3$$

plastic cross-section factor

a) about the central axis

$$\bar{\bar{W}} = 2 \cdot (6 \cdot 3 \cdot 2) = 72 \text{ cm}^3$$

b) about the neutral axis

$$\bar{\bar{W}} = 6 \cdot 3 \cdot 3 + 9 \cdot 2 \cdot 1 = 72 \text{ cm}^3$$

the shape coefficient:

$$k = \frac{\bar{\bar{W}}}{\bar{W}} = \frac{72}{40.8} = 1.76$$

Example 14.2

Determine the shape coefficient of the cross-section in Fig. 14.2.

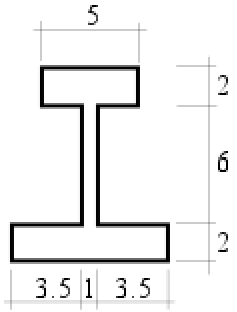

Fig. 14.2 Beam cross-section

Solution

the cross-section area, $A = 2 \cdot 5 + 6 \cdot 1 + 2 \cdot 8 = 32 \text{ cm}^2$

the position of centroid, $z_c = \frac{8 \cdot 2 + 6 \cdot 5 + 10 \cdot 9}{32} = 4.25$ cm

the position of neutral axis, $z_p = 2$ cm

the inertia moment, $I_y = \frac{8 \cdot 2^3}{12} + 16 \cdot 3.25^2 + \frac{1 \cdot 6^3}{12} + 6 \cdot 0.75^2 + \frac{5 \cdot 2^3}{12} + 10 \cdot 4.75^2 = 424.7 \text{ cm}^4$

the elastic cross-section factor, $\bar{W} = \frac{I_y}{z_{\max}} = \frac{424.7}{5.75} = 73.86 \text{ cm}^3$

the plastic cross-section factor (with respect to the central axis)

$$\bar{\bar{W}} = 2 \cdot 2 \cdot 8 \cdot 3.25 = 104 \text{ cm}^3$$

the plastic cross-section factor (with respect to the neutral axis)

$$\bar{\bar{W}} = 2 \cdot 8 \cdot 1 + 6 \cdot 3 + 10 * 7 = 104 \text{ cm}^3$$

the shape coefficient:

$$k = \frac{\bar{\bar{W}}}{\bar{W}} = \frac{104}{73.86} = 1.41$$

Example 14.3

A cross-section in Fig. below is loaded by a bending moment $M = \bar{M} + 0.5(\bar{\bar{M}} - \bar{M})$. Determine the stress distribution and the residual stress after unloading, $R_e = 480$ MPa.

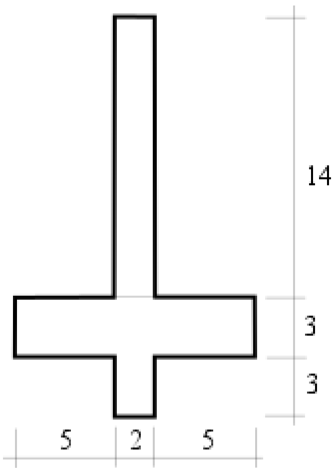

Fig. 14.3 Cross-section in bending

Solution

the cross-section area, $A = 14 \cdot 2 + 12 \cdot 3 + 2 \cdot 3 = 70$ cm^2

the position of centroid, $z_c = \frac{14 \cdot 2 \cdot 13 + 12 \cdot 3 \cdot 4.5 + 2 \cdot 3 \cdot 1.5}{70} = 7.643$ cm

the position of neutral axis, $z_p = 3 + \frac{35-6}{12} = 3 + 2.417 = 5.417$ cm

the inertia moment, $I_y = \frac{2 \cdot 20^3}{12} + 40 \cdot 2.357^2 + \frac{10 \cdot 3^3}{12} + 30 \cdot 3.143^2 = 1874$ cm^4

the elastic cross-section factor, $\bar{W} = \frac{I_y}{z_{\max}} = \frac{1874}{20-7.64} = 151.7$ cm^3

the plastic cross-section factor (with respect to the central axis)

$$\bar{\bar{W}} = 2 \cdot (6 \cdot 6.143 + 12 \cdot 2.417 \cdot 3.4345) = 272.9 \text{ cm}^3$$

the plastic cross-section factor (with respect to the neutral axis)

$$\bar{\bar{W}} = 6 \cdot 3.917 + 12 \cdot 2.417^2 \cdot 0.5 + 12 \cdot 0.583^2 \cdot 0.5 + 14 \cdot 2 \cdot 7.583 = 272.9 \text{ cm}^3$$

the value of applied bending moment

$$M = R_e\left[\overline{W} + 0.5 \cdot \left(\overline{\overline{W}} - \overline{W}\right)\right] = 480 \cdot 10^6[151.7 + 0.5(272.9 - 151.7)] \cdot 10^{-6} = 101.9 \text{ kNm}$$

the Bernoulli's hypothesis:

$$\varepsilon = \varepsilon_0 + \kappa z$$

where ε_0 – axis strain, κ – axis curvature

Using the position of neutral axis, z_0

$$\varepsilon(z = z_0) = 0 \rightarrow \ \varepsilon_0 = -\kappa z_0 \rightarrow \ \varepsilon = (z - z_0)\kappa$$

and the elastic range extent, ξ

$$|z - z_0| = \xi \rightarrow E\varepsilon(\xi) = R_e \rightarrow \ \varepsilon = \frac{z - z_0}{\xi} \cdot \frac{R_e}{E}$$

we are searching such values of z_0 and ξ, that the axial force and the bending moment would be:

$$\iint_A \sigma\, dA = 0, \qquad \iint_A \sigma z\, dA = 101.9 \cdot 10^3$$

The numerical procedure consists in several steps:

- divide the cross-section into rectangles with constant width
- check which part (if any) of each rectangle works in one of three range:
  - plastic compression,
  - elastic compression-tension, and
  - plastic tension.

From numerical solution, we get the parameters of the stress distribution:

$$z_0 = 5.586 \text{ cm}, \qquad \xi = 1.539 \text{ cm}$$

Partial results are given in the table below:

z_g	z_d	N_1	M_1	z_g	z_d	N_2	M_2	z_g	z_d	N_3	M_3
3	0	-6	36,86	4,048	3	-12,57	51,777	6	6	0	0
3	3	0	0	6	4,048	-8,5577	-38,15	7,124	6	1,4263	9,513
3	3	0	0	6	6	0	0	20	7,124	25,753	152,4

The unloading process means application of the elastic moment equal to the given elastic-plastic moment. The unload stress distribution is linear, so the extreme value of the unloading stress is:

$$\sigma_{\max} = \frac{M_{ul}}{\overline{W} R_e} = \frac{101.9 \cdot 10^3}{151.7 \cdot 10^{-6} \cdot 480 \cdot 10^6} = 1.399$$

it means, that the extreme value of the residual stress is equal to

$$\sigma_{res} = (1.399 - 1)R_e = 0.399R_e = 191.5 \text{ MPa}$$

The distribution of the loading, unloading and residual stress in the cross-section is given in Fig. 14.4

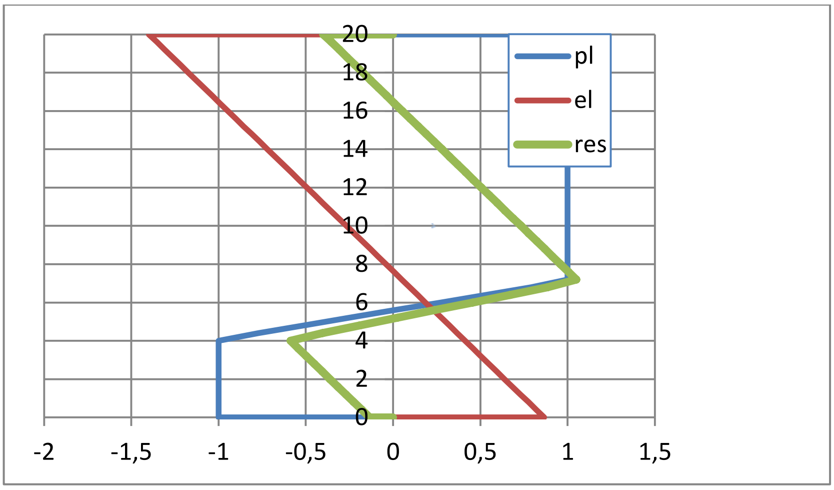

Fig. 14.4 Stress distribution (blue: loading, red: unloading, green: residual)

Review problems

For the given cross-section, determine the shape coefficient *k* (the relation of limit plastic factor to the limit elastic factor). Check your solution using the “przekroj” program.

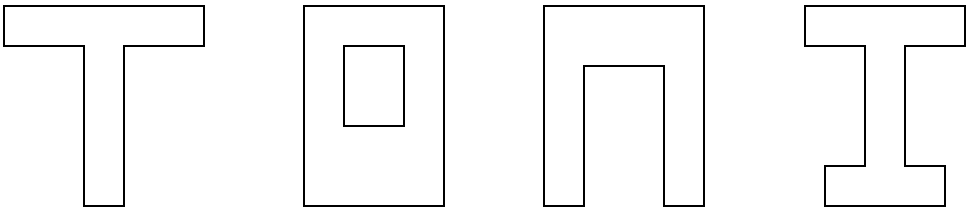

Fig. 14.5 Cross-sections with one symmetry axis