Tension – part 1

Practical formulae for straight prismatic bar with constant axial force

The state of stress is uniaxial and homogeneous:

$$\sigma_x = \frac{N}{A}$$

where *N* – axial force , *A* – cross-sectional area. The strain state is triaxial and homogeneous:

$$\varepsilon_x = \frac{N}{EA}, \quad \varepsilon_y = \varepsilon_z = -\nu\varepsilon_x$$

The bar elongation is:

$$\Delta l = \frac{Nl}{EA}$$

Non-mechanical factor – temperature

The temperature change generates stress field (thermal stress) only in the case of statically undetermined structures. The strain change and elongation depend on the coefficient of thermal expansion, α:

$$\varepsilon_x = \alpha\Delta t, \quad \Delta l = \alpha l \Delta t$$

Limit states

The Eurocodes are Limit State Design Codes. There are two main Limit States: Bearing Capacity (Strength) Limit State and Usability Limit State. The first one guarantees structures safety. The second one includes several prescriptions and requirements about stiffness, durability and so on. In the course of strength of materials, for the sake of simplicity, we ‘ll restrict usability requirements to structure stiffness only.

Ultimate Limit State

$$\max \sigma_x < R$$

where *R* is a calculation strength of a material (from a code directly or experiments).

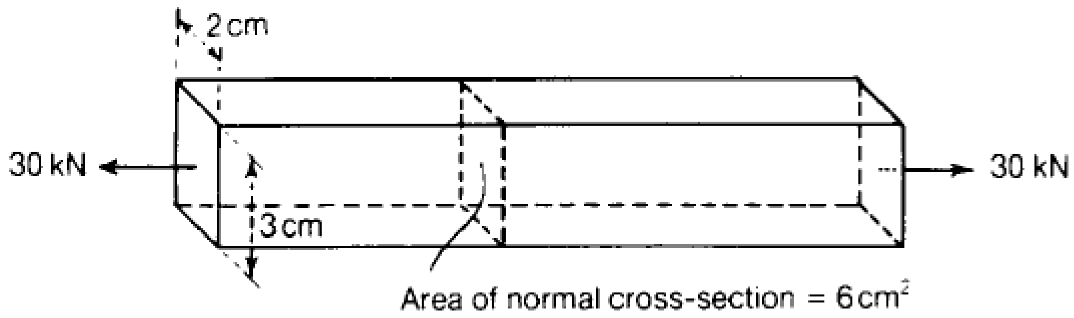
Usability Limit State

$$\Delta l < \Delta l_{acc}; \quad \varepsilon < \varepsilon_{acc}$$

Examples

Example 1

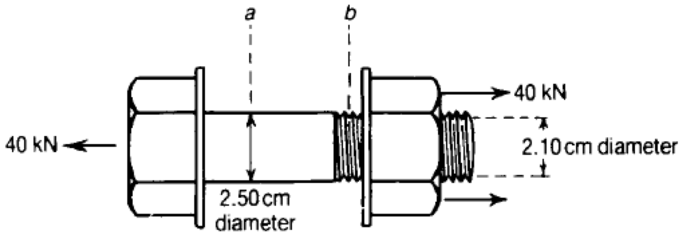
A steel bar of rectangular cross-section, 3 cm by 2 cm, carries an axial load of 30 **kN.** Estimate the average tensile stress over a normal cross-section of the bar.



Solution
The area of a normal cross-section of the bar is
$$A = 0.03 \cdot 0.02 = 0.6 \cdot 10^{-3}\ \mathrm{m}^2$$
The average tensile stress over this cross-section is then
$$\sigma = \frac{N}{A} = \frac{30 \cdot 10^3}{0.6 \cdot 10^{-3}} = 50\ \mathrm{MPa}$$

Example 2

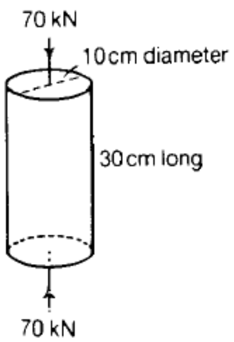
A steel bolt, 2.50 cm in diameter, carries a tensile load of 40 kN**.** Estimate the average tensile stress at the section *a* and at the screwed section *b*, where the diameter at the root of the thread is 2.10 cm.



Solution
The cross-sectional area of the bolt at the section a and b are
$$A_a = \frac{\pi}{4}(0.025)^2 = 0.491 \cdot 10^{-3}\ \mathrm{m}^2, \quad A_b = \frac{\pi}{4}(0.021)^2 = 0.346 \cdot 10^{-3}\ \mathrm{m}^2$$
The average tensile stress at a and b are then
$$\sigma_a = \frac{N}{A_a} = \frac{40 \cdot 10^3}{0.491 \cdot 10^{-3}} = 81.4\ \mathrm{MPa}, \quad \sigma_b = \frac{N}{A_b} = \frac{40 \cdot 10^3}{0.346 \cdot 10^{-3}} = 115.6\ \mathrm{MPa}$$

Example 3

A cylindrical block is 30 cm long and has a circular cross-section 10 cm in diameter. It carries a total compressive load of 70 kN**,** and under this load it contracts by 0.02 cm. Estimate the average compressive stress over a normal cross-section and the compressive strain.



Solution

The area of a normal cross-section is

$$A = \frac{\pi}{4}(0.10)^2 = 7.85 \cdot 10^{-3}\ \mathrm{m}^2$$

The average compressive stress over this cross-section is then

$$\sigma = \frac{N}{A} = \frac{70 \cdot 10^3}{7.85 \cdot 10^{-3}} = 8.92\ \mathrm{MPa}$$

The average compressive strain over the length of the cylinder is

$$\varepsilon = \frac{0.02 \cdot 10^{-2}}{30 \cdot 10^{-2}} = 0.67 \cdot 10^{-3}$$

Example 4

A tensile test is carried out on a bar of mild steel of diameter 2 cm. The bar yields under a load of 80 kN. It reaches a maximum load of 150 kN, and breaks finally at a load of 70 kN. Estimate:

- the tensile stress at the yield point;
- the ultimate tensile stress;
- the average stress at the breaking point, if the diameter of the fractured neck is 1 cm.

Solution

The original cross-section of the bar is

$$A_0 = \frac{\pi}{4}(0.020)^2 = 0.314 \cdot 10^{-3}\ \mathrm{m}^2$$

The average tensile stress at yielding is then

$$\sigma_Y = \frac{N_Y}{A_0} = \frac{80 \cdot 10^3}{0.314 \cdot 10^{-3}} = 254\ \mathrm{MPa}$$

where N_Y = load at the yield point

The ultimate stress is the nominal stress at the maximum load, i.e.,

$$\sigma_{ult} = \frac{N_{max}}{A_0} = \frac{150 \cdot 10^3}{0.314 \cdot 10^{-3}} = 477\ \mathrm{MPa}$$

The cross-sectional area in the fractured neck is

$$A_f = \frac{\pi}{4}(0.010)^2 = 0.0785 \cdot 10^{-3}\ \mathrm{m}^2$$

The average stress at the breaking point is then

$$\sigma_f = \frac{N_f}{A_f} = \frac{70 \cdot 10^3}{0.0785 \cdot 10^{-3}} = 892\ \mathrm{MPa}$$

where N_f = final breaking load.

Example 5

A circular bar of diameter 2.50 cm and length of 1 m is subjected to an axial tension of 20 kN. If the material is elastic with a Young's modulus E = 70 GPa, estimate the bar elongation.

Solution

The cross-sectional area of the bar is

$$A = \frac{\pi}{4}(0.025)^2 = 0.491 \cdot 10^{-3}\ \text{m}^2$$

The average tensile stress is then

$$\sigma = \frac{N}{A} = \frac{20 \cdot 10^3}{0.491 \cdot 10^{-3}} = 40.7\ \text{MPa}$$

The longitudinal tensile strain will therefore be

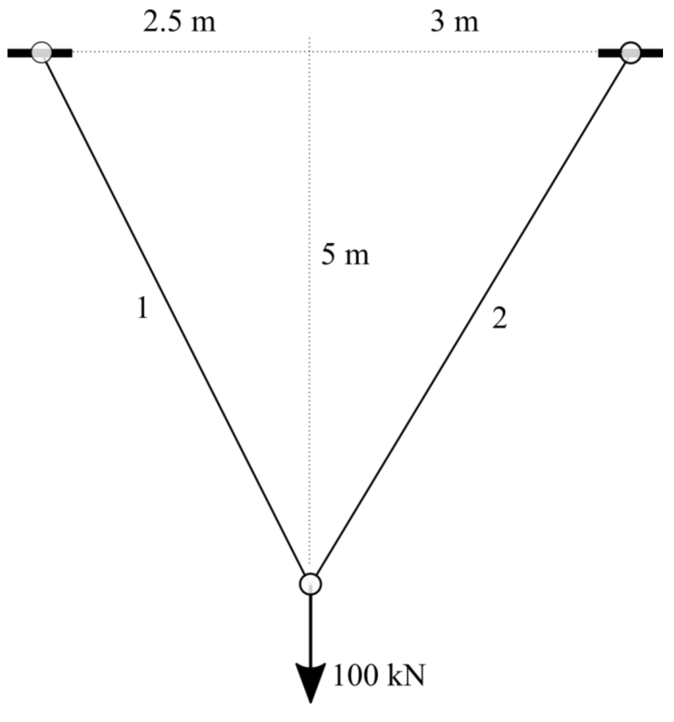
$$\varepsilon = \frac{\sigma}{E} = \frac{40.7 \cdot 10^6}{70 \cdot 10^9} = 0.582 \cdot 10^{-3}$$

The bar elongation will therefore be

$$\Delta l = \varepsilon l = 0{,}582 \cdot 10^{-3} \cdot 1 = 0{,}582 \cdot 10^{-3}\ \text{m} = 0.582\ \text{mm}$$

Example 6

Two trusses with identical geometry and made from the same material are loaded: a) by force 100 kN (see Figure below) and, b) by temperature change of 45°C. Determine the displacements of the bottom node in the both cases. Assume *E* = 210 GPa and the cross-sectional area A = 5 cm^2. The coefficient of thermal expansion is $14.5 \cdot 10^{-6}$ 1/C.



Solution

case a)

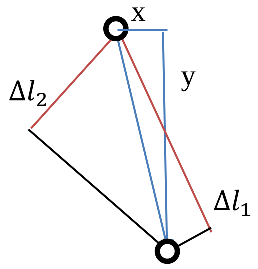
From the node balance equation we get: $N_1 = 60.98$ kN, $N_2 = 53.01$ kN.

The bars elongation will be

$$\Delta l_1 = \frac{N_1 l_1}{EA} = \frac{60.98 \cdot 10^3 \cdot 5.590}{210 \cdot 10^9 \cdot 5 \cdot 10^{-4}} = 3.247 \cdot 10^{-3}\ \mathrm{m} = 3.247\ \mathrm{mm}$$

$$\Delta l_2 = \frac{N_2 l_2}{EA} = \frac{53.01 \cdot 10^3 \cdot 5.831}{210 \cdot 10^9 \cdot 5 \cdot 10^{-4}} = 2.944 \cdot 10^{-3}\ \mathrm{m} = 2.944\ \mathrm{mm}$$

This is a problem with 2 DoF (horizontal and vertical translations).



We use projections of x and y onto Δl_1 and Δl_2

$$\overrightarrow{e_1}(0.4472, -0.8944),\ \ \overrightarrow{e_2}(-0.5145, -0.8575)$$

$$\overrightarrow{e_x}(1, 0),\ \ \overrightarrow{e_y}(0, -1)$$

and

$$\Delta l_1 = x \cdot \overrightarrow{e_x} \cdot \overrightarrow{e_1} + y \cdot \overrightarrow{e_y} \cdot \overrightarrow{e_1},\ \ \Delta l_2 = x \cdot \overrightarrow{e_x} \cdot \overrightarrow{e_2} + y \cdot \overrightarrow{e_y} \cdot \overrightarrow{e_2}$$

therefore

$$0.4472x + 0.8944y = 3.242 \cdot 10^{-3}$$

$$-0.5145x + 0.8575y = 2.944 \cdot 10^{-3}$$

with the solution

$$x = 1.74 \cdot 10^{-4}, y = 3.54 \cdot 10^{-3}$$

case b)

The bars elongations are

$$\Delta l_1 = \alpha l_1 \Delta t = 14.5 \cdot 10^{-6} \cdot 5.59 \cdot 45 = 3.647 \cdot 10^{-3}\ \mathrm{m} = 3.647\ \mathrm{mm}$$

$$\Delta l_2 = \alpha l_2 \Delta t = 14.5 \cdot 10^{-6} \cdot 5.831 \cdot 45 = 3.805 \cdot 10^{-3}\ \mathrm{m} = 3.805\ \mathrm{mm}$$

The equation set is

$$0.4472x + 0.8944y = 3.647 \cdot 10^{-3}$$

$$-0.5145x + 0.8575y = 3.805 \cdot 10^{-3}$$

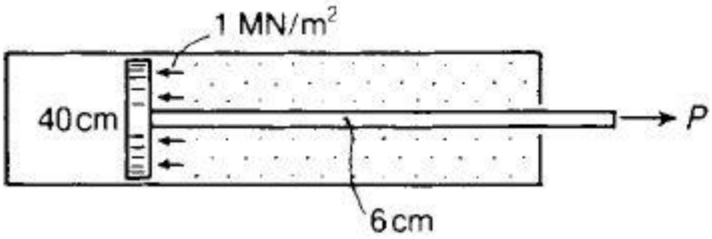
with the solution

$$x = -3.27 \cdot 10^{-4},\ \ y = 4.24 \cdot 10^{-3}.$$

Review problems

Problem 1

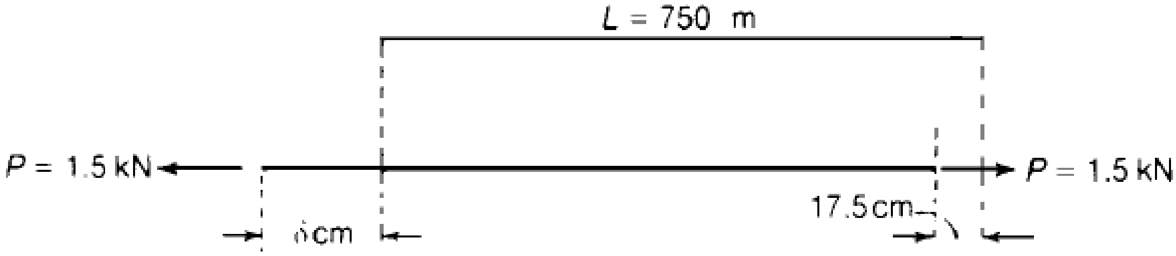
The piston of a hydraulic ram is 40 cm diameter, and the piston rod 6 cm diameter. The water pressure is 1 MN/m^2. Estimate the stress in the piston rod and the elongation of a length of **1** m of the rod when the piston is under pressure from the piston-rod side. Take Young's modulus **as** E = 200 GPa.



Answ.: $\sigma = 43.5$ MPa, $\Delta l = 0.0218$ cm

Problem 2

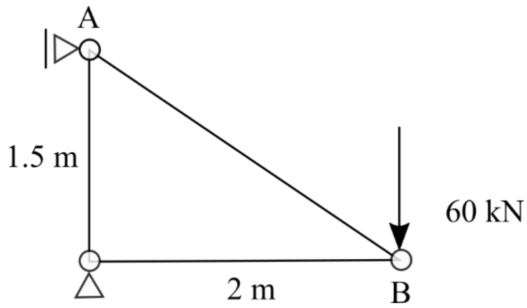
The steel wire working a signal is 750 m long and 0.5 cm diameter. Assuming a pull on the wire of 1.5 kN, find the movement which must be given to the signal-box end of the wire if the movement at the signal end is to be 17.5 cm. Take Young's modulus as 200 GPa.



Answ. $\delta = 46.2$ cm

Problem 3

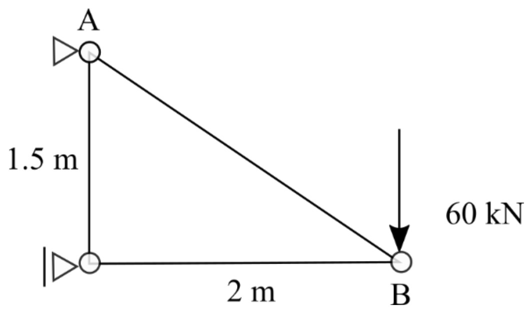
Determine needed cross-sectional area of the bar AB if the calculation strength is R = 250 MPa.



Answ. A = 4 cm^2.

Problem 4

Determine horizontal and vertical displacements of the node B. Assume E = 210 GPa and the cross-sectional area of all bars A = 4 cm^2.



Answ. $\delta(1.9, 7.5)$ mm (left and down).