Bending

Basic formulae

Stress distribution is given by the equation

$$\sigma_x = \frac{M_y}{I_y} z\,,$$

where the distance z is measured from the principal central axis.

The maximum absolute value of the stress is attained at the fibers which are the most distant from the neutral axis:

$$\max|\sigma_x| = \frac{|M_y|}{I_y} z_{\max} = \frac{|M_y|}{W_y}, \quad W_y = \frac{I_y}{z_{\max}}$$

Maximum stress is proportional to the bending moment and inversely proportional to the elastic section modulus.

From the ultimate limit state we have the design inequality:

$$\max|\sigma_x| \le R \quad \to \quad \frac{M_y}{W_y} \le R\,.$$

The curvature is proportional to the bending moment and inversely proportional to the bending stiffness:

$$\kappa = \frac{M_y}{EI_y}$$

Examples

Example 3.1

Determine the value of the parameter a of the cross-section of the beam in Fig. 3.1 if P = 140 kN, l = 2 m, b = 0.2 m, R = 150 MPa.

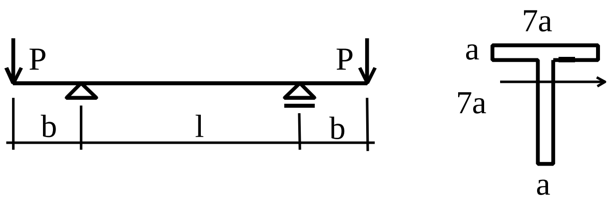


Fig. 3.1 Beam and the cross-section

Solution

– the maximum value of the bending moment is at the span and is $M_y = Pb = 28$ kNm
– position of the cross-section centroid: $y_0 = 5.5a$
– principal central inertia moment: $J_y = 85.17a^4$
– section modulus: $W_y = \dfrac{85.17a^4}{5.5a} = 15.48a^3$
– normal stress: $\sigma_x = \dfrac{M_y}{W_y} = \dfrac{28\cdot10^3}{15.48a^3} \le R = 150\cdot10^6$
– so, the parameter: $a^3 \ge \dfrac{28\cdot10^3}{15.48\cdot150\cdot10^6} = 1.206\cdot10^{-5}, \quad \to \quad a \ge 0.0229\,\text{m}$

Finally, we assume *a* = 2.3 cm, $J_y = 2380$ cm^4, $W_y = 188$ cm^3 and $\max|\sigma_x| = 149$ MPa.

The normal stress repartition can be presented by means of a stress solid or a stress diagram, Fig. 3.2.

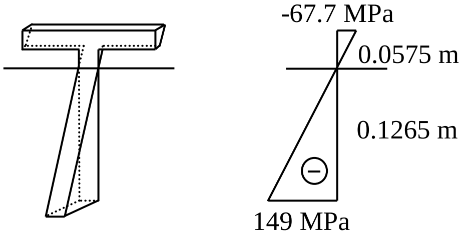


Fig. 3.2 Stress solid and stress diagram

Example 3.2

Knowing that for the extruded beam shown in Fig. 3.3, the allowable stress is 84 MPa in tension and 110 MPa in compression, determine the largest moment *M* that can be applied. The bottom side is tensioned.

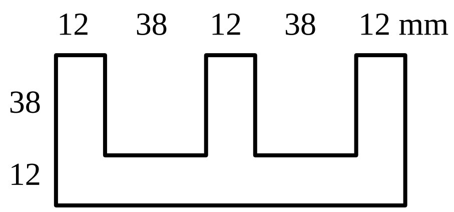


Fig. 3.3 Extruded cross-section

Solution

The cross-section characteristics:

– area: $A = (3\cdot12 + 2\cdot38)\cdot50 - 2\cdot38\cdot38 = 2712$ mm^2
– position of the centroid: $z_c = \dfrac{112\cdot50\cdot25 - 2\cdot38\cdot38\cdot31}{2712} = 18.61$ mm
– principal central inertia moment:

$$J_y = \frac{112\cdot50^3}{12} + 112\cdot50\cdot(25-18.61)^2 - 2\cdot\left[\frac{38^4}{12} + 38^2\cdot(31-18.61)^2\right] = 604500\,\text{mm}^4$$

– stress distribution:

$$\sigma_x = \frac{M_y}{J_y} z,$$

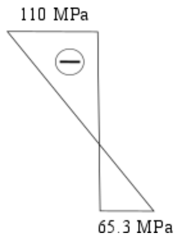
$$\sigma_x(z = -18.61) = \frac{-M_y}{604500 \cdot 10^{-12}}(-0.01861) \le 84 \cdot 10^6, \quad M_y \le 2.73 \text{kNm}$$

$$\sigma_x(z = 31.39) = \frac{|M_y|}{604500 \cdot 10^{-12}}(0.03139) \le 110 \cdot 10^6, \quad M_y \le 2.12 \text{ kNm}$$

and, the final answer is:

$$M_y \le 2.12 \text{ kNm}$$

The stress distribution:



Example 3

The cross section of a modified I-section beam is shown in Figure 3.4. A bending moment causes a maximum compressive flexural stress in the beam of magnitude 50 MPa at the top. Determine the magnitude of M, and the maximum tensile flexural stress for the cross section.

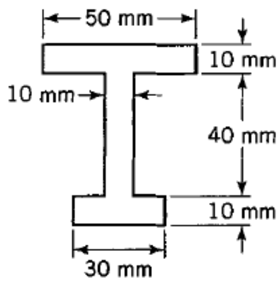


Fig. 3.4 Modified I-section

Solution

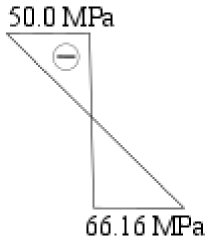
cross-section characteristics

$$A = 12 \text{ cm}^2, z_0 = 3.417 \text{ cm}, I_y = 53.92 \text{ cm}^4$$

stress distribution

$$\sigma_x = \frac{M_y}{I_y} z \to 50 \cdot 10^6 = \frac{M_y}{53.92 \cdot 10^{-8}}(0.06 - 0.03417) \to M_y = 1.044 \text{ kNm}$$

$$\sigma_x(z = -0.03417) = \frac{1044}{53.92 \cdot 10^{-8}} 0.03417 = 66.16 \text{ MPa}$$



Example 4

An aircraft wing strut is made of an aluminum alloy (E = 72 GPa and σ_0 = 300 MPa) and has the extruded cross section shown in Fig. 3.5. Strain gages, located at 55 mm and 75 mm below the top, measure axial strains of-0.00012 and 0.00080**,** respectively, and the bending moment is $M_y = 6$ kNm (tension at bottom). Determine the maximum tensile and compressive flexural stresses in the beam cross section, the location of the neutral axis, and the moment of inertia of the cross section.

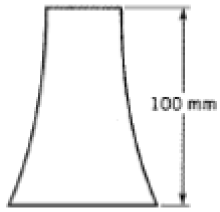


Fig. 3.5 Wing cross-section

Solution

(vertical coordinate inversed)

strain distribution: $\varepsilon = \varepsilon_0 + \kappa z$, with: $\varepsilon_x(z = 25) = 0.0008, \varepsilon_x(z = 45) = -0.00012$

$$\varepsilon = 1.95 \cdot 10^{-3} - 4.6 \cdot 10^{-5} \cdot z$$

so, at the extreme fibers

$$\varepsilon(0) = 1.95 \cdot 10^{-3}, \quad \varepsilon(100) = -2.65 \cdot 10^{-3}$$

the stresses from the Hooke's equation

$$\sigma_x = E\varepsilon \to \sigma_x(0) = 140.4 \text{ MPa}, \quad \sigma_x(100) = -190.8 \text{ MPa}$$

location of the neutral axis $\varepsilon(z_0) = 0 \to z_0 = \frac{1.95 \cdot 10^{-3}}{4.6 \cdot 10^{-6}} = 42.39$ mm

moment of inertia: $\sigma_x = \frac{M_y}{I_y} z \to I_y = \frac{M_y}{\sigma_x} z = \frac{6000}{140.4 \cdot 10^6} 42.39 \cdot 10^{-3} = 1.812 \cdot 10^{-6} \text{ m}^4 = 181.2 \text{ cm}^4$

Review problems

Problem 1

Determine the cross-section parameter a, Fig. 3.6, if the acceptable value of the normal stress is R = 240 MPa.

(Ans.: $a \geq 2.95$ [mm])

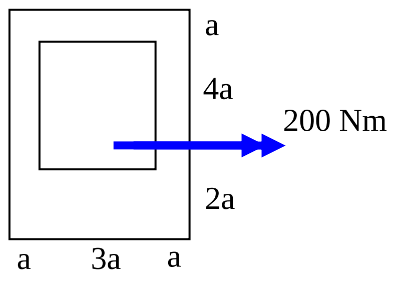


Fig. 3.6 Cross-section

Problem 2

Determine the maximum normal stress in the axle of the coal car, Fig. 3.7. Total weight is 72 tonnes, there are 4 axles of 120 mm diameter each, the wheel track 1435 cm, the span 1.3 m.

(Ans.: 35.1 MPa)

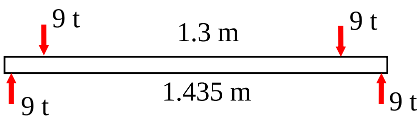


Fig. 3.7 Coal car and axle scheme

Problem 3

Determine the acceptable value of the bending moment, applied to the section in Fig. 3.13, knowing that the allowable stress is 80 MPa in tension and 105 MPa in compression. Draw the diagram of the stress repartition.

(Ans.: M≤370 kNm, compression determines)

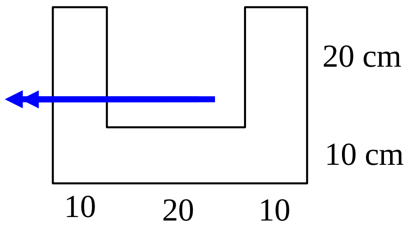


Fig. 3.13 Cross-section in bending

Problem 4

From the table of I-section profiles, chose an appropriate profile knowing that the acceptable value of the normal stress is 300 MPa and that the bending moment M = 120 kNm.

(Ans.: INP 260)