Cross-section core

Basis formulae

The intercept form of the neutral axis equation:

$$\frac{y}{b}+\frac{z}{c}=1$$

where the intercepts:

$$b\equiv-\frac{i_z^2}{y_N},\quad c\equiv-\frac{i_y^2}{z_N},$$

Two points parametric equation of the neutral axis:

$$\begin{aligned} y&=(y_2-y_1)t+y_1 \\ z&=(z_2-z_1)t+z_1 \end{aligned},$$

Construction of the cross-section core starts with the principal central inertia axes and cross-section geometric characteristics: area and principal central inertia radii. Next we made the convex outline of the cross-section. Lines and points of the outline correspond to lines and points of the core, respectively.

Examples

Example 6.1

Construct the core for the cross-section in Fig. 6.1.

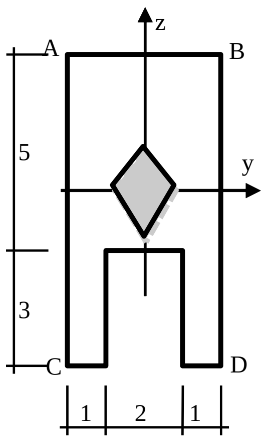

Fig. 6.1 Cross-section and the core

Solution

1. there is one vertical symmetry axis *z*, which is the principal central axis, the second axis is perpendicular to this first and passes by the centroid
2. the centroid position

$$z_0 = \frac{8\cdot 4\cdot 4 - 2\cdot 3\cdot 1.5}{8\cdot 4 - 2\cdot 3} = 4.58, \quad y_0 = 0$$

3. the principal central inertia moments

$$J_y = \frac{4\cdot 8^3}{12} + 4\cdot 8\cdot 0.58^2 - \frac{2\cdot 27}{12} - 6(4.58-1.5)^2 = 170.7 + 10.7 - 4.5 - 56.8 = 120\ \text{cm}^4$$

$$J_z = \frac{8\cdot 64}{12} - \frac{3\cdot 8}{12} = 42.7 - 2 = 40.7\ \text{cm}^4$$

3. the inertia radii squared

$$i_y^{\,2} = \frac{J_y}{A} = 4.62, \quad i_z^{\,2} = \frac{J_z}{A} = 1.56$$

4. the core curve:

A-B: $b = \pm\infty, \quad c = 3.42 \quad \Rightarrow \quad y_0 = 0, \quad z_0 = -1.35$

A-C: $b = -2, \quad c = \pm\infty \quad \Rightarrow \quad y_0 = 0.782, \quad z_0 = 0$

C-D: $b = \pm\infty, \quad c = -4.58 \quad \Rightarrow \quad y_0 = 0, \quad z_0 = 1.01$

(for the last point we use the symmetry property)

Example **6.2**

Determine the core for the section, Fig. 6.2.

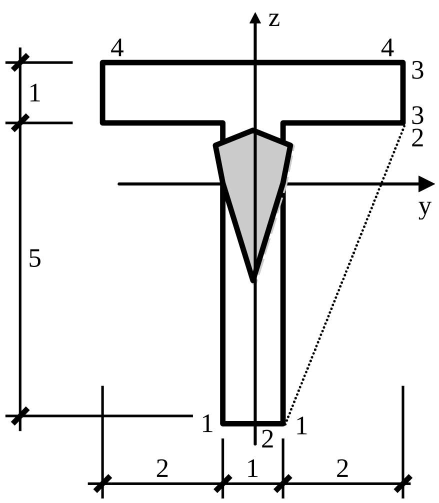

Fig. 6.2 T-section and its core

Solution

1. the centroid position

$$z_0 = \frac{5\cdot 1\cdot 2.5 + 5\cdot 1\cdot 5.5}{5+5} = 4.0, \quad y_0 = 0$$

2. the principal central inertia moments

$$J_y = \frac{1 \cdot 5^3}{12} + 5 \cdot (4 - 2.5)^2 + \frac{5 \cdot 1}{12} + 5 \cdot (5.5 - 4) == 10.42 + 11.25 + 0.42 + 11.25 = 33.33 \text{cm}^4$$

$$J_z = \frac{1 \cdot 125}{12} + \frac{5 \cdot 1}{12} = 10.42 + 0.42 = 10.83 \text{cm}^4$$

3. the inertia radii squared

$$i_y^{\ 2} = \frac{J_y}{A} = 3.33, \quad i_z^{\ 2} = \frac{J_z}{A} = 1.08$$

4. the core curve:

1-1: $b = \pm\infty, c = -4 \rightarrow y_N = 0, z_N = 0.83$

2-2: the line passes through two points: A(0.5, -4.0) and B(2.5, 1.0)

parametric equation:

$$y = (2.5 - 0.5)t + 0.5 = 2t + 0.5$$

$$z = \left(1 - (-4)\right)t - 4 = 5t - 4$$

the intercept y: $z = 0 \rightarrow t = 0.8 \rightarrow y = 1.6 + 0.5 = 2.1$

the intercept z: $y = 0 \rightarrow t = -0.25 \rightarrow z = -5.25$

$$b = 2.1, \quad c = -5.25 \quad \Rightarrow \quad y_0 = -0.516, \quad z_0 = 0.635$$

3-3: $b = 2, c = \pm\infty \rightarrow y_N = -0.433, z_N = 0$

4-4: $b = \pm\infty, c = 2 \rightarrow y_N = 0, z_N = -1.67$

(and, as previously, we use the symmetry property)

Example 6.3

Determine the core for the angle profile, Fig, 6.3.

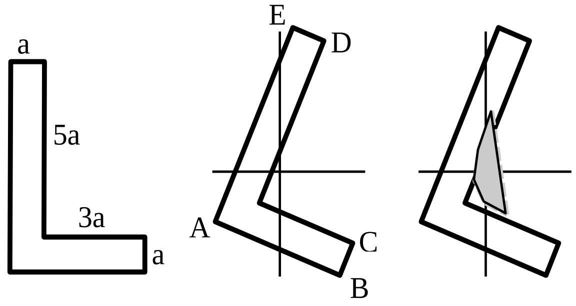

Fig. 6.3 Angle profile and its core

Solution

There is no symmetry axis.

Area

$$A = 9a^2$$

Centroid position

$y_C = \frac{5a^2 \cdot 0.5a + 4a^2 \cdot 2a}{9a^2} = 1.17, \;\; z_C = \frac{5a^2 \cdot 3.5a + 4a^2 \cdot 0.5a}{9a^2} = 2.17$

Central inertia moments

$$I_{yc} = \frac{5^3 a^4}{12} + 5a^2(3.5a - 2.17a)^2 + \frac{4a \cdot a^3}{12} + 4a^2(0.5a - 2.17a)^2 = 30.75a^4$$

$$I_{zc} = \frac{4^3 a^4}{12} + 4a^2(2a - 1.17a)^2 + \frac{5a \cdot a^3}{12} + 5a^2(0.5a - 1.17a)^2 = 10.75a^4$$

$$I_{yczc} = 0 + 5a^2(3.5a - 2.17a)(0.5a - 1.17a) + 0 + 4a^2(0.5a - 2.17a)(2 - 1.17a) = -10a^4$$

Principal central inertia moments and directions

$$I_y = \frac{30.75 + 10.75}{2}a^4 + \sqrt{\left(\frac{30.75 - 10.75}{2}\right)^2 + 10^2 \cdot a^4} = 34.98a^4$$

$$I_z = \frac{30.75 + 10.75}{2}a^4 - \sqrt{\left(\frac{30.75 - 10.75}{2}\right)^2 + 10^2 \cdot a^4} = 6.61a^4$$

$$\tan\alpha = \frac{34.98 - 30.75}{10} = 0.423 \rightarrow \;\; \alpha = 22.5°$$

Transformation of points coordinates to the principal central inertia system; the results are presented in Tab. 6.1.

Line	**b**	**c**	y_N	z_N
AB	-5.67 a	-2.35 a	0.13 a	1.65 a
BC	3.08 a	-7.33 a	-0.24 a	0.53 a
CD	1.85 a	12.37 a	-0.40 a	0.31 a
DE	9.89 a	4.15 a	-0.07 a	-0.94 a
AE	-1.27 a	3.07 a	0.58 a	-1.27 a

Tab. 6.1 Corners coordinates

Review problems

Determine the core for the cross-sections given in Fig. 7.8. Assume all needed geometry data.

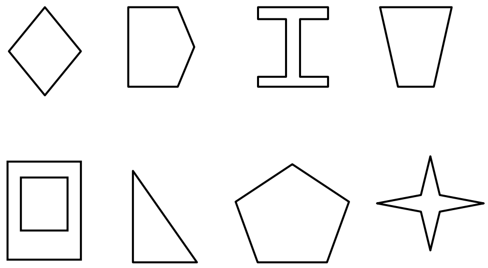

Fig. 6.4 Core - review problems