Beams deflections – Mohr's method

Introduction

An analogy between two differential equations is used. The analogy is full if the solutions of both equations are the same. The case takes a place when right sides of the equations are equal and the boundary conditions of both equations are compatible.
A fictive beam is constructed in such a way that the static boundary conditions correspond to the kinematic boundary conditions of the real beam. The fictive beam's load is the diagram of the bending moments of the real beam, divided by the flexural stiffness. The bending moments and shear forces of the fictitious beam are equal to the deflections and rotations of the real beam, respectively

$$M^f(x) \equiv w(x), \quad Q^f(x) \equiv w'(x).$$

Geometric formulae

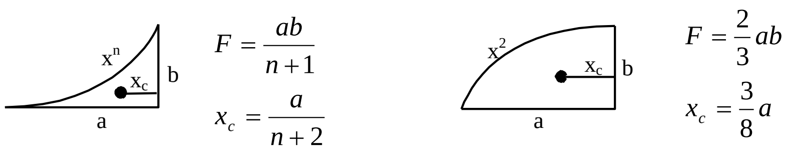

Fig. 9.1 Geometric formulae

Superposition

If the tangent to the bending moment diagram is not horizontal, use the superposition principle and add the component diagrams, Fig. 9.2

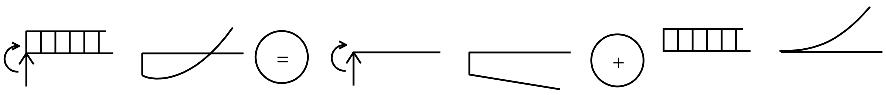

Fig. 9.2 Superposition principle application

Examples

Example 9.1

Determine the deflection and the rotation angles at the hinge of the beam in Fig. 9.3, $EJ = 200 \cdot 10^5$ Nm^2:

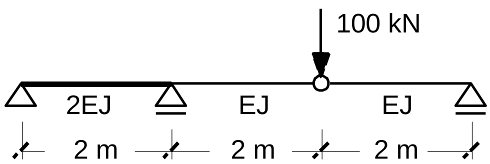

Fig. 9.3 Beam with load

Solution

The bending moment diagram:

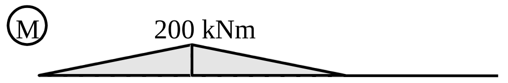

Fig. 9.4 Bending moment

The fictitious beam and decomposition into simple beams is presented in Fig. 9.5:

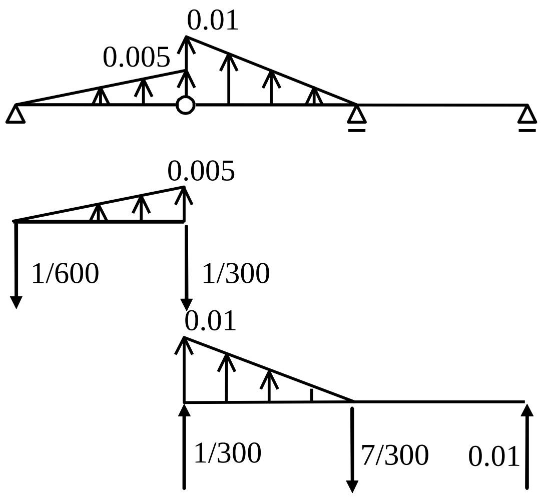

Fig 9.5. Fictitious beam

The fictitious moment at the pin support (the deflection of the hinge):

M^{f} = (2/3 + 2 2/3) 10^{-2} = 0.02 m = 2 cm,

the fictitious shear force from the left (the rotation angle):

Q^{f}_{L} =4/3 10^{-2} = 0.0133 $\Rightarrow$ α = 0.76°

the fictitious shear force from the right (the rotation angle):

Q^{f}_{R} =(4/3-7/3) 10^{-2} = -0.01 $\Rightarrow$ β = -0.57°

Fig. 9.6 presents the deflection sketch.

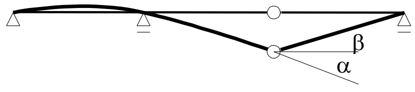

Fig. 9.6 Deflection sketch

Example 9.2

Determine the deflections of the beam in Fig. 9.7.

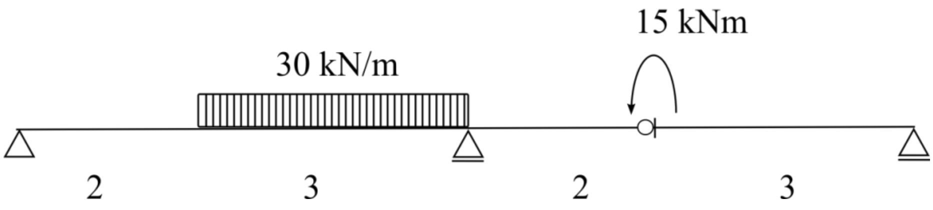

Fig. 9.7 Beam with load

Solution

statics: decomposition into the simple beams & constraints reactions

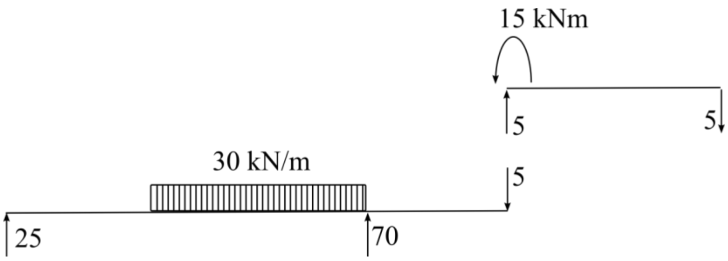

Fig. 9.8 Beam decomposition

diagram of bending moments:

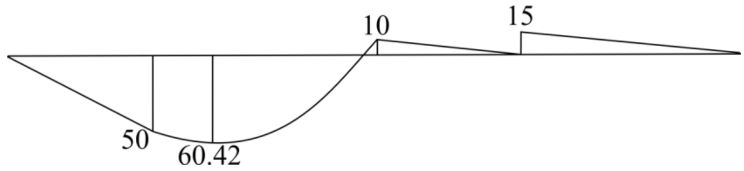

Fig. 9.9 Diagram of bending moments

$Q(x) = 25 - 30(x - 2) = 0 \quad \rightarrow \quad x = 2.833$ m, $M(2.833) = 60.42$ kNm

Fictitious beam

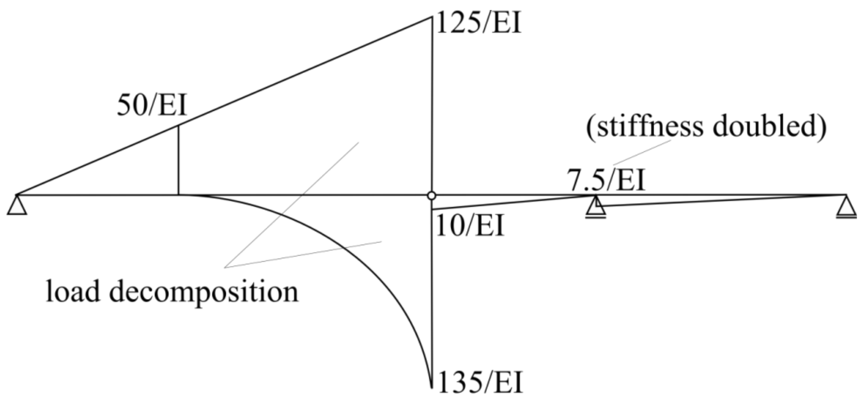

Fig. 9.10 Fictitious beam with the load

static scheme of beams

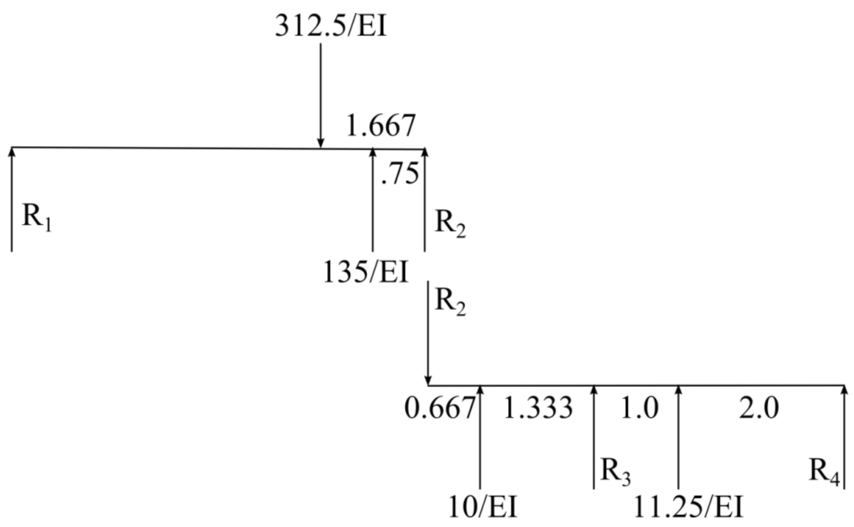

Fig. 9.11 Equivalent scheme of the beam

$$R_1=\frac{83.92}{EI},\quad R_2=\frac{93.58}{EI},\quad R_3=\frac{134.0}{EI},\quad R_4=-\frac{61.69}{EI}$$

$$Q^f(2^-)=-\frac{83.58}{EI},\quad Q^f(2^+)=\frac{50.42}{EI},\quad M^f(3)=R_4\cdot 3+\frac{11.25}{EI}\cdot 1=-\frac{173.8}{EI}$$

(the same results as from the Macaulay's method)

Example 9.3

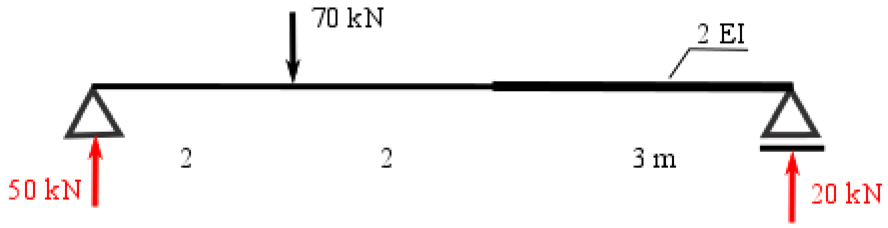

Fig. 9.12 Beam with load

Solution

Constraints reactions from the balance equations: $R_a=50$ kN, $R_b=20$ kN. The bending moments' diagram is presented in Fig. 9.13.

Fig. 9.13 Bending moments diagram

The fictitious beam should have the same static scheme as the real beam, load with the bending moments diagram, divided by the local flexural stiffness, Fig. 9.14.

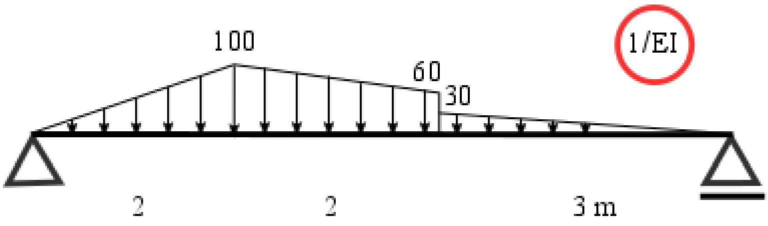

Fig. 9.14 Fictitious beam

Because we are interested in the solution values at particular points only, we replace continuous loading by point forces, Fig. 9.15.

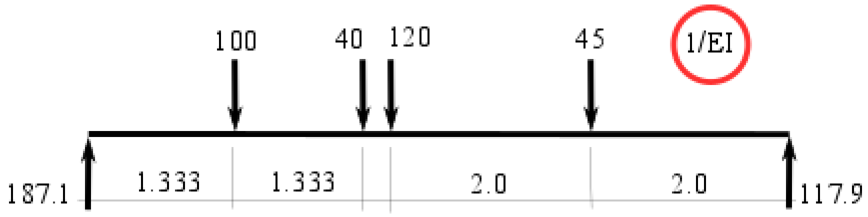

Fig. 9. 15 Fictitious beam with "discretized loads"

Using the above scheme, we calculate secondary bending moments and shear forces at two points:

$$M^f(x=2)=w(2)=\left(187.1\cdot 2-100\cdot\frac{2}{3}\right)\cdot\frac{1}{EI}=\frac{307.6}{EI}$$

$$M^f(x=4)=w(x=4)=w(x_1=3)=(117.9\cdot 3-45\cdot 1)\cdot\frac{1}{EI}=\frac{308.6}{EI}$$

$$Q^f(x=2)=w'(2)=(187.1-100)\cdot\frac{1}{EI}=\frac{87.1}{EI}$$

$$Q^f(x=4)=w'(x=4)=w'(x_1=3)=-117.9+45=-\frac{72.9}{EI}$$

The deflection diagram is shown in Fig. 9.16.

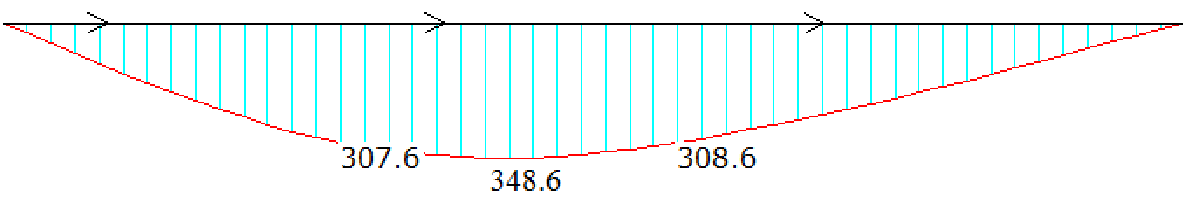

Fig. 9.16 Beam deflections

Example 9.4

Determine the beam deflections and rotations at points C and D, Fig. 9.17.

Solution

From calculations of the free body balance we get the constraints reactions: $R_a = 10$ kN, $R_b = 10$ kN, $M_d = 90$ kNm (clockwise).

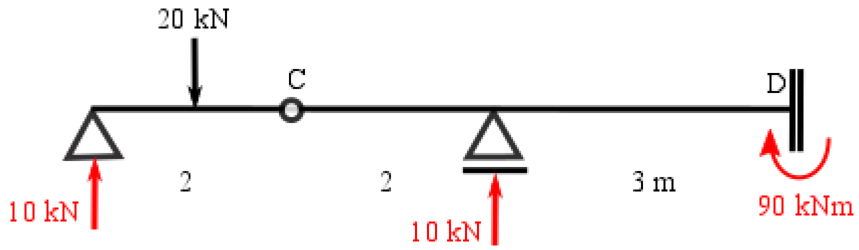

Fig. 9.17 Beam with the load and constraints reactions

From calculations of the bending moments we get the diagram, Fig. 9.18.

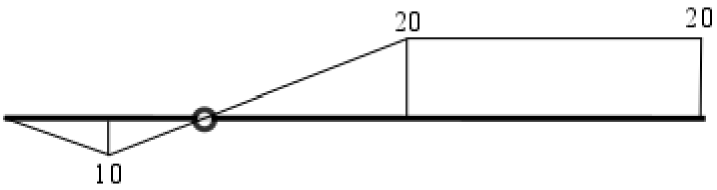

Fig. 9.18 Diagram of bending moments

The fictitious beam is loaded with the bending moments divided by the flexural stiffness, *EI*:

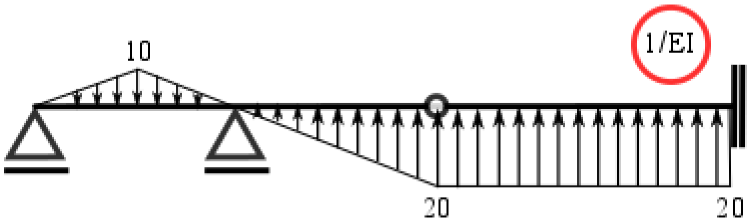

Fig. 9.19 Fictitious beam with loading

and the equivalent scheme is presented in Fig. 9.20:

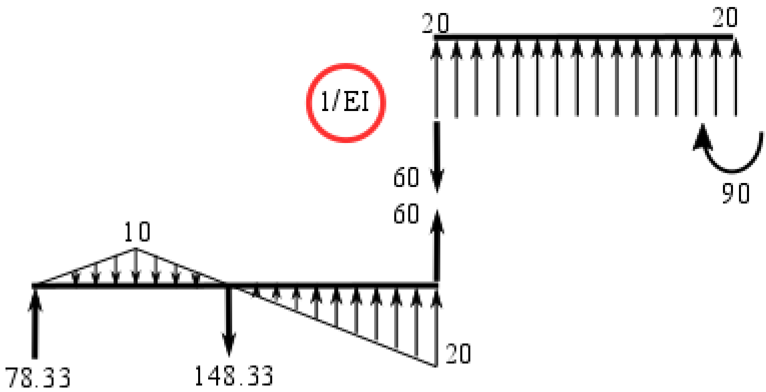

Fig. 9.20 Equivalent fictitious beam

$$M_C^f = w_C = \frac{78.33 \cdot 2 - 10 \cdot 1}{EI} = \frac{146.7}{EI}$$

$$M_D^f = w_D = -\frac{90}{EI}$$

$$Q_{C\,L}^f = w_C'^L = \frac{78.33 - 10}{EI} = \frac{68.33}{EI}$$

$$Q^f_{C\,R} = w'^R_C = \frac{-60 - 20}{EI} = -\frac{80}{EI}$$

Review problems

Determine the deflection at the point *K* of the beams in Fig. 9.21, using the Mohr's method.

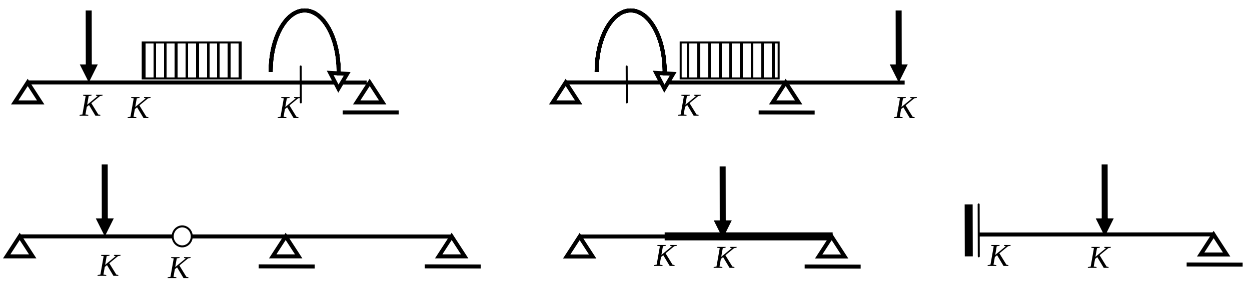

Fig. 9.21 Review problems