1. Preliminaries

The rules of obtaining the credit

The absence in excess of three (the limit) will result in lack of the credit.
During classes (except the first ones), there will be short tests estimated from 0 up to 5 points. The overall score should be over 55% to obtain the credit. The score below 45% means the lack of the credit.

What is the purpose of the course?

Although the main purpose of academic studies is to discuss ideas rather than carry out some "practical calculations", we will also focus our attention on some aspects of calculation techniques and their efficiency.
The first semester of the course is composed of two different parts:
− the statics, with a little theory and huge amount of problems; it constitutes about 90 % of the semester
− the linear elasticity, with a few problems and quite a considerable amount of theory.

How is the course organized?

The course has 15 units. Each unit, except this first, has five parts:
1. Introduction: basic ideas, definitions, theorems, formulae ready to use
2. Example: model of the solution with some explanation of calculation
3. Theme of the design exercise (the homework) without numerical data (please, adopt numerical data individually). The uniqueness of the theme will not be verified, but when, accidentally, more than one (exactly the same) set of data would be found, only the first one presented will be accepted. If you give a copy of your homework to someone, it may happen that his/her solution will be accepted and yours not. To have a unique set of data, the best way is to choose one particular datum (a piece of data), e.g. 12.37.
4. Problems to be solved unaided: typically 10 problems, which will be very similar to the problem on the test during the next class.
5. Addendum: additional remarks and tips, English – Polish glossary

Outline of the units

(navy blue – statics)
1. Preliminaries
2. Geometric stability (rigidity)
3. Cross-section forces
4. Calculation of constraints reactions
5. Simple beams
6. Simple beams – cont.
7. Slant beams
8. Continuous beams
9. Simple frames
10. Arches
11. Trusses
12. Combined structures
13. Cross-section characteristics
14. Stress state analysis
15. Strain state analysis

How to write a design report

A design report should be in the form of computer typesetting and contain:
− the author's name and surname
− all necessary data
− full documentation of all results; it means logic presentation of the solution way (with particular assumptions if necessary), the needed formulae, provided values and intermediate results
− verification of obtained results, if available
− all diagrams should be properly scaled

How to perform "engineering" calculations

Mathematical calculations:
− exact, but the way is more important than the result
− "internal beauty" and concise form of the equations matters; therefore, the rearrangement should often be done

Engineering calculations:
− not "exact", they are rather assessments, with 3-4 significant digits precision; see addendum
− "inline" change into SI fundamental units
− fewer calculations/transformations means fewer errors, the form suitable for practical use, not necessarily "beautiful"; the result is of crucial importance

We will use metric system of units, not imperial.

General assumptions

Every rule can be broken, but no rule may be ignored.
Engineering calculations include some over-simplifications resulting from admitted assumptions. Some of them are obvious, some of them are over-simplifications, but it should always be kept in mind and verified.

Material continuity

In macro-scale every structural material, namely concrete, steel and wood, may be considered as continuous. The scale may be called "macro" depending on some "characteristic length" of the material, it means the length sufficiently great to average material characteristics and sufficiently small to account for macro-changes:
− concrete: $l_0 = 1\,\text{dcm}$
− wood: $l_0 = 1\,\text{cm}$
− steel: $l_0 = 1\,\text{mm}$

Often, this property is explained trough so-called Representative Volume Element (RVE).

Stability of the structure

It is assumed that the structure considered is stable and geometrically rigid. This should always be verified, otherwise, the consequences might be catastrophic (e.g. disaster of the exhibition hall at Katowice in 2006). The phenomenon is very dangerous because it usually occurs instantaneously without any sign of warning. For further reading, see unit 2.

Principle of rigidity

In civil engineering, the structures are very rigid, so we assume that the (insignificant) deformations do not influence static quantities.

Superposition principle

Iff[1] the equations are linear and homogeneous the superposition is valid:

The effect of independently but simultaneously acting causes is equal to the sum of effects resulting from each individual cause.

Static scheme

It is the drawing of the structure including only the information necessary to perform static calculations. From this point of view "continuum", "structure" and "set" are synonyms. We start the static calculations with the static scheme. In "real life" the static scheme should be carefully chosen, so that it corresponds with the considered structure. Usually it is very easy but sometimes it is a very difficult problem, demanding broad knowledge and routine.

Addendum

Some hints on the static results

Tip: To solve the bar set (structure, construction) means to make any necessary diagrams of so-called cross-section forces. In statics, except rare cases, the results are the cross-section forces diagrams. In engineering, the diagrams make sense only if drawn to scale.

Tip: Always use the described methodology rigorously. The best way is the most efficient one: the shortest and the most reliable. Be careful not to take certain short cuts (unreliable, not checked).

Tip: Not the calculations but the ability to subsequent verification of the results makes you an engineer.

Moment about a point

When you calculate a moment about a point resulting from an acting force, keep in mind that the calculation of moment directly from the definition is the worst way. Please remember that the force vector is so-called "sliding vector". Its point of application matters only for the decision whether it will be considered. After this, the point of application doesn't matter. The vector could be translated along its action line to a more convenient point and decomposed. In Fig. 1.1, the lever length d is not convenient to calculate while a may be easily calculated (if not already known).
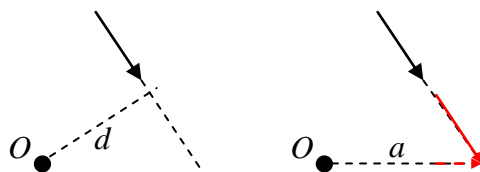

Fig. 1.1 Moment about a point

The moment as a vector is so-called "free" vector: can be freely translated at the plane.

Continuous loading

In Fig. 1.2, all resultants are $R = qa$, in a) and b) the intensity of loading q is in kN per meter, in c) q is in kN per running meter.
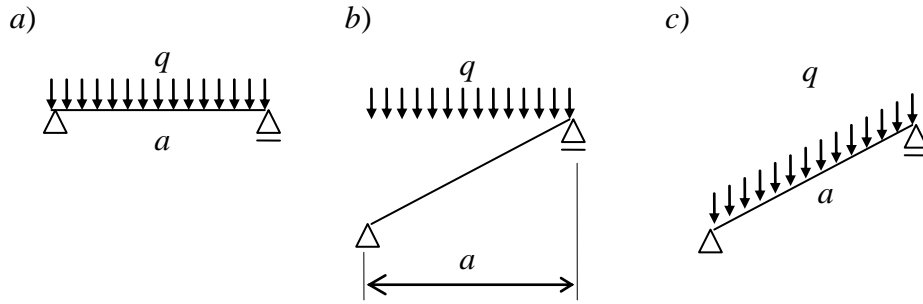
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[1] if and only if

a)   b)   c)

Fig. 1.2 Continuous loading

In Fig. 1.3, the loadings of both beams are equivalent (the same resultant and action line).
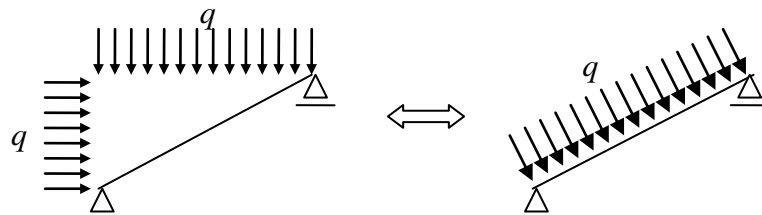
Fig. 1.3 Equivalent loadings
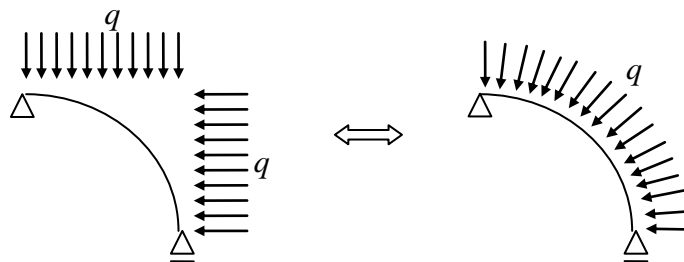
The same is true if the bar axis is curvilinear:

Fig. 1.4 Equivalent loadings on circular arches

In statics we often have to calculate the resultant of some part of loading. In most cases the problem is reduced to the calculation of figure area and gravity centre. It is easy for simple figures such as rectangle or triangle. When the part of a triangle becomes a trapezoid we have to either use the specific formulae or to decompose the figure into simpler ones. The trapezoidal loading, c.f. Fig. 1.5, may be replaced with other equivalent loadings, depending on the purpose. The cases *a)* and *b)* (the rectangle plus/minus the triangle) are suitable for the calculation of cross-section functions from left to right or from right to left, respectively, and *c)* (two rectangles) for the calculation of reactions.
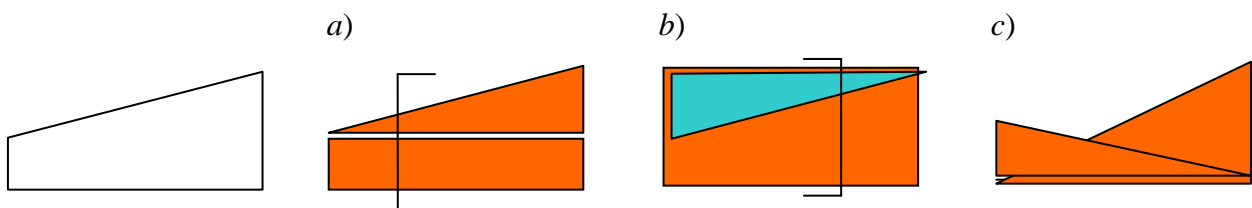
a)   b)   c)

Fig. 1.5 Decomposition of trapezoidal loading

For an arbitrary loading intensity we will use the following formulae for the resultant and its moment about the point of section x, c.f. Fig. 1.6:

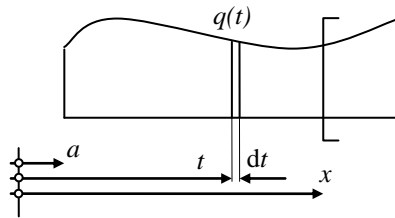$$R(x) = \int_a^x q(t)\,dt, \quad M(x) = \int_a^x q(t)(x-t)\,dt$$

Fig. 1.6 Arbitrary intensity of loading

Significant digits number

The accuracy of the solution of a problem depends upon two items: the accuracy of the given data and the accuracy of the computations performed. The solution cannot be more accurate than the less accurate of these two items.

In engineering problems, the data are seldom known with accuracy greater than 0.1%. It is therefore seldom justified to write the answers with greater accuracy.

Normalized scientific notation has the form of $a \cdot 10^b$, where the exponent b is chosen in a way that the absolute value of a remains at least one but less than ten ($1 \le |a| < 10$).

Engineering notation is similar to scientific notation with the exponent 0 or multiple of 3 (so that the range of values would be more visible).

In the following example all numbers have 3 significant digits:
123000, 1.23, 0.123, 0.000123, 0.00120 (0 at the end is significant)

In-line application of SI units

Given the data set:
Q = 12 kN, S_y = 1600 cm^3, J_y = 28000 cm^4, b = 20 mm
we calculate from the formula:

$$\tau_{xz} = \frac{Q \cdot S_y}{J_y \cdot b} = \frac{12 \cdot 10^3 \cdot 1600 \cdot 10^{-6}}{28000 \cdot 10^{-8} \cdot 20 \cdot 10^{-3}} \left[\frac{N \cdot m^3}{m^4 \cdot m}\right] = 3.43 \cdot 10^6 \left[\frac{N}{m^2} = Pa\right] = 3.43\,MPa$$

Example of equation final form

An equation of bending moment:

$$M(x) = 120 \cdot x - \frac{40}{2}(x-4)^2 - 30 \cdot (x-4) = \ldots = -40 \cdot x^2 + 530 \cdot x - 640$$

When we have to calculate the value of the bending moment at the end of interval (x = 4), the first form *in statu nascendi* is more useful. The transformation to the "final" form is more complicated and risky (due to possible mistakes!).

Workshop theme

For the situations below, Fig. 1.7, determine the moments about the point O. The coordinates of points *A, B, C* and *O* as well as the values of the forces are given. Specify the action of each moment: clockwise or counterclockwise.
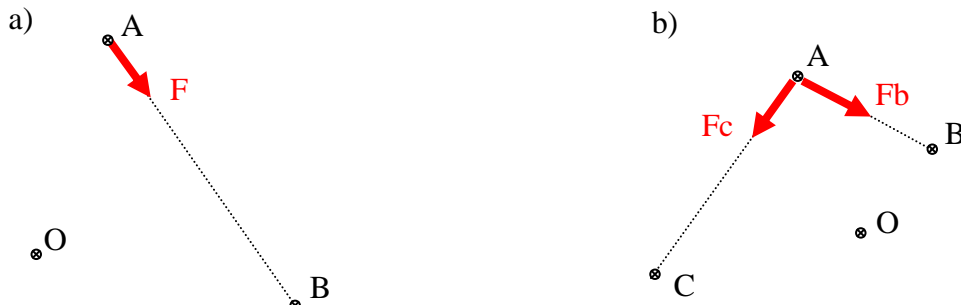
a)

A

F

O

B

b)

A

Fb

Fc

B

O

C

Fig. 1.7 (workshop's problem)

Example of design exercise report

Author: John Kowalsky, group #01
date: November 16, 2013
Design exercise #4
Title: Simply supported beam
Theme: Write cross-section equations and draw their diagrams for the beam in Fig. 1.8. The dimensions are in [m] and the angle is 45 degrees.
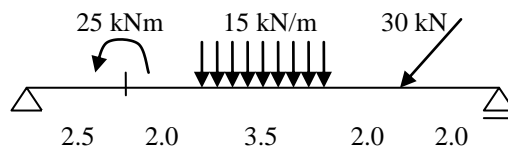

25 kNm   15 kN/m   30 kN

2.5   2.0   3.5   2.0   2.0

Fig. 1.8 Simply supported beam

Solution

Geometric rigidity:
Beam has internal geometric stability (1 shield)
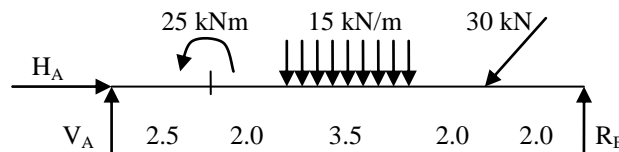Beam has geometric rigidity as a whole (2ST).
Beam reactions:

H_A

25 kNm   15 kN/m   30 kN

V_A 2.5   2.0   3.5   2.0   2.0 R_B

Fig. 1.9 Beam with reactions

$$\sum M_B = 0 \implies V_A = \frac{25 + 15 \cdot 3.5 \cdot 5.75 + 30\sqrt{2}/2 \cdot 2}{12} = 30.78\,\text{kN}$$

$$\sum M_A = 0 \implies R_B = \frac{-25 + 15 \cdot 3.5 \cdot 6.25 + 30\sqrt{2}/2 \cdot 10}{12} = 42.94\,\text{kN}$$

$$\sum X = 0 \implies H_A = 21.21\,\text{kN}$$

Verification:

$\sum Y = 15 \cdot 3.5 + 30\sqrt{2}/2 - 30.78 - 42.94 = 52.5 + 21.21 - 73.72 = 73.71 - 73.72 = 0.01 \approx 0$, OK!

Cross-section forces equations:
1st interval:

$0 < x < 2.5$
$$\begin{cases} M(x) = 30.78x, \quad M(0) = 0, \quad M(2.5) = 76.95\,[\text{kNm}] \\ Q(x) = 30.78\,[\text{kN}] \\ N(x) = -21.21\,[\text{kN}] \end{cases}$$

2nd interval

$2.5 < x < 4.5$
$$\begin{cases} M(x) = 30.78x - 25, \quad M(2.5) = 51.95, \quad M(4.5) = 113.5\,[\text{kNm}] \\ Q(x) = 30.78\,[\text{kN}] \\ N(x) = -21.21\,[\text{kN}] \end{cases}$$

3rd interval

$4.5 < x < 8$
$$\begin{cases} M(x) = 30.78x - 25 - 15\dfrac{(x - 4.5)^2}{2}, \quad M(4.5) = 113.5, \quad M(8) = 129.4\,[\text{kNm}] \\ Q(x) = 30.78 - 15 \cdot (x - 4.5), \quad Q(4.5) = 30.78, \quad Q(8) = -21.72\,[\text{kN}] \\ N(x) = -21.21\,[\text{kN}] \end{cases}$$

(due to change of shear force's sign, we calculate the extreme bending moment)
$Q(x) = 0 \;\rightarrow\; x = 6.552, \quad M(6.552) = 145.1\,[\text{kNm}]$
4th interval
(we use continuous loading resultant)

$8 < x < 10$
$$\begin{cases} M(x) = 30.78x - 25 - 15 \cdot 3.5 \cdot (x - 6.25), \quad M(8) = 129.4, \quad M(10) = 85.93\,[\text{kNm}] \\ Q(x) = 30.78\text{-}15 \cdot 3.5 = \text{-}21.72\,[\text{kN}] \\ N(x) = -21.21\,[\text{kN}] \end{cases}$$

5th interval
(for the last interval we use another coordinate, x_1)

$0 < x_1 < 2$
$$\begin{cases} M(x_1) = 42.94x_1, \quad M(0) = 0, \quad M(2) = 85.88\,[\text{kNm}] \cong M(x = 10) \\ Q(x_1) = -42.94\,[\text{kN}], \quad \text{ver.} : Q(x = 10\,)\text{-}P_V = -21.72 - 30\sqrt{2}\,/\,2 = -42.93 \cong -42.94, \text{OK} \\ N(x_1) \equiv 0 \end{cases}$$

Cross-section forces diagrams:
The diagrams result from the computer program "statyka", ver. 2007.
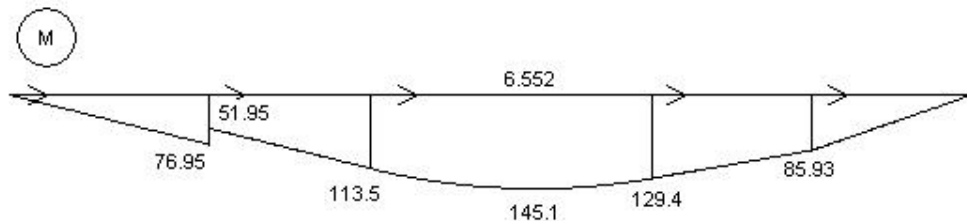Bending moments:

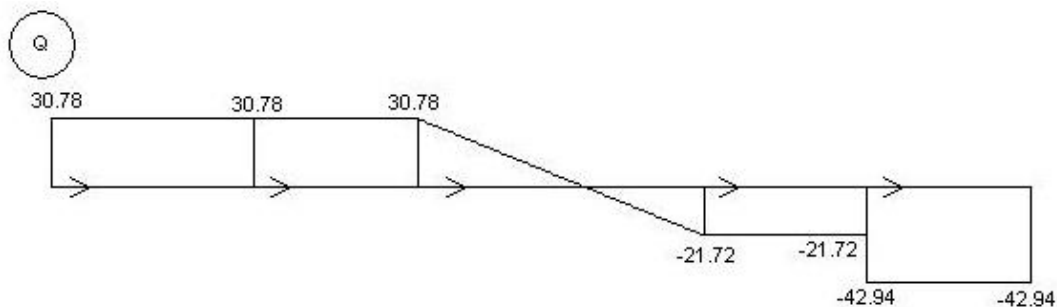Fig. 1.10 Bending moments diagram

Shear forces:
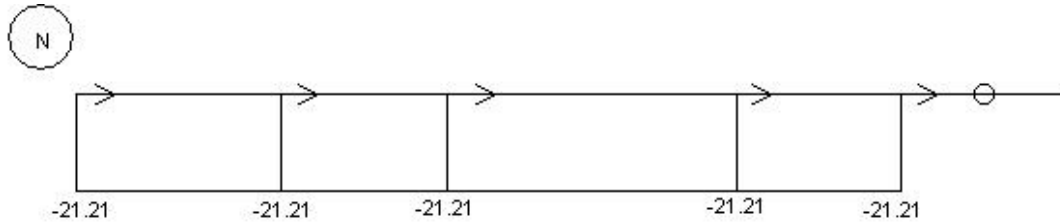

Fig.1.11 Shear forces diagram

Axial forces:

Fig.1.12 Axial forces diagram

Glossary
bar – pręt
section, cut, cutting – przekrój, cięcie
sliding vector – wektor przesuwny
free vector – wektor swobodny
continuous loading – obciążenie ciągłe
intensity of continuous loading – gęstość (intensywność) obciążenia ciągłego, kN/m
per meter – na metr (w rzucie)
per running meter – na metr bieżący (pręta, mierzony wzdłuż jego osi)
principle of rigidity – zasada zesztywnienia
superposition principle – zasada superpozycji (addytywności wyników)
static, statical – statyczny
stability – stateczność
stable – stateczny, stabilny