2. Geometric stability (rigidity)

Introduction

Definitions

degrees of freedom (DOF) – a number of independent parameters that describe the movement of the structure

mechanism – a structure with at least one DOF

bar – an element with length much greater than two other dimensions

shield – a set of elements rigidly linked (reciprocal movements of all elements are excluded); from the kinematic point of view, a shield can be considered as a bar and vice versa

hinge – a connection that can be considered as a connection by two bars pinned at the point

virtual velocity – a velocity consistent with imposed constraints

virtual displacement – a displacement consistent with virtual velocity (the same direction and sense)

instantaneous centre of rotation (ICR) – a centre of rotation following the virtual displacements

geometric rigidity – a rigidity of the whole structure

free-body geometric rigidity – a rigidity of the structure considered in isolation from constraints (internal or local rigidity)

Theorems

A shield has only one ICR.

If two points of a shield are immobilized, the whole shield is immobilized.

(Two shields theorem, 2ST)

The rigid connection between two shields consists in the connection by three bars, the direction of which don't intersect at one point (including so-called improper point – at infinity)

Instead of three bars, the connection of two shields may consists in one hinge and one bar not passing by the hinge.

(Three shields theorem, 3ST)

The rigid connection of three shields consists in the connection of each pair of shields by one hinge and these hinges are not collinear.

It means that the connection "in triangle" is geometrically rigid.

Note! In fact this means a connection of three elements, not the shape of the set (see Fig. 2.1).
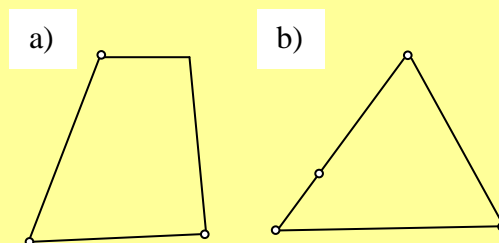

Fig. 2.1 a) the quadrangle shape made of three elements is geometrically rigid,
b) the triangular shape made of four elements is not geometrically rigid

Unfortunately, in English (as well as in Polish) there is no such word as "three-lateral".

(Virtual velocities theorem, VVT)

A structure is geometrically rigid if and only if there is no consistent field of virtual velocities.

The opposite theorem is also valid.
To prove that the whole structure is rigid, it should be proved that every point of the structure is blocked (unmovable).

Methods to determine geometric rigidity

The use of two or three shields theorem is easy but not always effective. The theorem of virtual velocities is always effective, but – usually – not the easiest, because it should be proved for all elements of a structure considered.
Usually we proceed in two steps: free-body rigidity and rigidity of the whole structure (overall rigidity). Each step of analysis is important and may be fruitful.
The analysis of free-body rigidity suggests the way of reactions calculation. Free-body stiff structure means (there will be) no problem with reactions calculation. Free body unstable system usually requires some particular effort: hinge equation (zero hinge's moment), additional intersections and so on. In extreme cases reaction calculation may be very difficult.
The rigidity of the whole structure is needed for static calculations. The mechanisms (the structures unstable) should be analyzed dynamically, with the use of inertia (d'Alembert's) forces.
Although in particular cases unstable structures can be solved by static equations only, we deliberately limit scope of static considerations to the problems with zero (or less) DOF.
If the rigidity of some subset is proved we may replace it by a shield in the sequel.

Example

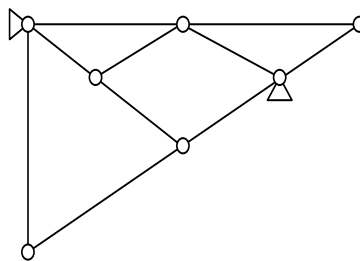Let's determine geometric stability of the structure from the figure below:

Fig. 2.2 The structure considered

Solution by 3 shields theorem

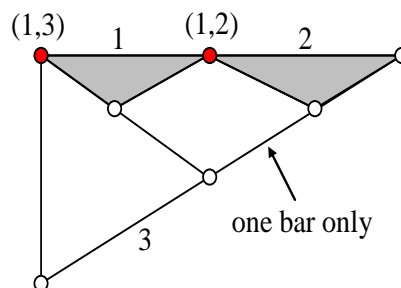1. We determine the free-body rigidity

Fig. 2.3 Free-body rigidity

We detect two sets "in triangle" – there are two shields. Using the theorem of three shields we find that the free-body structure is not rigid: one bar is missing between the shields 1 and 3.

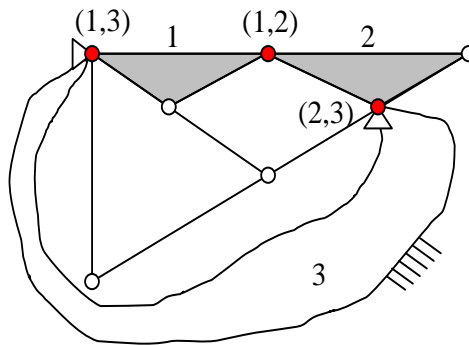2. We determine the whole structure rigidity

Fig. 2.4 Overall rigidity of the structure

We detect three shields (one of them is the bedrock) which are correctly connected: three hinges of connection are not collinear, so they are blocked.
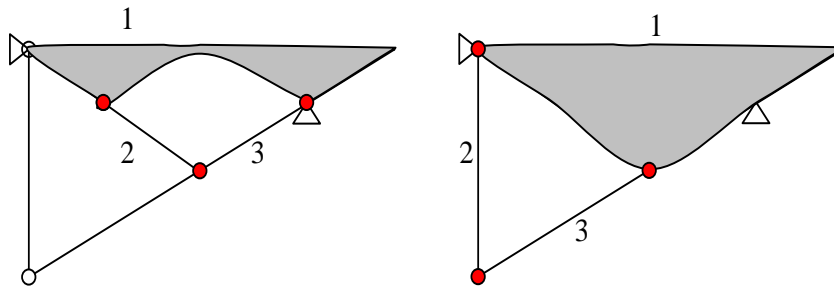

Fig. 2.5 Overall rigidity of the structure, cont.

We find three shields properly connected (Fig. 2.5 from the left) and next – considering the rigid subset as a shield – we come to the same conclusion once more (the Fig. 2.5 from the right). In this way we make certain that all elements of the structure are immobilized.

This authorizes us to claim that the structure is free-body unstable and entirely (it means with constraints) stable.

The solution by virtual velocities method

For free-body rigidity, we start with the three-laterals, one of them tentatively immobilized. The points of the blocked shield are ICR of adjacent bars and the shield. There is no contradiction in given velocities sketch so movement is possible.
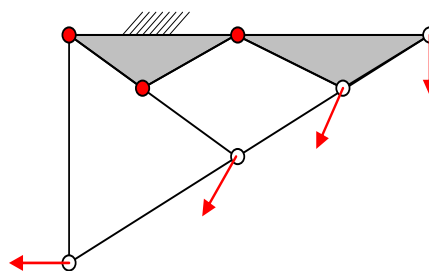

Fig. 2.6. Free-body rigidity by VV method

For the whole structure rigidity, we consider two ICRs at the pins. Two different velocities result at the point of two shields' connection. This contradiction vanishes if and only if both velocities are zero, so the point should be fixed, see Fig. 2.7
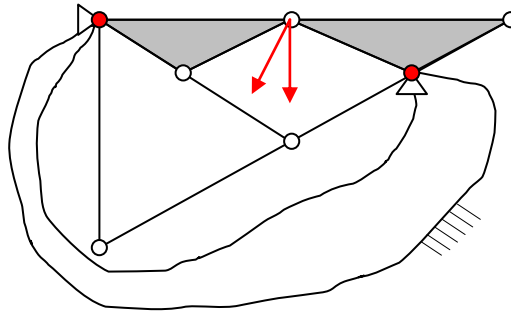

Fig. 2.7 Contradiction at the connection point

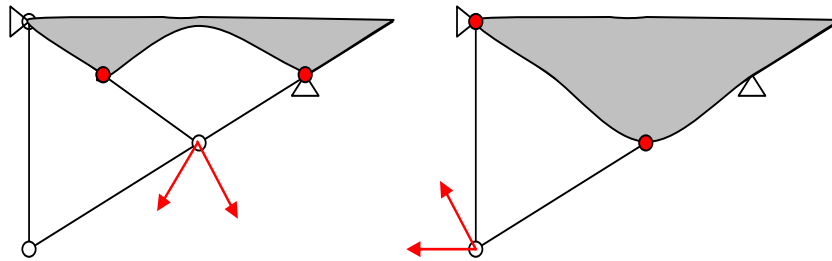Repeating the reasoning, as successive figure 2.8 show, we find the rigidity of the whole structure.

Fig. 2.8 Contradictions at next points

The conclusion is the same as before: the structure is free-body unstable and stable as a whole.

Workshop theme

For the structures below, determine their stability using known methods, but one for each reasoning (using 2ST only, 3ST only and VVT only, 3×3 = 9 solutions for free-body stability and, similarly 9 solutions for the stability of the entire structures).

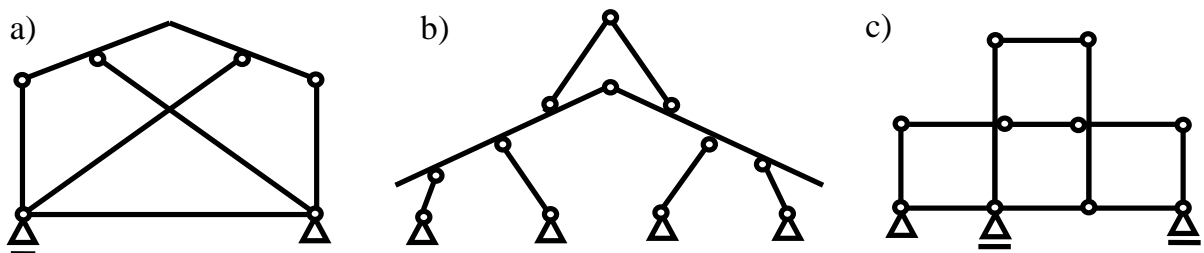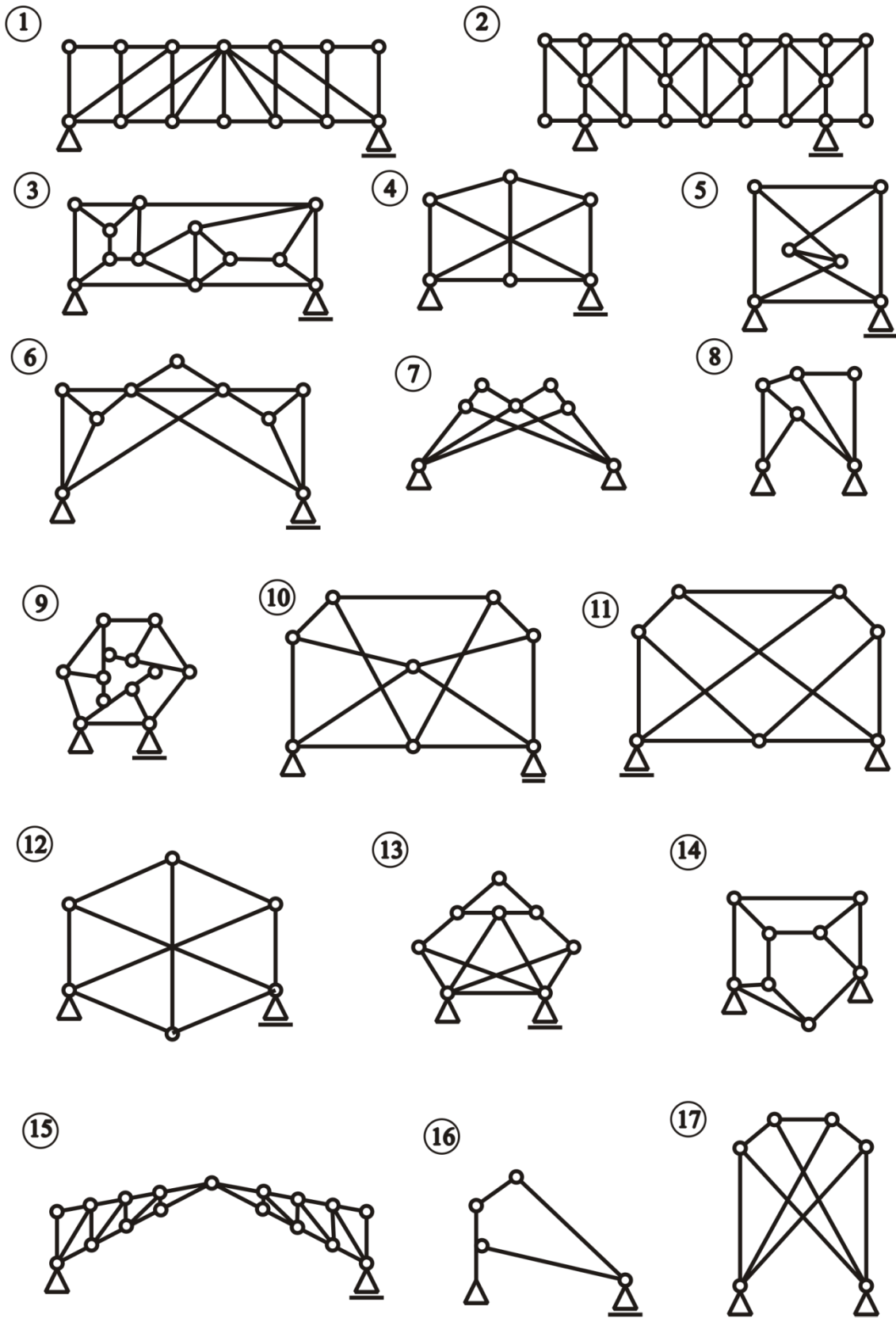Fig. 2.9 Free-body and overall stability problems

Review problems

Fig. 2.10 Free-body and overall stability review problems.

Addendum

Displacements

There are three different kinds, see the figure below:
– real (true), calculated or measured,
– allowed (possible), that may be anticipated
– virtual, that agree with virtual velocities directions.
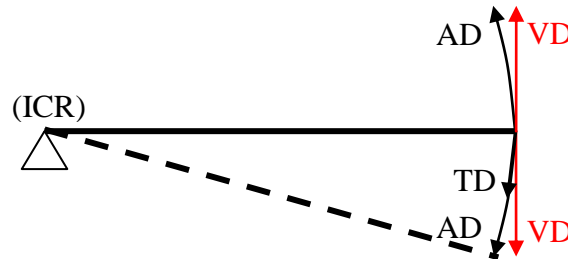

Fig. 2.11 Displacement: true, possible and virtual

Looking at Fig. 2.9, some people think that the virtual displacement – as an approximation of the real displacement – should be infinitesimal (sufficiently small), especially as the real displacements in civil engineering are close to infinitesimal. It is not true: it is not an approximation and its order is arbitrary (and may be finite).

Glossary

"geometric rigidity", mechanical stability – geometryczna niezmienność
(Visibly, there is no English equivalent. Common use of "rigidity" or "stability" might be misleading. Both words have a different meaning in structural mechanics. "Rigidity", as opposite to flexibility, means "hardly flexible" or "stiff". "Stability" is connected with loss of stability – sudden and very dangerous phenomena (which will be considered in the second semester of our course)
free-body rigidity – geometryczna niezmienność wewnętrzna
(the whole structure, overall) rigidity – geometryczna niezmienność (zewnętrzna, całości)
bar – pręt
hinge – przegub
shield – tarcza
instantaneous centre of rotation (ICR) – chwilowy środek obrotu
bedrock – ostoja