3. Reactions of constraints

Definitions

mechanism – a structure which has some degrees of freedom and can be analyzed dynamically only
stable (rigid) structure – a structure which has zero degree of freedom
statically indeterminate problem – a type of static analysis problem in which internal forces are not calculable on considering statical equilibrium alone

Constraints and their reactions

We consider 2D case only.

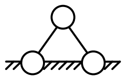
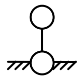
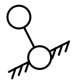
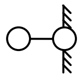
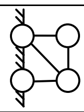
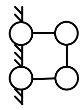
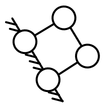
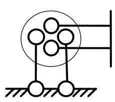
name	scheme	kin. sch.	DOF[1]	reactions[2]
pin			1 (R)	1 V, 1 H
roller			2 (R, T)	1 V
slanted roller			2 (R, T)	1 P
vertical roller			2 (R, T)	1 H
fixing, fixed end			0	1 V, 1 H, 1 M
moving fixing guided support			1 (T)	1 H, 1 M
moving slanted fixing			1 (T)	1 P, 1 M
no rotation (parallelogram)			2 (T)	1 M

Table 3.1 Constraints in 2D

Equilibrium equations

3D case

$$\sum X = 0, \quad \sum Y = 0, \quad \sum Z = 0, \quad \sum M_X = 0, \quad \sum M_Y = 0, \quad \sum M_Z = 0$$

2D case

1st form:

$$\sum M_A = 0, \quad \sum M_B = 0, \quad \sum M_C = 0, \quad A, B, C - \text{not collinear}$$

[1] R – rotation, T - translation
[2] V – vertical, H – horizontal, P – perpendicular, M – fixing moment

2nd form:

$$\sum M_A = 0, \quad \sum M_B = 0, \quad \sum L = 0, \quad L \text{ not parallel to } \overline{AB}$$

3rd form:

$$\sum X = 0, \quad \sum Y = 0, \quad \sum M_O = 0$$

A set of convergent forces

$$\sum X = 0, \quad \sum Y = 0, \quad X \text{ not parallel to } Y \text{ (2 equations only)}$$

A set of parallel forces

$$\sum M_A = 0, \quad \sum L = 0, \quad L \text{ not parallel to the forces direction (2 equations only)}$$

Additional equations

The most common case of additional balance equations is so-called hinge equation (zero moment of forces from one side of a hinge).

Rules of calculations

The equilibrium of a structure can be ensured if and only if the structure is rigid (stable).
Application of reactions means that constraints were replaced by their actions. Strictly proceeding, the constraints and their reactions shouldn't be drawn in the same figure.
Due to the solidification assumption, the results do not depend on the structure shape. The position of constraints and the hinges as well as the action lines of loadings matter only.
The numerical results of constraints calculations should be correct. It is absolutely essential. Incorrect values of reactions disqualify the whole solution, which will not be even further verified. So, careful verification should be provided.

Examples

Example of free-body stable structure

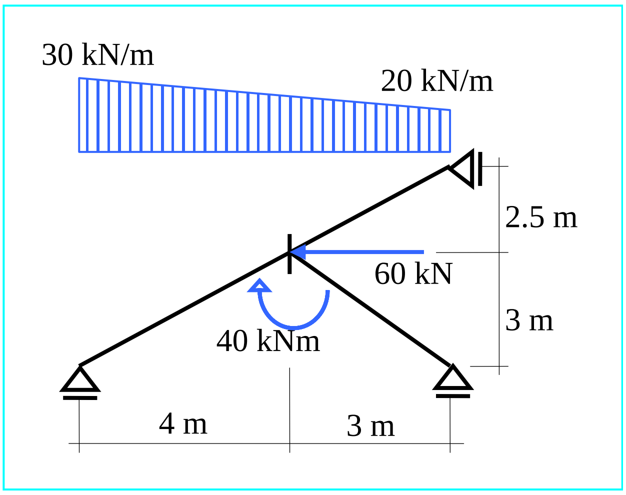


Fig. 3.1 Structure with the load

Solution:

We replace the constraints with the reactions forces, Fig. 3.1 and exchange the trapezoid loading into two triangles.

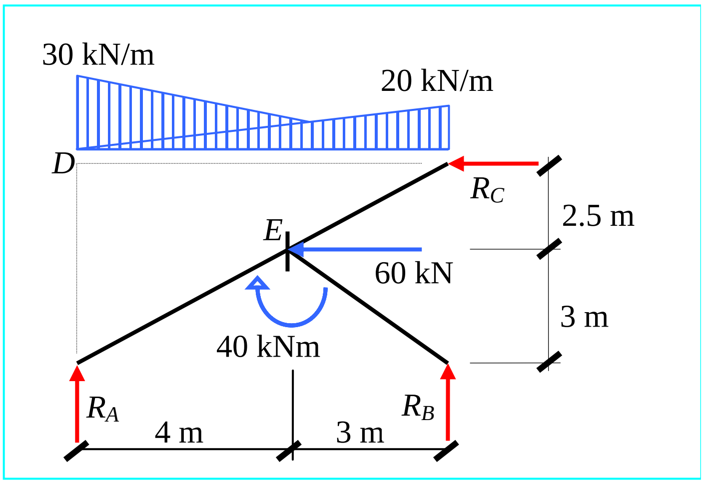


Fig. 3.2 Structure with reactions

We choose the set of balance equations (uncoupled):

$$\sum M_C = 0 \quad \rightarrow \quad R_A = \frac{\frac{1}{2}\cdot 30\cdot 7\cdot \frac{2}{3}\cdot 7 + \frac{1}{2}\cdot 20\cdot 7\cdot \frac{1}{3}\cdot 7 - 60\cdot 2.5 - 40}{7} = 66.19\,\text{kN}$$

$$\sum M_D = 0 \quad \rightarrow \quad R_B = \frac{\frac{1}{2}\cdot 30\cdot 7\cdot \frac{1}{3}\cdot 7 + \frac{1}{2}\cdot 20\cdot 7\cdot \frac{2}{3}\cdot 7 + 60\cdot 2.5 + 40}{7} = 108.81\,\text{kN}$$

$$\sum X = 0 \quad \rightarrow \quad R_C = -60\,\text{kN}$$

Verification:

(Due to simplicity of 3rd equation we verify values R_A and R_B only.)

$\sum Y = 66.19 + 108.81 - 25\cdot 7 = 175 - 175 = 0$, OK

Example of 3-hinges structure

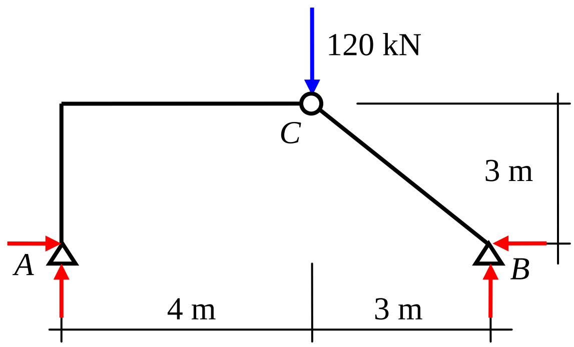


Fig. 3.3 Structure with reactions

Solution

Because the points *A* and *B* are at the same level, the balance equations can be partially uncoupled.

$$\sum M_B = 0 \quad \rightarrow \quad V_A = \frac{120\cdot 3}{7} = 51.43\,\text{kN}$$

$$\sum M_A = 0 \quad \rightarrow \quad V_B = \frac{120\cdot 4}{7} = 68.57\,\text{kN}$$

$$\sum X = 0 \quad \rightarrow \quad H_A = H_B$$

hinge equation:

$$\sum M_C^R = 0 \quad \rightarrow \quad 3H_B = 3V_B \quad \rightarrow \quad H_B = V_B = H_A = 68.57\,\text{kN}$$

Verification:

2nd hinge equation:

$\sum M_C^L = 4V_A - 3H_A = 4 \cdot 51.43 - 3 \cdot 68.57 = 0.01 \approx 0$, OK

Example of a structure analogous to 3-hinges structure

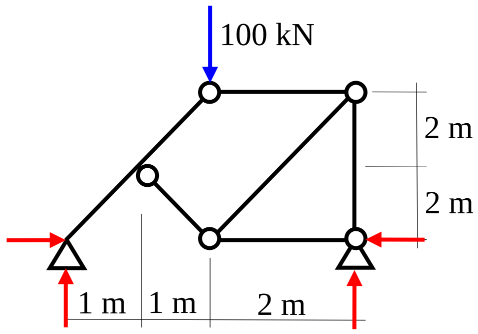


Fig. 3.4 Structures with reactions

Solution
(However a solution is possible, the way presented below is not the shortest one)
We decompose the structure cutting through the hinges, Fig. 3.5.

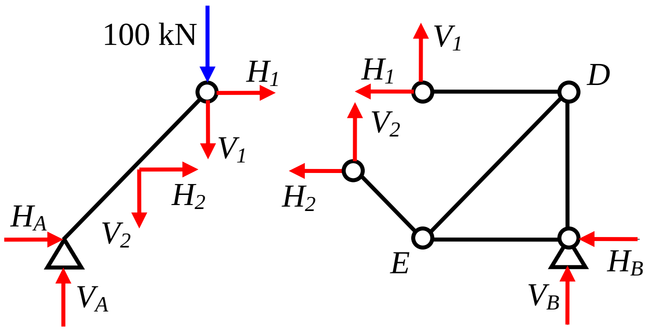


Fig. 3.5 Structure cut through the hinges *1* and *2*.

$\sum M_B = 0 \quad \rightarrow \quad V_A = \frac{100 \cdot 2}{4} = 50 \text{ kN}$

$\sum M_A = 0 \quad \rightarrow \quad V_B = 50 \text{ kN}$

$\sum M_D^L = 0 \quad \rightarrow \quad V_1 = 0$

$\sum M_E^L = 0 \quad \rightarrow \quad V_2 = H_2$

Left part balance:

$\sum Y^L = 0 \quad \rightarrow \quad V_A - V_2 - V_1 - 100 = 0 \quad \rightarrow \quad V_2 = -100 - 0 + 50 = -50 \text{ kN}, \ H_2 = -50 \text{ kN}$

$\sum M_A^L = 0 \quad \rightarrow \quad 2H_1 + 100 \cdot 2 + H_2 \cdot 1 + V_2 \cdot 1 = 0 \quad \rightarrow \quad H_1 = \frac{1}{2}(-200 + 50 + 50) = -50 \text{kN}$

$\sum X^L = 0 \quad \rightarrow \quad H_A + H_2 + H_1 = 0 \quad \rightarrow \quad H_A = 50 + 50 = 100 \text{ kN}$

and, finally

$H_B = 100 \text{kN}$

Verification:

$\sum Y = V_A + V_B - 100 = 50 + 50 - 100 = 0$

(It is clearly visible, that the moment about any point on the vertical "symmetry" axis equals zero).

Workshop theme

Chose data within given intervals. Determine the reactions of the constraints. If possible use uncoupled set of equations. Verify the results.

1.

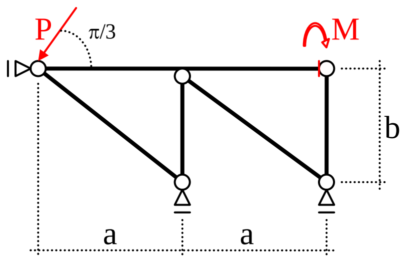


Fig. 3.6 Structure (free-body stable)

M = ……. kNm (15÷45), P = ……. kN (5÷35), a = ……. m (0,8÷5,2)., b = ……. m (1,2÷4,8).

2.

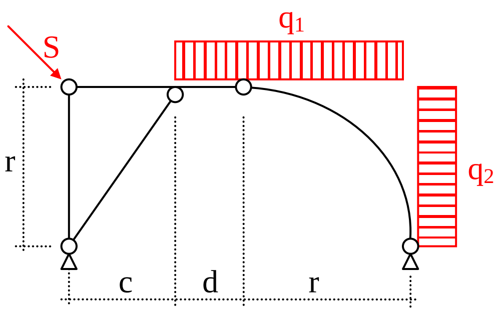


Fig. 3.7 Structure (free-body unstable)

S = …….. kN (12÷35), q_1 = …….kN/m (10÷50), q_2 = ……. kN/m(10÷50), c = ……. m (0,2÷3,6), d = ……. m (1,2÷4), r = ……. m (1÷3).

3.

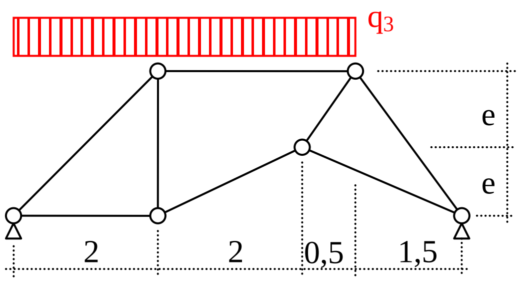


Fig. 3.8 Structure (analogous to 3-higes structure)

q_3 = ……. kN/m (8÷48), e = ……. m (1,2÷2,8).

Reviews problems

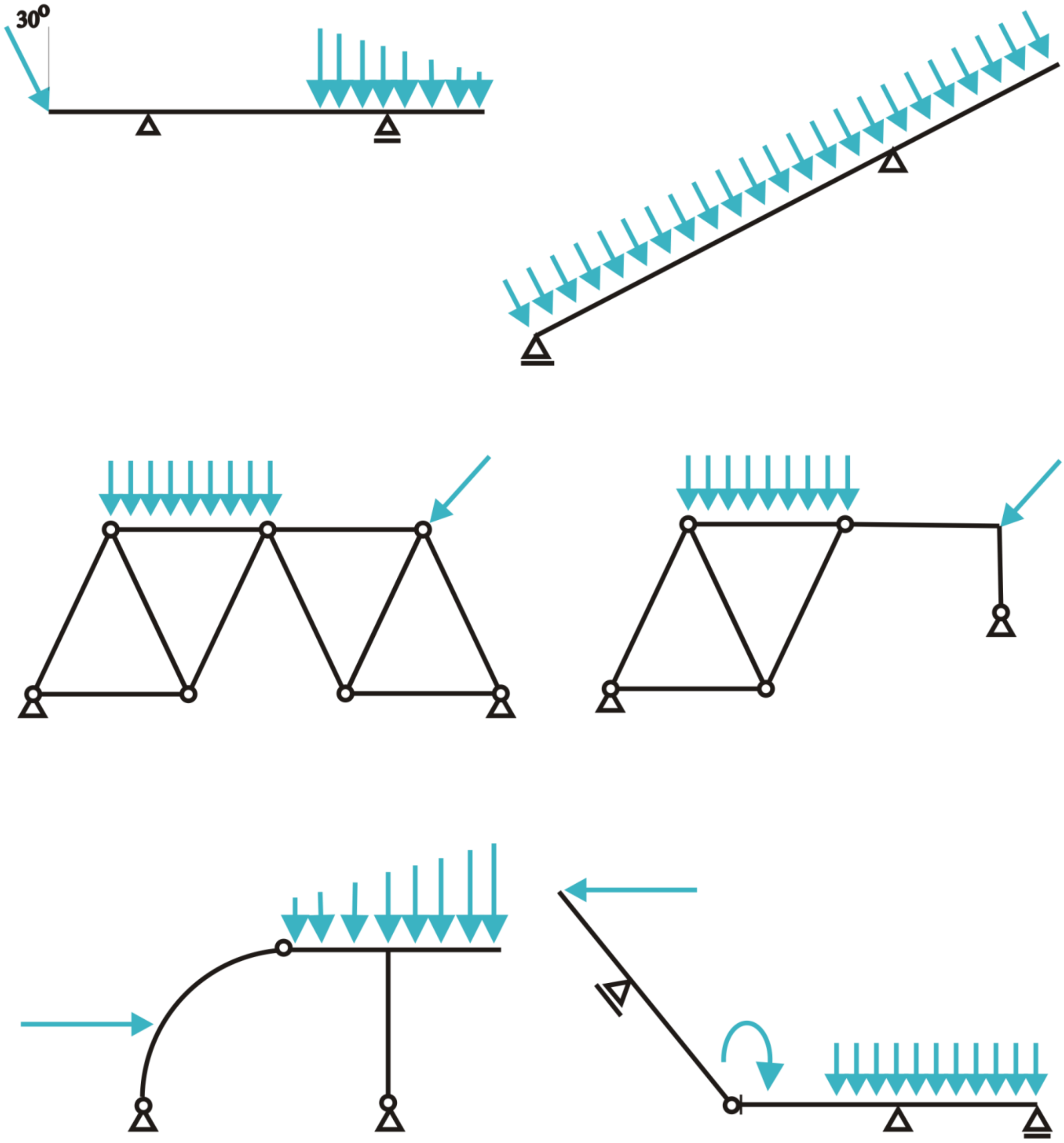


Fig. 3.9 Review problems

Addendum

Calculation hints

The best way of reactions calculations is to use a set of uncoupled balance equations. It means the equations that have only one unknown reaction each. Such set of equations:

- can be easily solved
- a solution of one equation does not depend on other equations solutions
- in case of correction, only the wrong equation with its solution has to be corrected.

Although the uncoupled set of equations needs some additional effort, the aforesaid advantages prevail, so, always try to write the uncoupled set. In some cases it is not possible.
For verification, the rule is opposite: use as many reactions as possible in one equation. Never repeat an equation previously written.

Glossary

static equilibrium – równowaga statyczna
statically equivalent – statycznie równoważne
convergent forces – siły zbieżne
support – podpora
constraints – więzy
pin – podpora nieprzesuwna
roller – podpora (poziomo) przesuwna
slanted roller – podpora przesuwna pod kątem
vertical roller – podpora pionowo przesuwna
fixed end, fixing – utwierdzenie
moving fixing – utwierdzenie (pionowo) przesuwne
moving slanted fixing – utwierdzenie ukośnie przesuwne
no rotation – odebrany obrót
parallelogram – równoległowód (także równoległobok)
hinge equation – równanie przegubu
3 hinges structure – układ trójprzegubowy