5. Simple beams

Introduction

Differential relationships between cross-section forces

In the case of straight beam we have the boundary value problem (BVP)[1]:

$$\frac{\mathrm{d}M(x)}{\mathrm{d}x}=Q(x),\quad \frac{\mathrm{d}Q(x)}{\mathrm{d}x}=-q(x),\quad \Rightarrow\quad \frac{\mathrm{d}^2M(x)}{\mathrm{d}x^2}=-q(x)$$

The order of the bending moment equation is two orders greater than the continuous loading, see the table below.

continuous loading	bending moment equation (diagram)
q = 0, no continuous loading	linear
$q(x)$ = const.	2nd order (nonlinear, 2nd order parabola)
$q(x)$ is linearly variable	3rd order (nonlinear, 3rd order parabola)

It results from the sign convention, that the bending moment diagram convexes in the sense of the continuous loading. Moreover, the bending moment maximum is attained at the section where the shear force vanishes.

Example

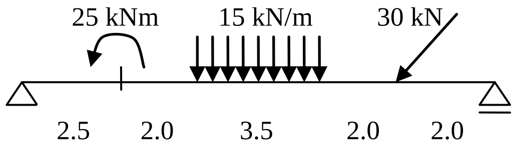

Fig. 5.1 Simply supported beam

Write cross-section equations and draw their diagrams for the beam in Fig. 5.1. If not stated, the dimensions are in [m] and the angle is 45 degrees.

Solution

Beam reactions:

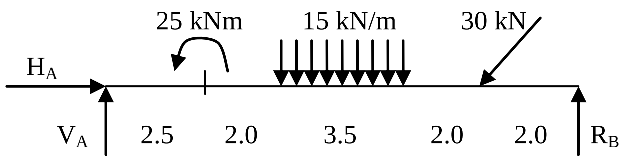

Fig. 5.2 Beam with reactions

$$\sum M_B=0 \quad\Rightarrow\quad V_A=\frac{25+15\cdot 3.5\cdot 5.75+30\sqrt{2}/2\cdot 2}{12}=30.78\,\mathrm{kN}$$

$$\sum M_A=0 \quad\Rightarrow\quad R_B=\frac{-25+15\cdot 3.5\cdot 6.25+30\sqrt{2}/2\cdot 10}{12}=42.94\,\mathrm{kN}$$

$$\sum X=0 \quad\Rightarrow\quad H_A=21.21\,\mathrm{kN}$$

Verification:

[1] Boundary value problem – a differential equation (or set of equations) with the boundary conditions (BCs). For the equation of *n*-th order there are *n* BCs.

$\sum Y = 15 \cdot 3.5 + 30\sqrt{2}/2 - 30.78 - 42.94 = 52.5 + 21.21 - 73.72 = 73.71 - 73.72 = 0.01 \approx 0$, OK!

Cross-section forces equations:

$0 < x < 2.5$
$$\begin{cases} M(x) = 30.78x, \quad M(0) = 0, \quad M(2.5) = 76.95\,[\text{kNm}] \\ Q(x) = 30.78\,[\text{kN}] \\ N(x) = -21.21\,[\text{kN}] \end{cases}$$

$2.5 < x < 4.5$
$$\begin{cases} M(x) = 30.78x - 25, \quad M(2.5) = 51.95, \quad M(4.5) = 113.5\,[\text{kNm}] \\ Q(x) = 30.78\,[\text{kN}] \\ N(x) = -21.21\,[\text{kN}] \end{cases}$$

$4.5 < x < 8$
$$\begin{cases} M(x) = 30.78x - 25 - 15\dfrac{(x-4.5)^2}{2}, \quad M(4.5) = 113.5, \quad M(8) = 129.4\,[\text{kNm}] \\ Q(x) = 30.78 - 15\cdot(x-4.5), \quad Q(4.5) = 30.78, \quad Q(8) = -21.72\,[\text{kN}] \\ N(x) = -21.21\,[\text{kN}] \end{cases}$$

(due to change of shear force's sign, we calculate the bending moment extremum)

$Q(x) = 0 \quad \to \quad x = 6.552, \quad M(6.552) = 145.1\,[\text{kNm}]$

(we use continuous loading resultant)

$8 < x < 10$
$$\begin{cases} M(x) = 30.78x - 25 - 15\cdot 3.5\cdot(x-6.25), \quad M(8) = 129.4, \quad M(10) = 85.93\,[\text{kNm}] \\ Q(x) = 30.78 - 15\cdot 3.5 = -21.72\,[\text{kN}] \\ N(x) = -21.21\,[\text{kN}] \end{cases}$$

(for the last interval we use another coordinate, x_1)

$0 < x_1 < 2$
$$\begin{cases} M(x_1) = 42.94x_1, \quad M(0) = 0, \quad M(2) = 85.88\,[\text{kNm}] \cong M(x=10) \\ Q(x_1) = -42.94\,[\text{kN}], \quad \text{ver.}: Q(x=10) - P_V = -21.72 - 30\sqrt{2}/2 = -42.93 \cong -42.94, \text{OK} \\ N(x_1) \equiv 0 \end{cases}$$

Cross-section forces diagrams:

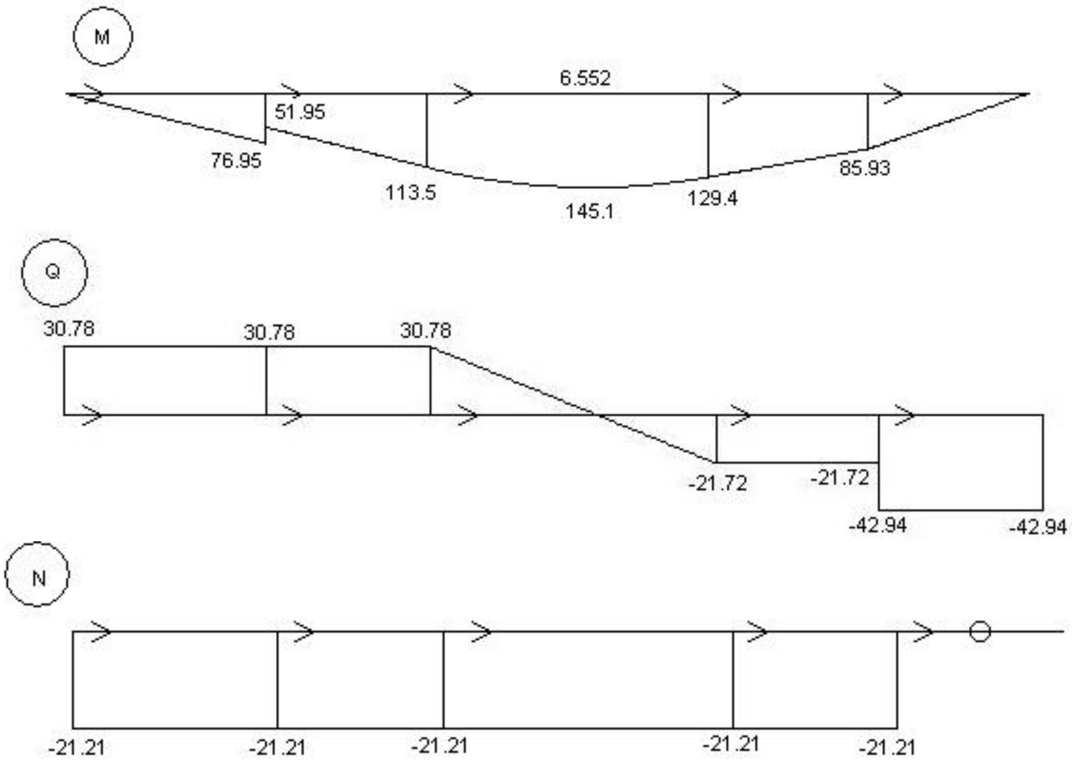

Fig. 5.3 Cross-section forces diagrams

Workshop theme

Construct the cross-section forces diagrams for the beam in Fig. 5.4.

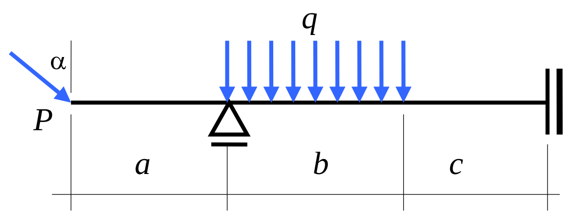

Fig. 5.4 Simple beam

Input data:

P = ...(10÷150 kN), α = ...(15°÷75°), q = ...(10÷80 kN/m), a = ..., b = ..., c = ...(1÷3.5 m)

Review problems

Simple beams

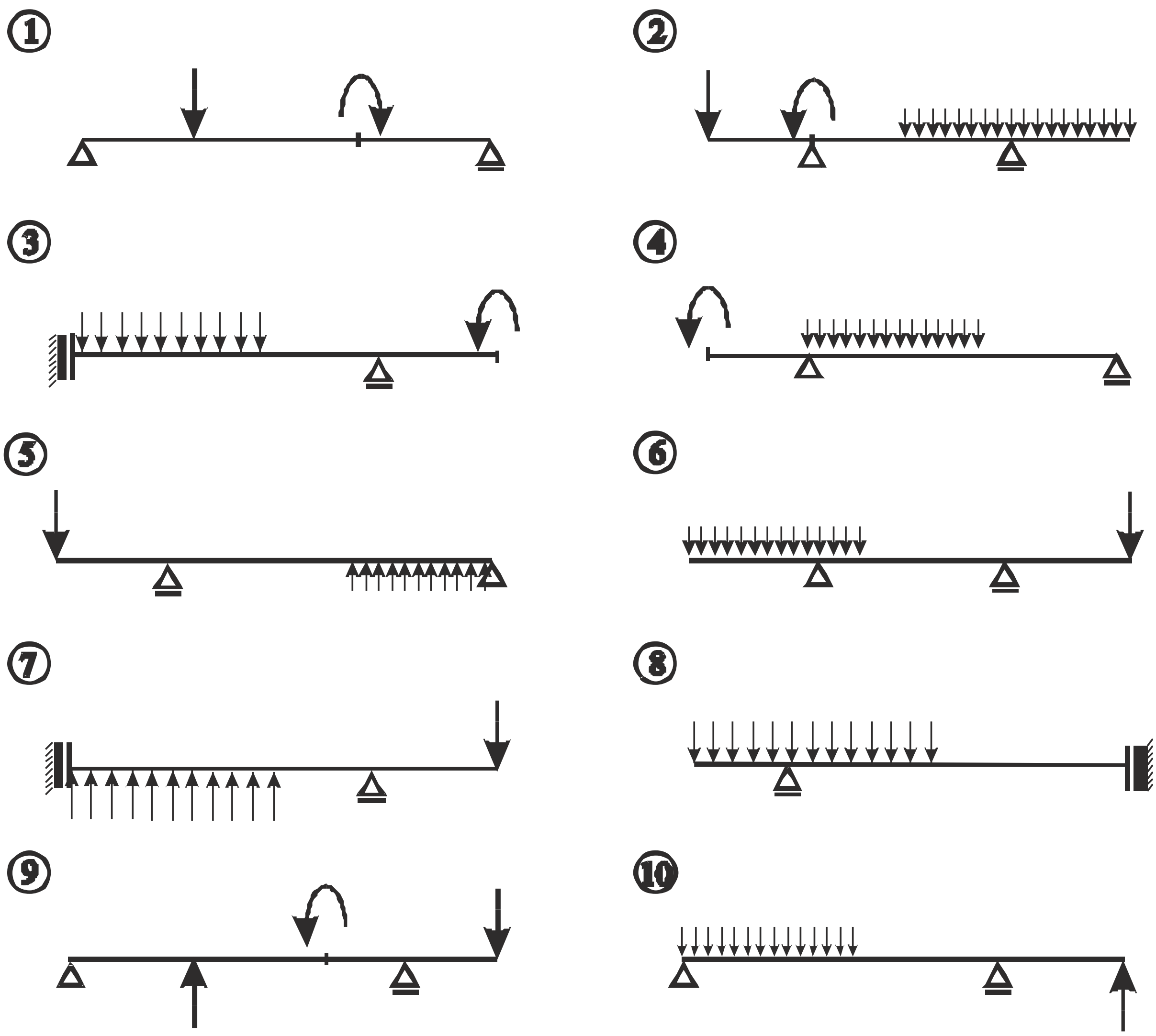

Fig. 5.5 Simple beams – review problems

Addendum

Hints

Tip: There are two typical students' errors:

- an incomplete section; section does not determine the subsets properly, in effect the subsets are not disjoint
- a section without suitable internal forces or cross-section forces

Tip: The best proportion of the diagram is the height/length ratio = 1/3 (approx.)