6. Beams – cont.

Introduction

Continuous loading with variable intensity

Triangular loading with increasing intensity

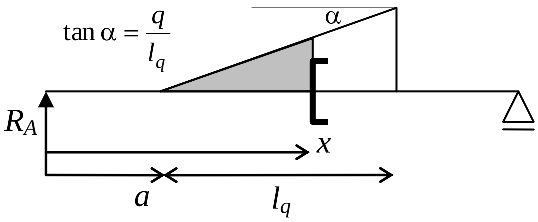


Fig. 5.1 Triangular loading, increasing intensity

$$M(x) = R_A x - \underbrace{\underbrace{\underbrace{\frac{q}{l_q}(x-a)}_{q(x)\text{ value}}\underbrace{\frac{x-a}{2}}_{\text{base/2}}}_{\text{resultant}}\underbrace{\frac{x-a}{3}}_{\text{lever}}}_{\text{moment}} = R_A x - \frac{q}{l_q}\frac{(x-a)^3}{6}$$

Triangular loading with decreasing intensity

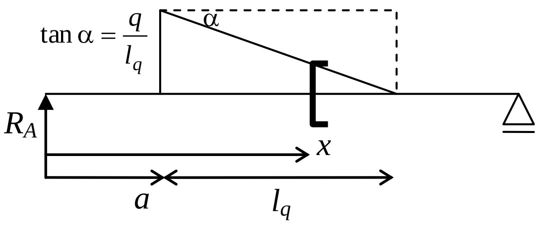
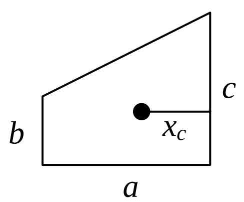


Fig. 5.2 Triangular loading, decreasing intensity

$$M(x) = R_A x - \underbrace{q\frac{(x-a)^2}{2}}_{\text{rectangle}} + \underbrace{\underbrace{\underbrace{\frac{q}{l_q}}_{\tan\alpha}(x-a)}_{q(x)\text{ value}}\underbrace{\frac{x-a}{2}}_{\text{base/2}}\underbrace{\frac{x-a}{3}}_{\text{lever}}}_{\text{triangle}} = R_A x - q\frac{(x-a)^2}{2} + \frac{q}{l_q}\frac{(x-a)^3}{6}$$

Trapezoidal loading

Gravity centre of a trapezoid:



$$x_c = a\frac{2b+c}{3(b+c)}$$

$$b = c \quad\rightarrow\quad x_c = \frac{a}{2}$$

$$b = 0 \quad\rightarrow\quad x_c = \frac{a}{3}$$

$$c = 0 \quad\rightarrow\quad x_c = \frac{2}{3}a$$

Fig. 5.3 Gravity centre of a trapezoid

Let's consider a bending moment from trapezoidal loading, Fig.5.4

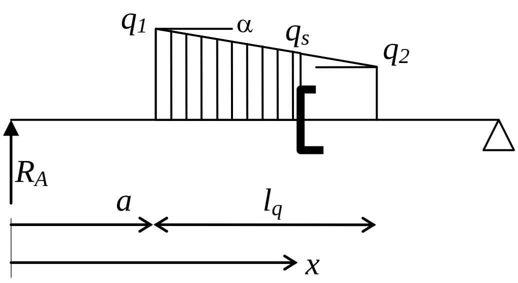


Fig. 5.4 Trapezoidal loading

1) Direct way, not recommended

calculation of q_s: $q_s = q_1 - \underbrace{\frac{q_1 - q_2}{l_q}}_{\tan a}(x_s - a)$

moment calculation: $M(x) = R_A x - \underbrace{\underbrace{\frac{q_1 + q_2}{2}}_{\text{ar.mean}}(x-a)}_{\text{resultant}} \underbrace{\frac{2q_1 + q_s}{3(q_1 + q_s)}}_{\text{lever}}$

2) The way not quite natural, but recommended

a) $q_1 > q_2$

$$M(x) = R_A x - \left[\underbrace{q_1 \frac{(x-a)^2}{2}}_{\text{rectangle}} - \underbrace{\underbrace{\frac{q_1 - q_2}{l_q}}_{\tan\alpha}(x-a)\frac{x-a}{2}\frac{x-a}{3}}_{\text{triangle}} \right]$$

b) $q_1 < q_2$

$$M(x) = R_A x - \left[\underbrace{q_1 \frac{(x-a)^2}{2}}_{\text{rectangle}} + \underbrace{\frac{q_2 - q_1}{l_q}\frac{(x-a)^3}{6}}_{\text{triangle}} \right]$$

Combining both results, we write the result in general form:

$$M(x) = R_A - q_1 \frac{(x-a)^2}{2} + \frac{q_1 - q_2}{l_q}\frac{(x-a)^3}{6}$$

Rules of drawing cross-section diagrams without calculation

Bending moment diagram

– the pointed tip is always in the same sense as the point force
– the bending moments are zero at the ends of a beam, unless the point moment is applied
– the point moment load draws aside the bending moment diagram; the jump is equal in sense and value with the applied load
– there is no jump on the bending moment diagram unless the point moment load is applied
– the continuous loading produces non-linear bending moment diagram; the convexity of the diagram follows the sense of applied load
– there is no pointed tip at the ends of applied continuous loading interval

Shear force diagram

– the jump of shear force is equal in the sense and value to the point force
– there is no visual effect of acting point moment load on shear force diagram

– there is no jump of the shear force at the ends of the continuous loading interval
– the rate of shearing force coincides with the continuous loading intensity

Reverse problem

The beam loading can be reconstructed from the bending moment diagram, exactly. The beam loading can be reconstructed from shear force diagram, exactly to constant bending moment.

Example

Drawing diagrams without calculation

Sketch the cross-section forces diagrams of the beam in Fig. 5.5 without exact calculations. Consider all possibilities.

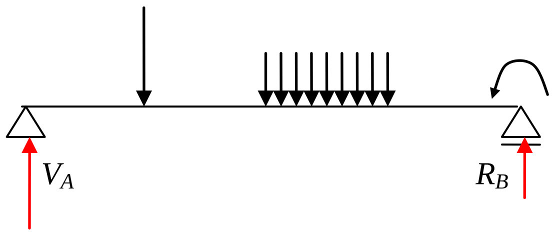


Fig. 5.5 Beam with loads

Solution
1. Reactions: The reaction R_A should be upward, because it is upward for each load. In the first interval bottom fibers are tensioned.
2. The sense of R_B is not clear. For huge point moment load it may be downwards.
3. Otherwise it is upwards.
4. Bending moments at the ends: at point A it should be zero
5. Bending moment at point B is equal to the applied point moment, bottom fibers tensioned.
6. Bending moment in the interval of continuous loading applied should be convex downward, the question is: is there an extremum? It depends, whether shear force changes sign; at point of zero value the bending moment attains extremum (this time it should be maximum).
7. Other cases are: shear force is positive in the interval or
8. shear force is negative in the interval.

Connecting properly earlier points we get the sketch of cross-section forces diagrams, Fig. 5.6, Fig. 5.7 and Fig. 5.8.

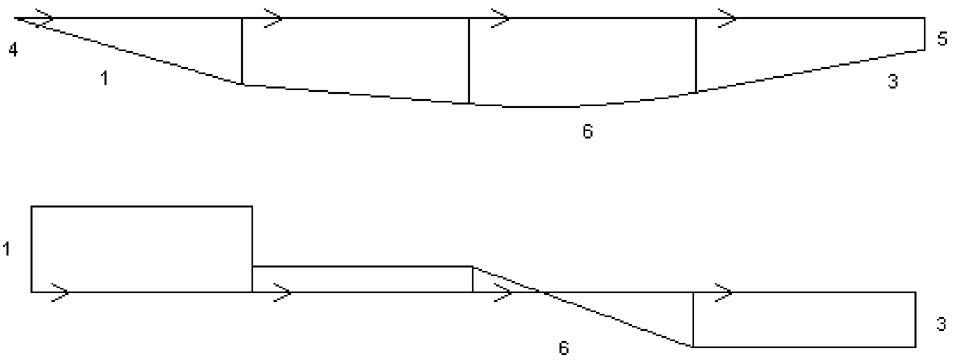


Fig. 5.6 Reactions upwards, maximum of bending moment

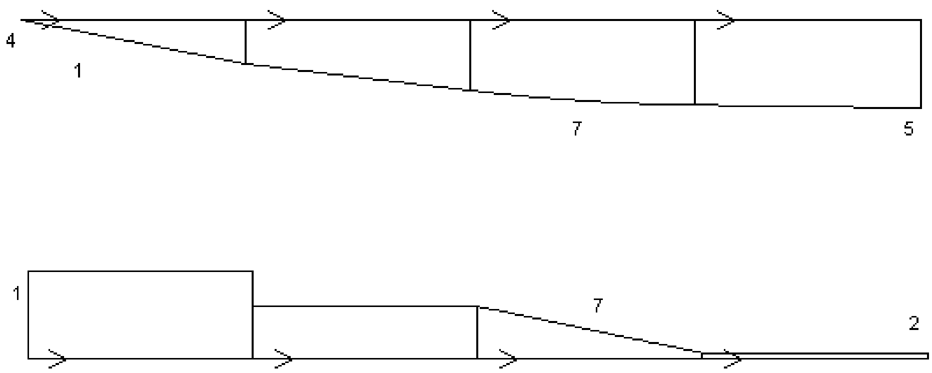


Fig. 5.7 Reactions upward and downward, no extreme moment

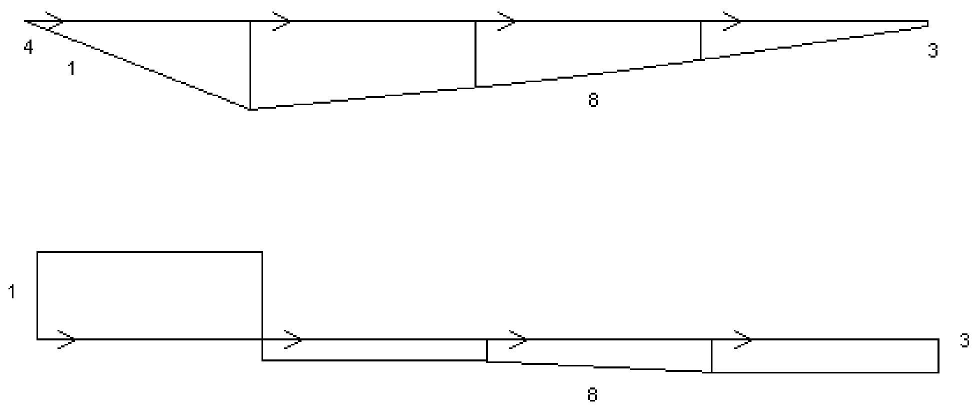


Fig. 5.8 Reactions upward, no extreme moment

Reverse problem

Reconstruct the load applied to the beam from the bending moment diagram in Fig.5. 9.

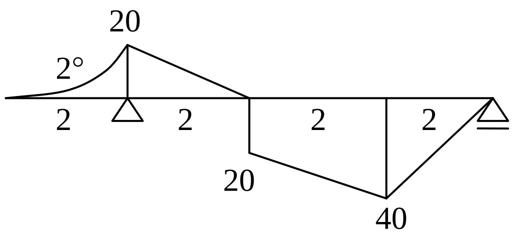


Fig. 5.9 Bending moment diagram

Solution

We begin by determining load type:

1. The first interval (a cantilever) has constant continuous load (the parabola of 2^{nd} order)
2. Remaining intervals have no continuous loading (straight lines of diagram)
3. In the next characteristic points there are point loads: upward force of reaction, point moment clockwise (bottom fibers in tension), downward point force and upward point force of reaction, Fig. 5.10. There is no point force applied with point moment because lines are parallel (lack of bend).

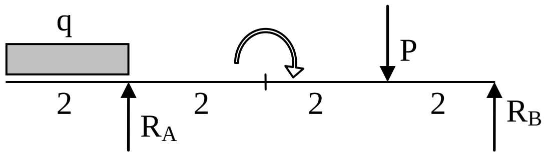


Fig. 5.10 Loadings applied to the beam

4. From extreme intervals we find: $M = \frac{qa^2}{2} = 20, \quad a = 2 \quad \rightarrow \quad q = 10$ and $R_B = 20$.
5. From moment equations for next points (from left and right) we find: $R_A = 20$, $P = 30$. In this way we reconstructed the entire loading of the beam.

Workshop theme

For the beam in Fig. 5.11 calculate the constraints reactions, write down the cross-section forces functions and draw their diagrams.

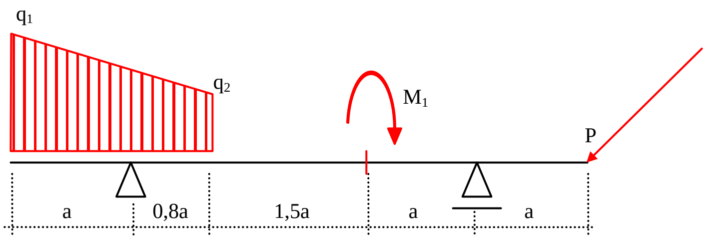


Fig. 5.11 Loaded beam

Input data:
a = ……. m (1,2÷4,8)
P = …….. kN (20÷80)
M_1 = …….. kNm (10÷50)
q_1 = ……. kN/m (12÷45)
q_2 = ……. kN/m (14÷40)

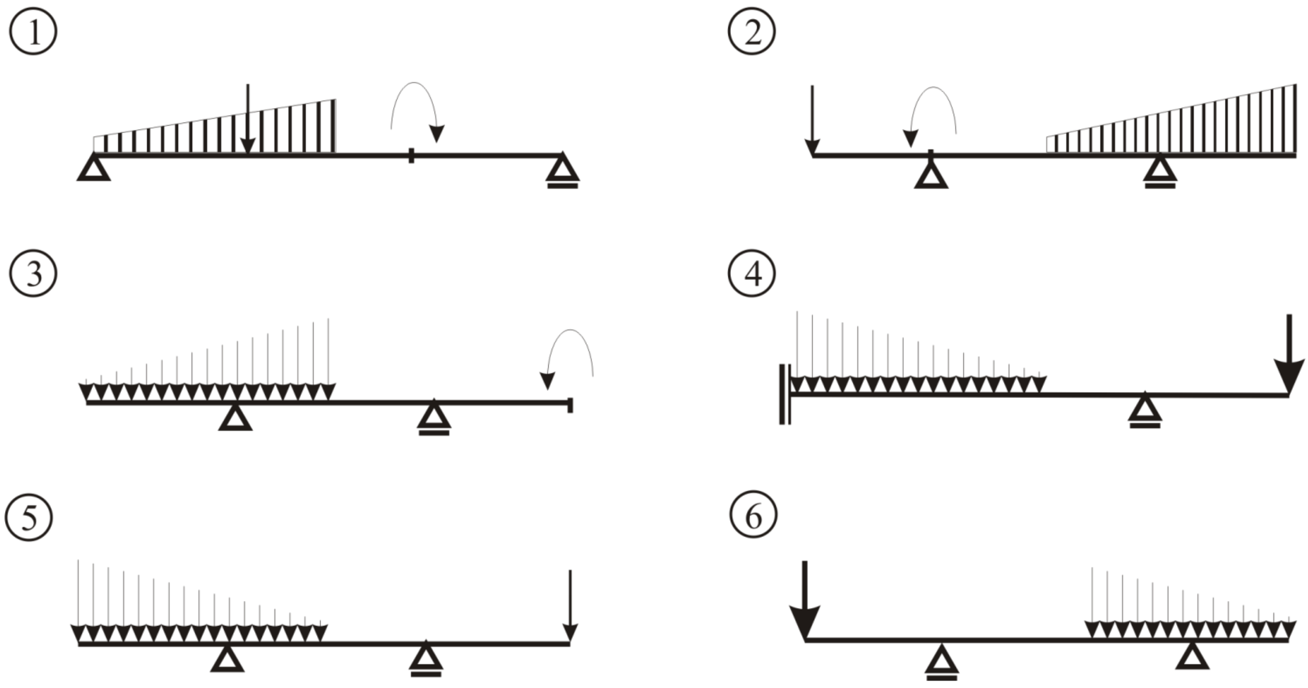
Addendum

Tip: Try to guess the cross-section forces diagrams for several beams and next verify your prediction by using a computer program. Carefully analyse your mistakes (if any).

Tip: Remember the diagrams should be drawn in scale.

Tip: Solve the problems with real numbers obtaining numerical results. Don't use parameters.

Review problems



reconstruct the loads

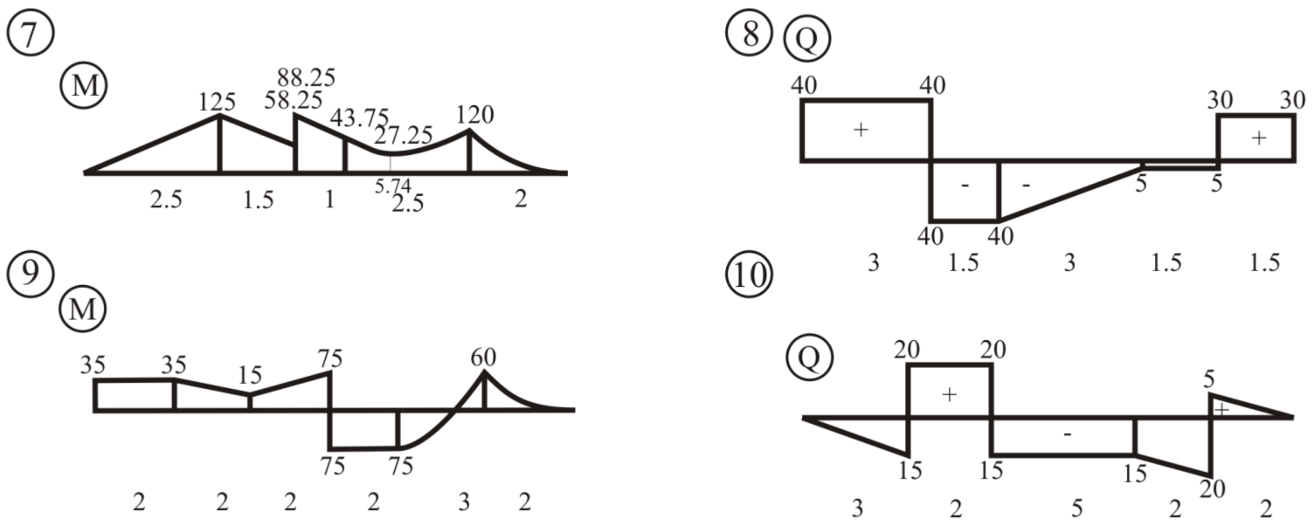


Fig. 5.11 Review problems – beams