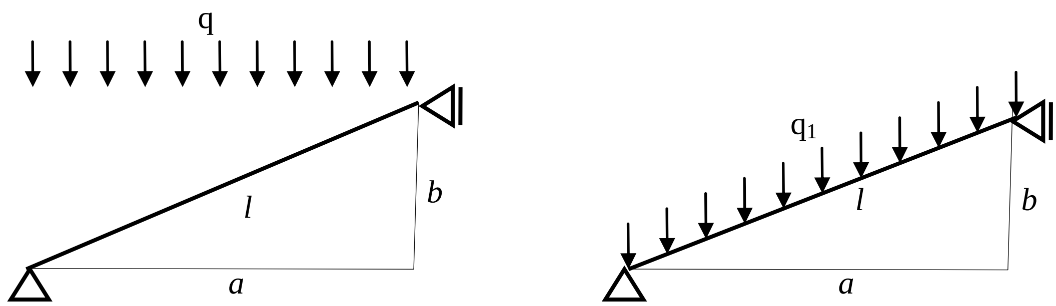
7. Slant beams

Introduction

The intensity of continuous loading may be determined in two ways:
- as intensity per meter
- as intensity per running meter.

The variants are drawn differently, c.f. Fig. 7.1.



resultant = qa resultant = $q_1 l$

Fig. 7.1 Different definitions of continuous loading intensity

The intensity can be recalculated at any time, provided that: $qa = q_1 l$.

Examples

Example 1

The intensity of vertical continuous loading is given per meter, Fig. 7.2.

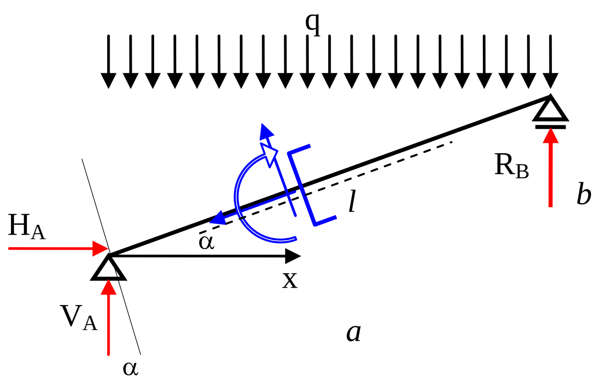


Fig. 7.2 Slant beam with vertical continuous loading

$$M(x) = V_A x - H_A y - \frac{qx^2}{2}, \qquad \left(= V_A x - H_A x \underbrace{\frac{b}{a}}_{\tan\alpha} - \frac{qx^2}{2} \right)$$

$$Q(x) = V_A \cos\alpha - H_A \sin\alpha - qx\cos\alpha$$

$$N(x) = -V_A \sin\alpha - H_A \cos\alpha + qx\sin\alpha$$

Example 2

The intensity of horizontal continuous loading is given per meter, Fig. 7.3.

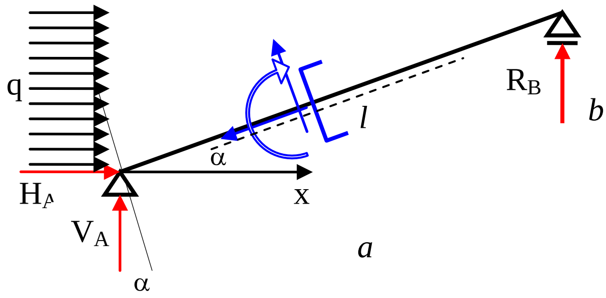


Fig. 7.3 Slant beam with horizontal continuous loading

$$M(x) = V_A x - H_A y - \frac{qy^2}{2}$$
$$Q(x) = V_A \cos\alpha - H_A \sin\alpha - qy \sin\alpha$$
$$N(x) = -V_A \sin\alpha - H_A \cos\alpha - qy \cos\alpha$$

Example 3

The intensity of vertical continuous loading is given per running meter, Fig. 7.4.

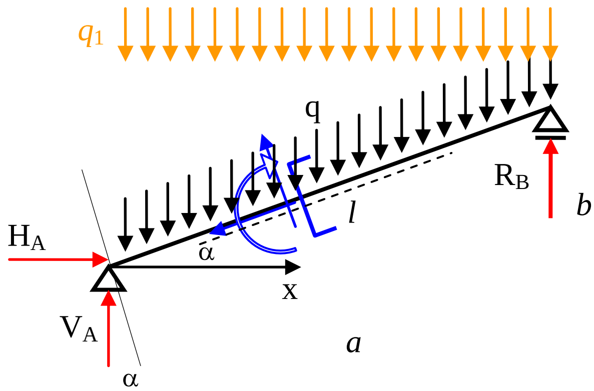


Fig. 7.4 Slant beam with vertical continuous loading

The loading intensity per running meter can be recalculated into the loading intensity per meter: $q_1 = \frac{l}{a} q$,

and further calculations are the same as in Ex. 1.

However, we can determine the cross-section forces "directly" (not recommended way):

$$M(x) = V_A x - H_A y - \underbrace{qx\frac{l}{a}}_{\text{resultant}} \underbrace{\frac{x}{2}}_{\text{lever}}$$
$$Q(x) = V_A \cos\alpha - H_A \sin\alpha - qx\frac{l}{a}\cos\alpha$$
$$N(x) = -V_A \sin\alpha - H_A \cos\alpha + qx\frac{l}{a}\sin\alpha$$

Example 4

The continuous loading perpendicular to the beam axis, Fig. 7.5

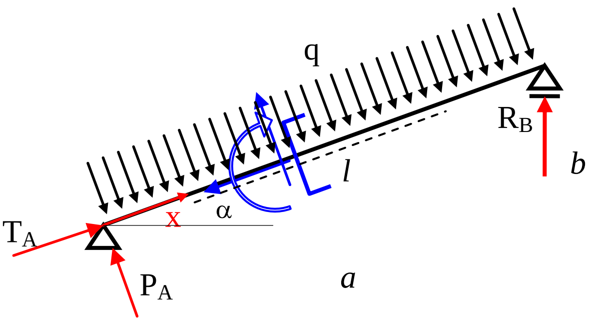


Fig. 7.5 Slant beam with perpendicular loading

The continuous loading perpendicular to the bar axis can be replaced by two continuous loadings of the same intensity: vertical and horizontal (cf. unit 1). Nevertheless, the problem can be solved as simple supported beam which has been turned by an angle α. Because the components of reaction at the point A can be chosen arbitrarily, the easiest way is to adopt their directions – as well as the coordinate system direction – in accordance with the beam axis (P_A – perpendicular, T_A – tangent).

$$M(x) = P_A x - \frac{qx^2}{2}$$

$$Q(x) = P_A - qx$$

$$N(x) = -T_A$$

To exercise cross-section forces' writing, an analogous solution can be found without coordinate rotation, Fig. 7.6.

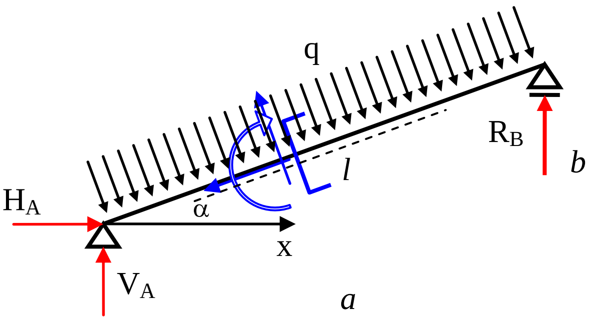


Fig. 7.6 Coordinate set without rotation

$$M(x) = V_A x - H_A y - \frac{qx_1^2}{2} = V_A x - H_A y - \frac{q}{2}\left(x\frac{l}{a}\right)^2$$

$$Q(x) = V_A \cos\alpha - H_A \sin\alpha - qx_1 \cos\alpha = V_A \cos\alpha - H_A \sin\alpha - qx\frac{l}{a}\cos\alpha$$

$$N(x) = -V_A \sin\alpha - H_A \cos\alpha + qx\frac{l}{a}\sin\alpha$$

Let's verify the shearing force. Because $V_A \cos\alpha - H_A \sin\alpha = P_A$ and $qx\frac{l}{a}\cos\alpha = qx\frac{l}{a}\frac{a}{l} = qx$, the solution is exactly as before.

Workshop theme

Determine the reactions, write down and draw the diagrams of the cross-section forces.

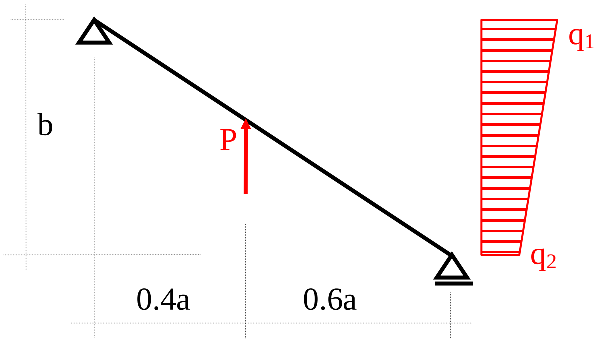


Fig. 7.7 Slant beam

Input data (arbitrary but within given intervals):
a = ……. m (2.0÷4.0 m)
b = ……. M (1.0÷3.0 m)
P = ……. kN (20÷50 kN)
q_1 = ……. kN/m (10÷60 kN/m)
q_2 = ……. kN/m (0.2÷0.8q_1)

Review problems

Slant beams

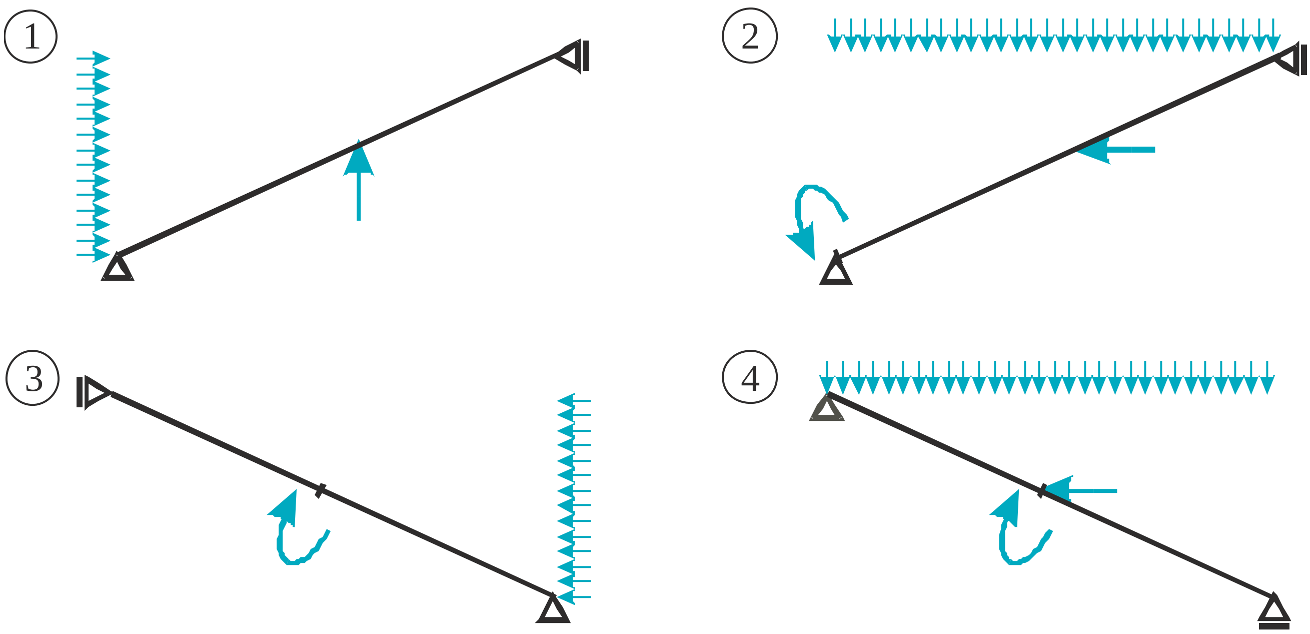


Fig. 7.8 Slant beams – review problems

Addendum

Hints

Tip: Writing the functions of shear and axial cross-section forces we have to think only once. Passing from shear cross-section force to axial one, or from vertical reaction to horizontal one, the trigonometric functions change into co-functions.

Note: This is not true for the signs of the terms. Each of them should be determined separately.

Glossary

slant (slanted, sloping) beam – belka ukośna
per meter – na metr (w rzucie poziomo albo pionowo)
per running meter – na metr bieżący (w rzucie na oś belki)