9. Simple frames

Introduction

Definitions

node – a rigid joint of the bars
simple node – a node joining two perpendicular bars
complex node – a node which is not simple
frame – a system of straight bars joined at (rigid) nodes
simple frame – a frame composed of few straight bars

Simple frames solution

The cross-section forces of simple frames can be determined by solving each bar separately. Next, collected diagrams of the cross-section forces should be verified by checking the balance of each complex node. The node balance means that all cross-section forces determined at the node form a zero force system.
Cutting out the node we draw the acting cross-section forces on the basis of the diagrams: the bending moments are drawn on the tensioned side and the axial and shear forces follow the sign convention. Because the dimensions of the cut out node (and thus the forces' levers) are zero, instead of checking equilibrium equations for the cross-section forces together, we can check the moments' equilibrium and the forces' equilibrium separately.

Example

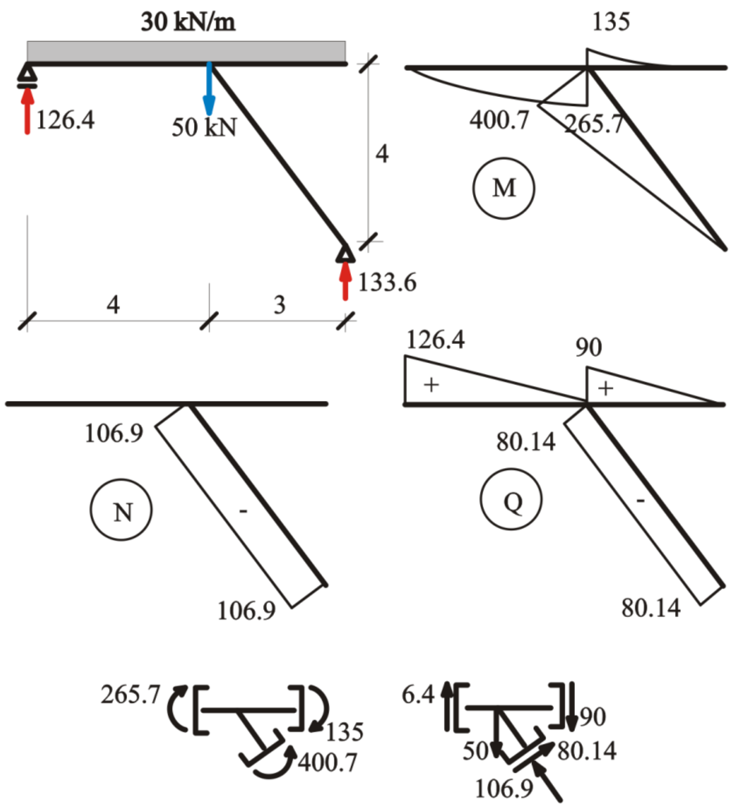

Fig. 9.1 Simple frame

Solution:

Constraints reactions:

$$\sum M_B = 0 \quad \rightarrow \quad R_A = \frac{30\cdot 7\cdot 3.5 + 50\cdot 3}{7} = 126.4\,\text{kN}$$

$$\sum M_A = 0 \quad \rightarrow \quad R_B = \frac{30\cdot 7\cdot 3.5 + 50\cdot 4}{7} = 133.6\,\text{kN}$$

$$H_B = 0$$

We calculate the bars starting from their ends opposite to the node.

a) horizontal beam from the left

$Q(x) = 126.4 - 30\cdot x, \quad Q(0) = 126.4, \quad Q(4) = 126.4 - 120 = 6.4$ (no change of the sign, no moments' extremum)

$$M(x) = 126.4\cdot x - 30\frac{x^2}{2}, \quad M(0) = 0, \quad M(4) = 256.6$$

b) cantilever from the right

$$M(x) = -30\frac{x^2}{2}, \quad M(0) = 0, \quad M(3) = 135$$

$$Q(x) = 30\cdot x, \quad Q(0) = 0, \quad Q(3) = 90$$

c) sloped bar

$$M(x) = 133.6\cdot x, \quad M(0) = 0, \quad M(3) = 400.7$$

$$Q(x) = -133.6\cdot \underbrace{0.6}_{\cos\alpha} = -80.16$$

$$N(x) = -133.6\cdot \underbrace{0.8}_{\sin\alpha} = -106.9$$

Having done the diagrams of cross-section forces, we check the node static equilibrium:

a) moments: $265.7 + 135 - 400.7 = 0$, OK

b) forces: $\sum X = 80.14\cdot 0.8 - 106.9\cdot 0.6 = -0.028 \approx 0$, OK, $\sum Y = \underbrace{-50}_{\text{at node}} + 6.4 - 90 + 80.14\cdot 0.6 + 106.9\cdot 0.8 = 0.004 \approx 0$, OK

Workshop theme

Determine the cross-section forces of the frame in Fig. 9.2 and verify the node balance.

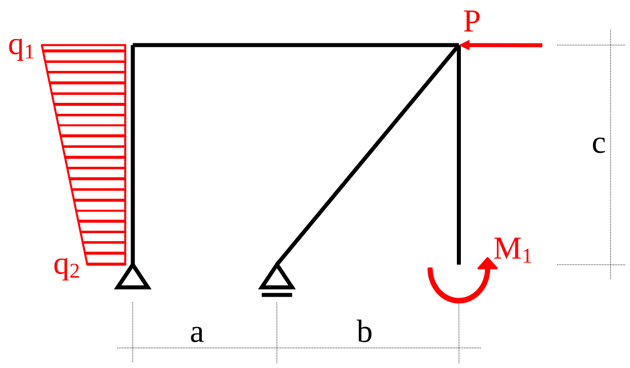

Fig. 9.2 Simple frame

Input data:

a = ……. m (2÷4 m), b = ……. m (2÷5 m), c = ……. m (4÷6 m)

P = ……. kN (40÷80 kN), M_1 = ……. kNm (30÷70 kNm)

q_1 = ……. kN/m (20÷80 kN/m), q_2 = ……. kN/m (0.3÷0.7 q_1)

Review problems

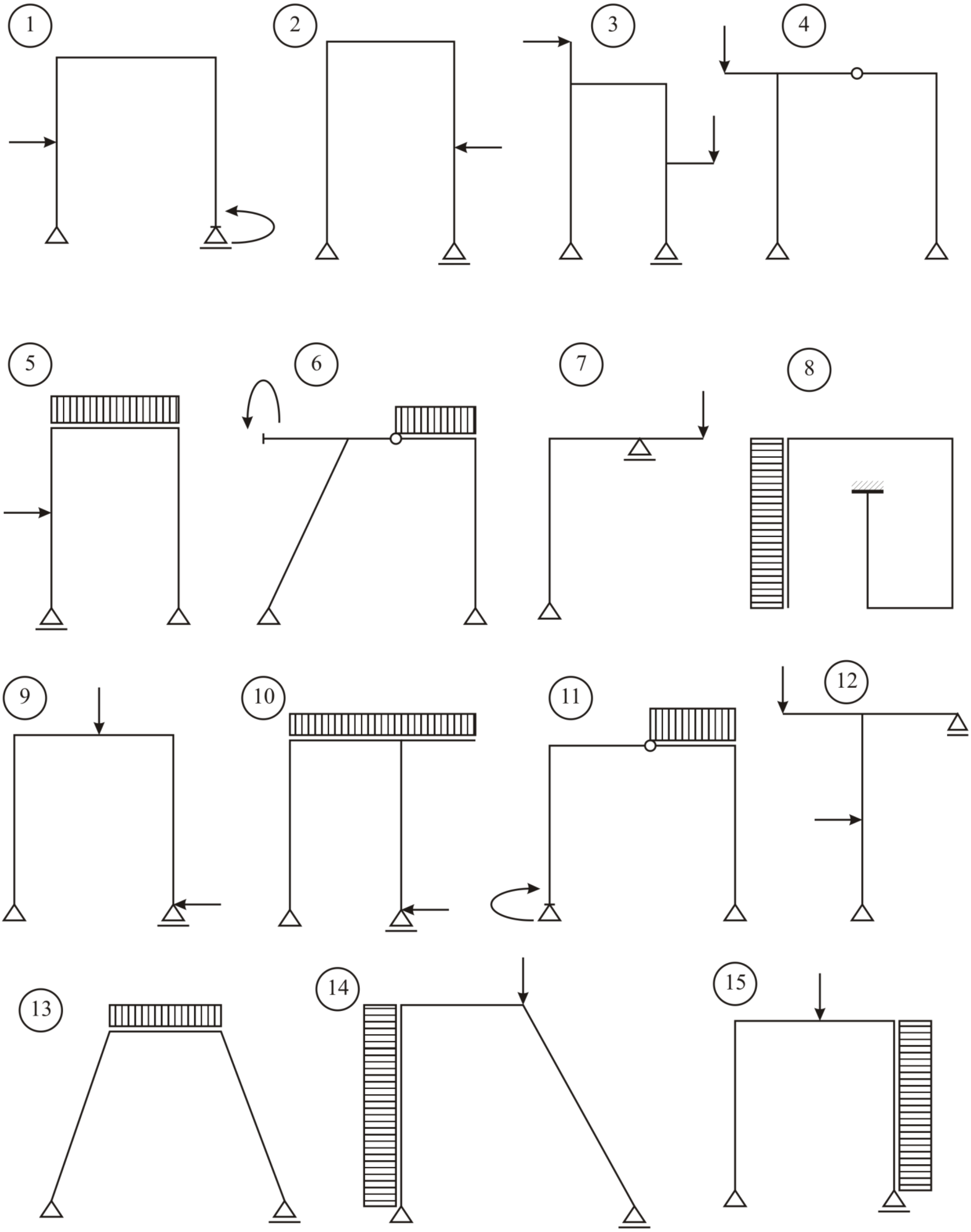

Fig. 9.3 Frames – review problems

Addendum

Hints

Tip: The node balance checking is the final verification of cross-section forces diagrams. Therefore, the checking should be based on the diagrams and not the earlier results.

Glossary

node – węzeł
simple node – węzeł prosty
complex node – węzeł złożony
frame – rama
node balance – równowaga węzła
spandrel (beam) – rygiel
column – słup