11. Trusses

Introduction

Definitions

truss – a pin-jointed structure made of straight bars, loaded by point forces at hinges only
truss bar – an element of a truss: a straight bar with the hinges at its ends, all loads are applied at the joints; there is neither the point moment load nor continuous loading
strut – a truss member in compression
tie – a truss member in tension

Design economy

To demonstrate the design economy let's consider the Eiffel Tower which stands 324 meters tall. If the whole metal structure were melted down, it would fill the 125-meter square base to the depth of ... 6 cm only.

Cross-section forces set

Contrary to the frames, where each bar has the cross-section forces given by three different functions, all trusses' bars have only one constant axial force.
This can be proved easily considering a truss bar free-body balance, in Fig. 11.1.

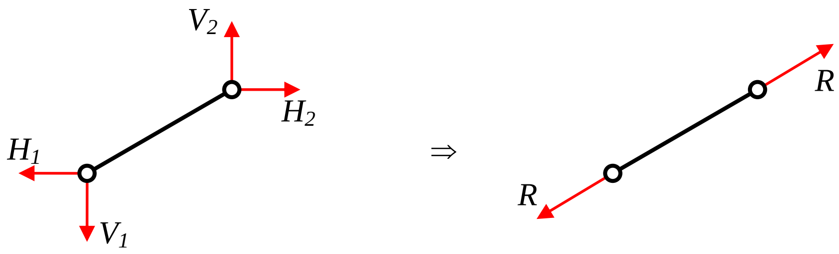


Fig. 11.1 Free-body balance of a truss bar

At the hinged ends there is no moment. The free-body balance needs the resultant forces at the ends to pass through the opposite hinge. It means that the resultants have the same directions as the bar, therefore, bending moment and shear force are identically equal zero and axial force is constant. The two forces are applied at the ends of the member and are necessarily equal, opposite and collinear for equilibrium. Hence the English name: two-force members.
Note: Due to design peculiarity (explained later on), the sign of the axial force is more important than its value and should be clearly indicated.
It will be assumed that sign "plus" means tension and "minus" means compression of the bar. Otherwise, each time the axial force sign would have to be cleared up by words, cf. Fig. 11.2.

Tip: When you use cross-section force sense consistently with the outward normal, no clarification is needed.

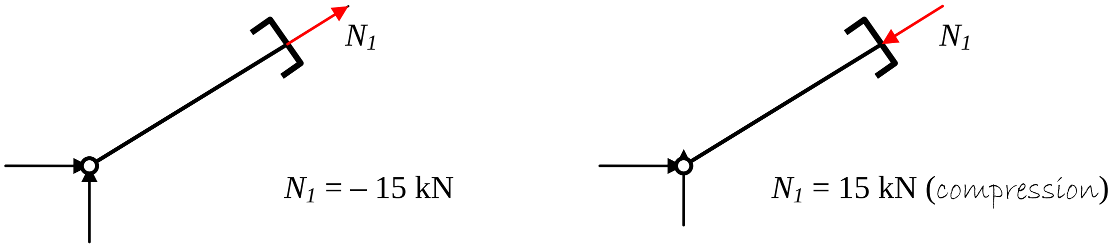


Fig. 11.2 Sign of axial force – the convention

Tip: If there is no doubt, the hinges can be omitted in the static scheme drawing.

Theorems of zero force members (ZFMT)

1. Two truss bars connected at a hinge which is not loaded are zero force members.
2. If two truss bars are connected at a hinge which is loaded in the direction of one bar, the second bar is a zero force member.
3. If three truss bars are connected at a hinge which is not loaded and two of them are collinear, then the third bar is a zero force member.

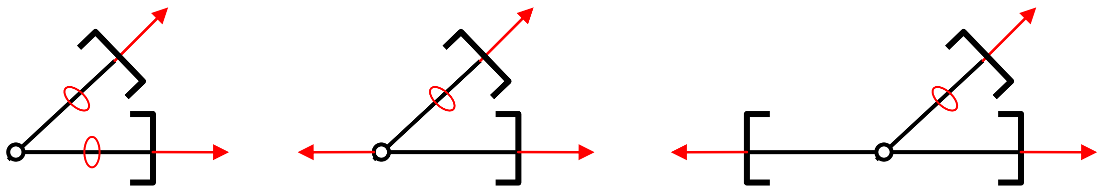


Fig. 11.3 Theorems of zero force members

Tip: The analysis of truss begins with the theorems of zero force members.

The zero force members use is needed for reasons of mechanical or element stability. In Fig. 11.4, the triangular cantilever has several zero force bars "supporting" lower (compressed) strut.

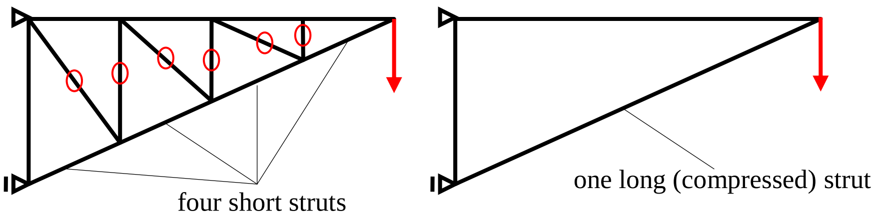
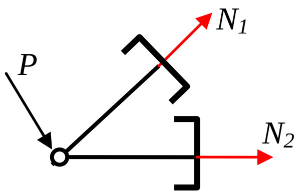


Fig. 11.4 Use of zero force bars

Method of joints

The method consists of isolating each joint of the framework in the form of a free-body diagram and then, by considering equilibrium at each of these joints, the forces in the members of the framework can be determined; before analyzing each joint, it should be ensured that each joint does not have more than two unknown forces.



$$\left.\begin{aligned}\sum X = 0\\ \sum Y = 0\end{aligned}\right\} \rightarrow N_1, N_2$$

Fig. 11.5 Joints method

Note: There are only two independent balance equations for convergent forces set.

Ritter method

The method consists in a section through three bars whose axial forces are unknown and can be determined from the balance equations.

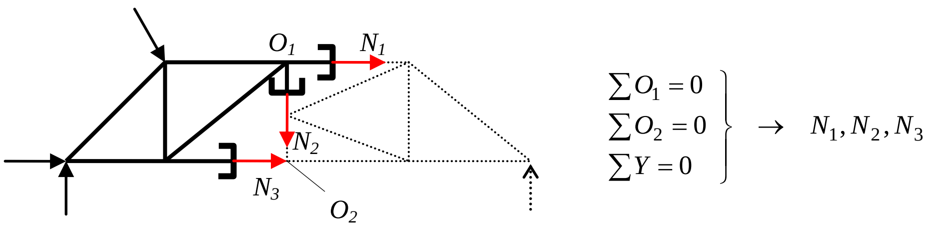


Fig. 11.6 Section method

Henneberg's method (of bar conversions)

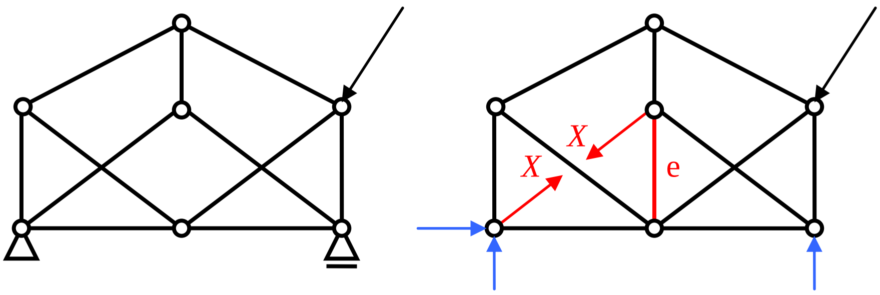


Fig. 11.7 Bar exchange

We replace one bar by its axial force X. Because the structure becomes unstable, we add one extra bar (at another place) to ensure mechanical stability. The axial force in the extra bar should be zero. Using this condition we can calculate the force in the removed bar.
Here, $N_e(P)$ means the force due to external loadings with appropriate reactions, $N_e(X=1)$ is the force caused by self-equilibrated forces X (with no reaction), and we used the superposition principle twice:

$$N_e = \underbrace{N_e(P) + N_e(X)}_{\text{superpositon}} = N_e(P) + \underbrace{XN_e(X=1)}_{\text{superpositon}} = 0 \quad \rightarrow \quad X = -\frac{N_e(P)}{N_e(1)}$$

Examples

Example of zero force members identification

The numbers in Fig. 11.8 indicate the sequence of nodes where the ZFMT would be applied. The zero force bars are indicated by "zero" signs.

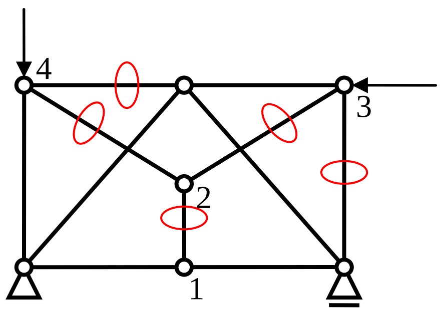


Fig. 11.8 Zero force bars identification

Example of the joint method

The method is suitable for small structures with the limited amount of the members.

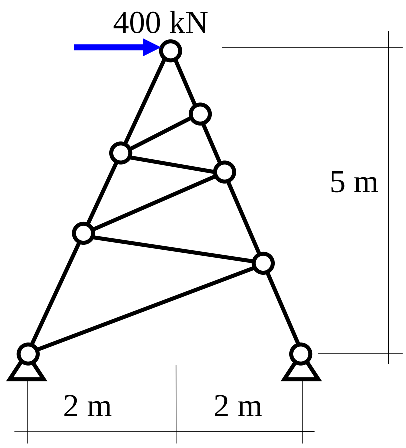


Fig. 11.9 Simple truss

We start with identifying zero-force members, Fig. 11.10. Beginning from the highest sloping bar to the lowest, we find that in principle only two lateral bars are "working".

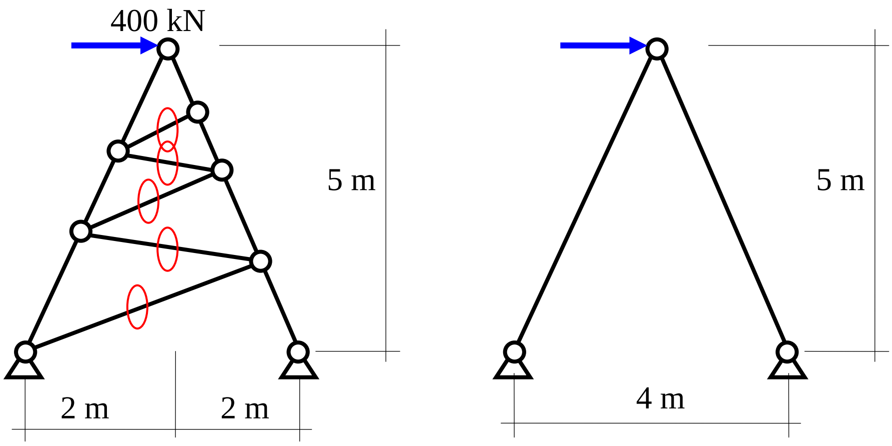


Fig. 11.10 Zero force bars and working bars

From the top hinge balance, Fig. 11.11:

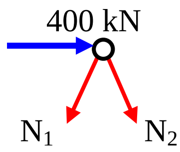


Fig. 11.11 Node balance

we get:

$$\sum Y = 0 \quad \rightarrow \quad N_1 = -N_2$$

$$\sum X = 0 \quad \rightarrow \quad 400 + (-N_1 + N_2)\cos\alpha = 400 + 2N_2 \frac{2}{\sqrt{2^2+5^2}} = 400 + 0.7428 N_2 = 0 \quad \rightarrow \quad N_2 = -538.5, \quad N_1 = 538.5\,\text{kN}$$

Example of section method

Determine the axial force in the indicated bar, Fig. 11.12.

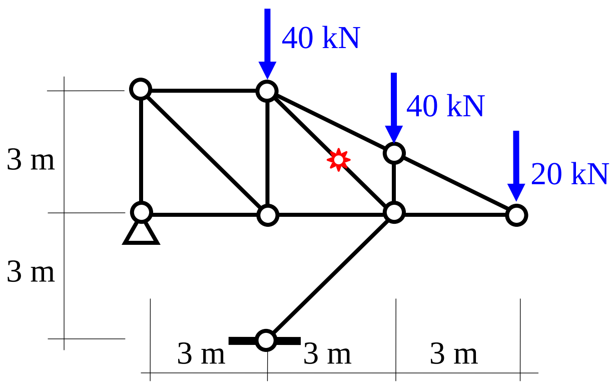


Fig. 11.12 Truss with a bar in question

For the solution we need the force in the lowest sloping bar, not necessarily all constraint reactions. From the hinge equation for the lower part of the structure we know that the reaction should be along the bar direction, pointing to the hinge. Moving this free vector to the position of the hinge, we determine the axial force from the balance condition of the upper part of the structure, Fig. 11.13.

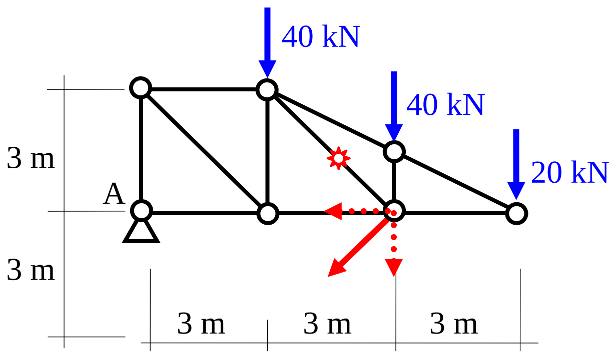


Fig. 11.13 Axial force in the lower bar

$$\sum M_A = 0 \quad \rightarrow \quad 40\cdot 3 + 40\cdot 6 + 20\cdot 9 + N\frac{\sqrt{2}}{2}\cdot 6 = 0 \quad \rightarrow \quad N = -127.3\,\text{kN}$$

The section, shown in Fig 11.14, gives:

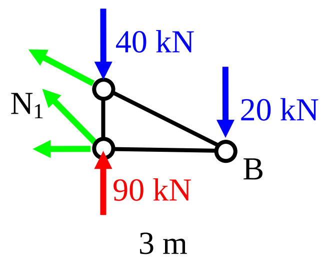


Fig.11.14 Section method

$$\sum M_B = 0 \quad \rightarrow \quad N_1 \frac{\sqrt{2}}{2} \cdot 3 + 90 \cdot 3 - 40 \cdot 3 = 0 \quad \rightarrow \quad N_1 = -70.71 \text{kN}$$

Answer: the axial force in the indicated bar is equal to –70.71 kN (compression).

Example of Henneberg's method

Determine the axial force in the indicated bar.

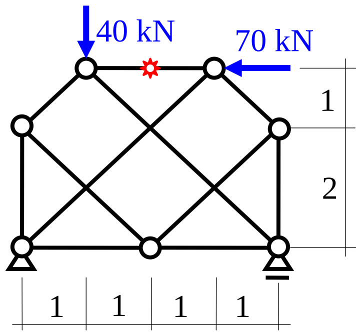


Fig. 11.15 Structure in question

The structure can be analyzed neither by the joint nor the section methods without a rather large system of equations. So, we are doomed to the Henneberg's method.

First, we consider mechanical stability of the structure, Fig. 11.16. From 3ST we find that the structure is free-body and overall stable.

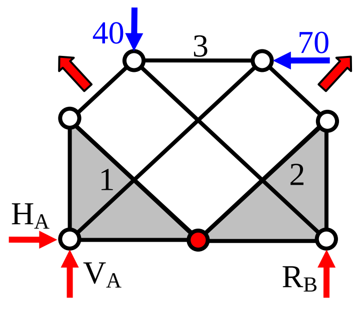


Fig. 11.16 Mechanical stability

Reaction calculation gives:

$$V_A = \frac{40 \cdot 3 + 70 \cdot 3}{4} = 82.5 \text{kN}, \ R_B = \frac{40 \cdot 1 - 70 \cdot 3}{4} = -42.5 \text{kN}, \ H_A = 70 \text{kN}$$

We remove the bar and replace its action by self-equilibrated forces X. To ensure mechanical stability of the structure we add a substitute bar, assuming its axial force to be zero. An equivalent static scheme with the analysis of free-body mechanical stability is drawn in Fig. 11.17 where consecutive shields have been drawn.

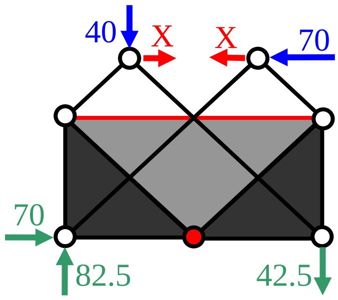


Fig. 11.17 Equivalent scheme

We will calculate the axial force in the indicated bar twice: for the so-called basic load (in blue and green in the Fig. 11.17) and for self-equilibrated forces (in red). Based on the superposition principle, axial force in the bar due to all loads is a sum of forces in the bar due to basic load and *X*-times unit self-equilibrated forces: $N = N^{(P)} + N^{(X)} = N^{(P)} + X \cdot N^{(1)}$.

Solution for basic load

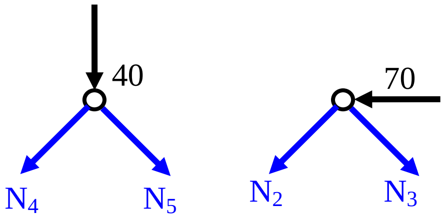


Fig. 11.18 Solution for basic load

$$\sum Y = 0 \quad \rightarrow \quad N_2 = -N_3, \quad \sum X = 0 \quad \rightarrow \quad -\frac{\sqrt{2}}{2} N_2 + \frac{\sqrt{2}}{2} N_3 - 70 = 0 \quad \rightarrow \quad \sqrt{2} N_3 = 70 \quad \rightarrow \quad N_3 = 49.5\,\text{kN}$$

$$\sum X = 0 \quad \rightarrow \quad N_4 = N_5, \quad \sum Y = 0 \quad \rightarrow \quad 40 + \frac{\sqrt{2}}{2} N_4 + \frac{\sqrt{2}}{2} N_5 = 0 \quad \rightarrow \quad N_5 = -\frac{40}{\sqrt{2}} = -28.28\,\text{kN}$$

and then, cf. Fig. 11.19, we have:

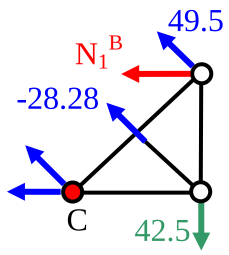


Fig. 11.19 Solution for basic load – cont.

$$\sum M_C = 0 \quad \rightarrow \quad \frac{\sqrt{2}}{2} N_5 \cdot 2 + \frac{\sqrt{2}}{2} N_3 \cdot 4 + N_1^B \cdot 2 = 0 \quad \rightarrow \quad N_1^B = 28.28 \frac{\sqrt{2}}{2} - 49.5 \cdot \sqrt{2} + 42.5 = -7.5\,\text{kN}$$

Solution for unit self-equilibrated load

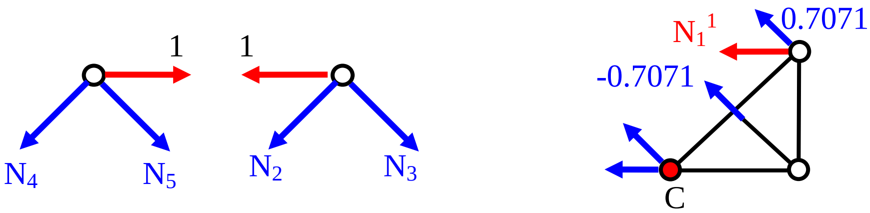


Fig. 11.20 Solution for unit load

$$\sum Y=0 \;\rightarrow\; N_2=-N3,\;\; \sum X=0 \;\rightarrow\; -\frac{\sqrt{2}}{2}N_2+\frac{\sqrt{2}}{2}N_3-1=0 \;\rightarrow\; \sqrt{2}N_3=1 \;\rightarrow\; N_3=0.7071\,\text{kN}$$

$$\sum Y=0 \;\rightarrow\; N_4=-N_5,\;\; \sum X=0 \;\rightarrow\; 1-\frac{\sqrt{2}}{2}N_4+\frac{\sqrt{2}}{2}N_5=0 \;\rightarrow\; N_5=-\frac{1}{\sqrt{2}}=-0.7071\,\text{kN}$$

$$\sum M_C=0 \;\rightarrow\; \frac{\sqrt{2}}{2}N_5\cdot 2+\frac{\sqrt{2}}{2}N_3\cdot 4+N_1^1\cdot 2=0 \;\rightarrow\; N_1^1=-\frac{\sqrt{2}}{2}\cdot 0.7071=-0.5\,\text{kN}$$

We have, eventually:

$$N_1=-\frac{N_1^B}{N_1^1}=-\frac{-7.5}{-0.5}=-15\,\text{kN}$$

Workshop theme

Determine the forces in the indicated truss member:

a) by the section method (Ritter's method)

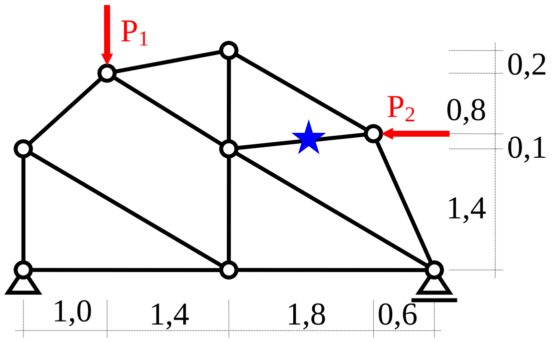


Fig. 11.21 Truss to be considered by section method

Input data: P_1 = ……. kN, P_2 = ……. kN, dimensions in [m].

b) by the Henneberg's method

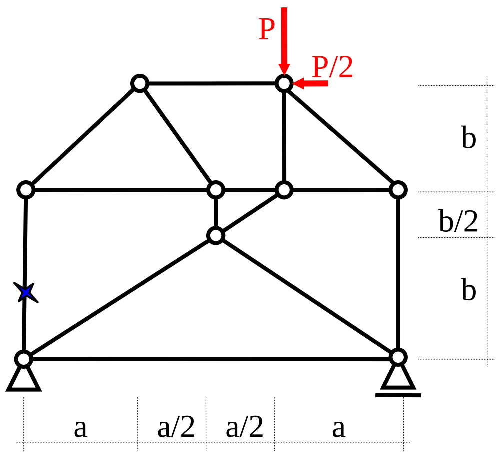


Fig. 11.22 Truss to be considered by Henneberg's method

Input data:

a = ……. m (1÷4 m)

b = ……. m (1÷3 m), where: $\frac{a}{b} \neq 0.625$

P = ……. kN (50÷250 kN)

Review problems

Determine the axial force in the indicated element (c.f. Fig. 11.21).

Addendum

[1]As opposed to a solid block, a truss:
- uses less material;
- puts less gravity load on other parts of the structure;
- leaves space for other things of interest (e.g., cars, cables, wires, people).

You can notice trusses in bridges, radio towers, and large-scale construction equipment. Early airplanes were flying trusses[2]. Bamboo trusses have been used as scaffoldings for millennia. Birds have had bones whose internal structure is truss-like since they were dinosaurs.
Trusses are practical sturdy light structures.
Trusses can carry big loads with little use of material and can look nice. They are used in many structures. Why don't engineers use trusses for all structural designs? Here are some reasons to consider not using a truss:
- trusses are relatively difficult to build, involving many small parts and thus requiring much time and effort to assemble,
- trusses can be sensitive to damage when forces are not applied at the anticipated joints; they are especially sensitive to loads on the middle of the bars,
- trusses inevitably depend on the tension strength in some bars; some common building materials (e.g., concrete, stone, and clay) crack easily when pulled,
- trusses often have little or no redundancy, so failure in one part can lead to total structural failure,
- the triangulation that trusses require can use space that is needed for other purposes (e.g., doorways, rooms),
- trusses tend to be stiff, and sometimes more flexibility is desirable (e.g., diving boards, car suspensions),
- in some places some people consider trusses unaesthetic. (e.g., the Washington Monument is not supposed to look like the Eiffel Tower).

None-the-less, for situations where you want a stiff, light structure that can carry known loads at pre-defined points, a truss is often a great design choice.

Hints

Tip: Always try to write uncoupled equations set.

Tip: Remember that the bar force is a sliding vector.

Glossary

truss, framework, lattice, latticework – kratownica
truss bar – pręt kratowy
zero force bar – pręt zerowy
hinges balance method, joints method – metoda równoważenia węzłów
Ritter method, sections method – metoda Rittera

[1] excerpt from: *Introduction to statics and dynamics*, by Andy Ruina and Rudra Pratap, Oxford Univ. Press (Preprint), 2008, p. 249 and 251

[2] and, more recently, space vehicles and spacecrafts like Lunar Module and Mars Pathfinder (A.Z.)

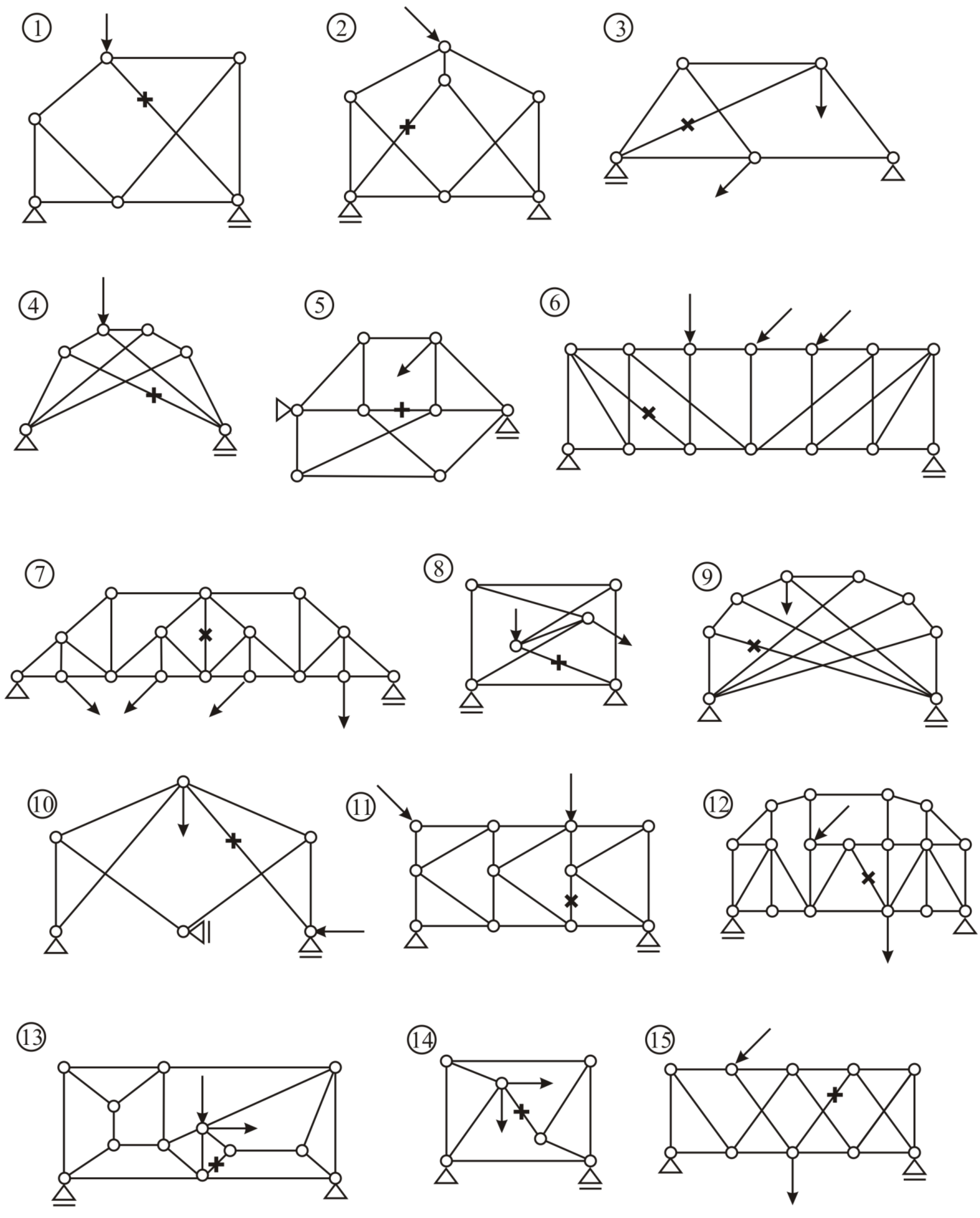


Fig 11.21 Review problems

Henneberg's method, bars exchange method – metoda wymiany prętów (Henneberga)
truss post – słupek
cross brace – krzyżulec

top chord – pas górny
bottom chord – pas dolny
truss web configuration – konfiguracja siatki kratownicy
strut – rozpórka, zastrzał
tie, tie rod, bowstring – ściąg
wind bracing – wiatrownica
sway brace – stężenie poprzeczne
substitute bar – pręt zastępczy
self-equilibrated – samozrównoważony