12. Combined structures

Introduction

Definitions

combined structure – a structure with parts of different kinds: frame, arch, truss
truss bar – a bar with constant axial force only
beam (frame) bar – a bar with full set of cross-section forces (usu. not constant)

Solution

The first very important step consists of the geometric stability analysis and truss/beam bars recognition. The importance of the geometric rigidity analysis is obvious: it predicts reaction calculation. If the structure is free-body rigid, reaction calculation should be easy and "standard" (the use of equilibrium equations only). If not, some additional effort should be made (at least the hinge's equation).
Next step consists in the distinction between beam and truss bars. The truss members' forces are determined first. Having the beam bars forces also determined, we construct the cross-section forces diagrams as a final result. Additional verifications complete the work.

Example

Free-body stable structure

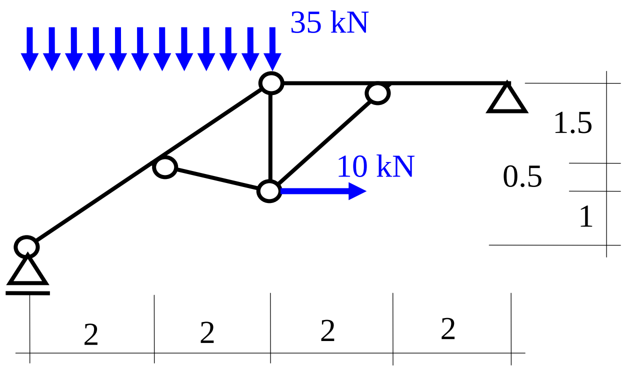

Fig. 12.1 Free-body stable structure

We distinguish the beam elements by making them thicker, Fig. 12.2.

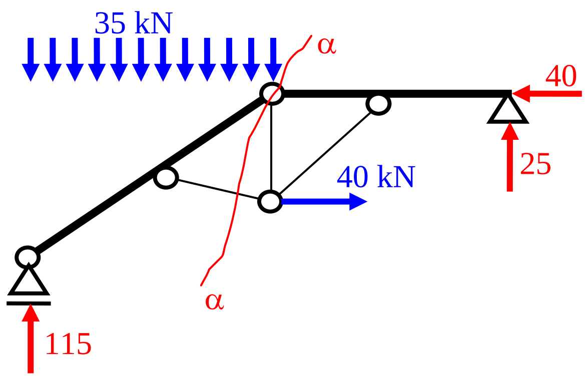

Fig. 12.2 Beam and truss elements and section

To calculate the truss bar forces we cut the structure through the hinge and one truss bar, Fig. 12.3.

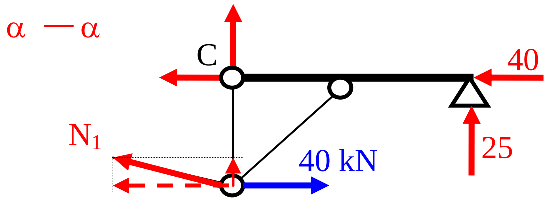

Fig. 12.3 Right part of the structure

$$\sum M_C = 0 \quad \rightarrow \quad -25 \cdot 4 - 40 \cdot 2 + 2 \cdot N_1 \cdot \frac{2}{\sqrt{2^2 + 0.5^2}} = 0 \quad \rightarrow \quad N_1 = 92.77\,\text{kN}$$

Next we consider equilibrium of the truss bars' joint:

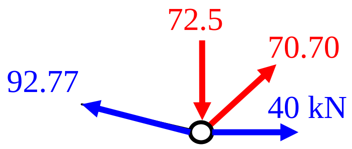

Fig. 12.4 Joint equilibrium

Now, we return to the beam elements, Fig. 12.5, and construct the diagrams of cross-section forces.

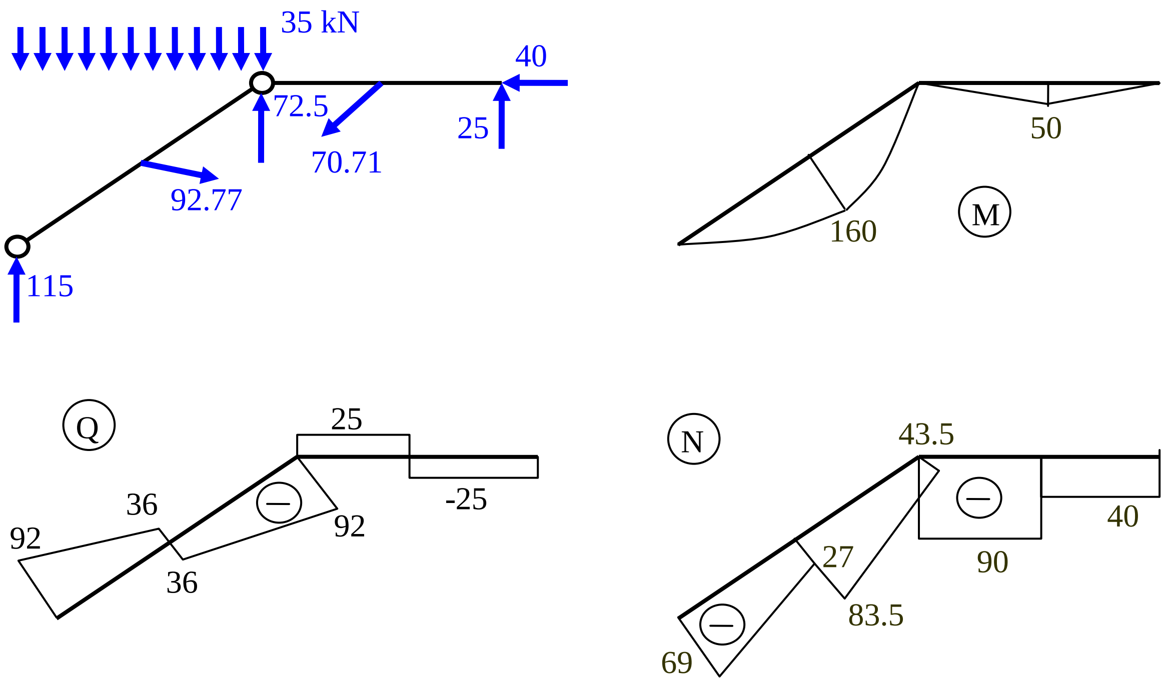

Fig. 12.5 Cross-section forces diagrams

Free-body unstable structure

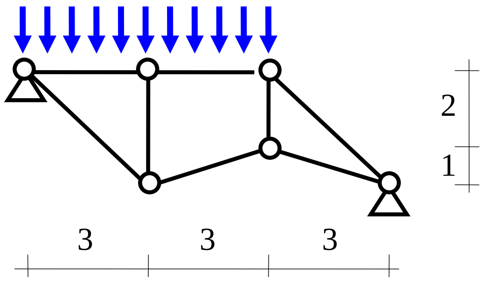

Fig. 12.6 Free-body unstable structure

First, we distinguish the beam elements. Analysis of geometric stability suggests the main idea of the solution. Leaving the shields of three elements “untouched”, we cut the structure through the hinges and the same truss element, Fig. 12.7.

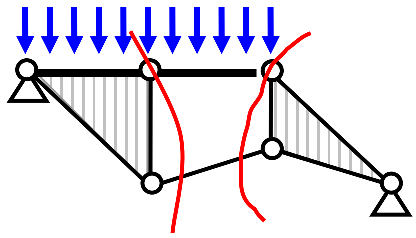

Fig. 12.7 Beam elements and suggested sections

For the entire structure we have:

$\sum M_A = 0 \quad \rightarrow \quad 35 \cdot 6 \cdot 3 - 9 \cdot V_B + 3 \cdot H_B = 0$

For the structure parts, Fig. 12.8, we have:

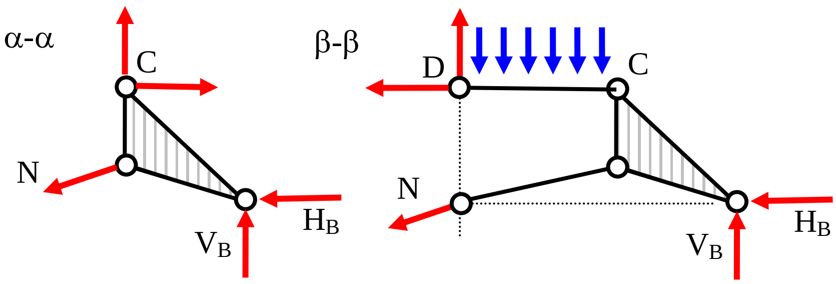

Fig. 12.8 Reactions calculation

$$\sum M_C = 0 \quad \rightarrow \quad 2 \cdot N \cdot \frac{3}{\sqrt{3^2 + 0.5^2}} + 3 \cdot H_B - 3 \cdot V_B = 0$$

$$\sum M_D = 0 \quad \rightarrow \quad 35 \cdot 3 \cdot 1.5 + 3 \cdot H_B - 6 \cdot V_B + 3 \cdot N \cdot \frac{3}{\sqrt{3^2 + 0.5^2}} = 0$$

We get the system of equations:

$$\begin{bmatrix} 0 & -9 & 3 \\ 1.973 & -3 & 3 \\ 2.959 & -6 & 3 \end{bmatrix} \bullet \begin{bmatrix} N \\ V_B \\ H_B \end{bmatrix} = \begin{bmatrix} -630 \\ 0 \\ -157.5 \end{bmatrix} \quad \rightarrow \quad \begin{matrix} N = 79.85 \\ V_B = 78.75 \\ H_B = 26.25 \end{matrix}$$

We compute the constraints reactions, the axial forces in truss bars and consider the beam elements, obtaining all data needed to construct the cross-section forces diagrams.

Workshop theme

Determine the cross-section forces of the structures in Fig. 12.9 and Fig. 12.10 and draw their diagrams.
a)

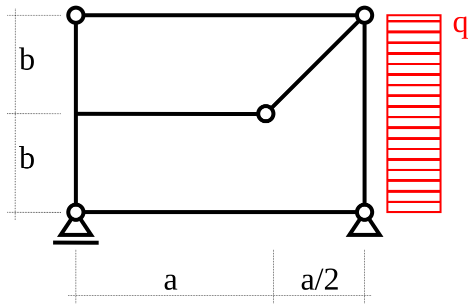

Fig. 12.9 Free-body stable structure

Input data:
a = ……. m   (2÷5 m)
b = ……. m   (1÷3 m)
q = ……. kN/m   (5÷50 kN/m)

b)

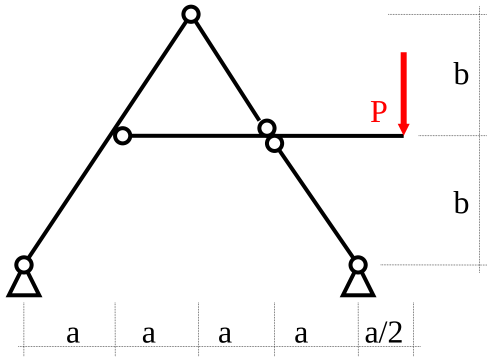

Fig. 12.10 Free-body unstable structure

Input data:
a = ……. m   (1,5÷4 m)
b = ……. m   (2÷6,5 m)
P = ……. kN   (100÷400 kN)

Review problems

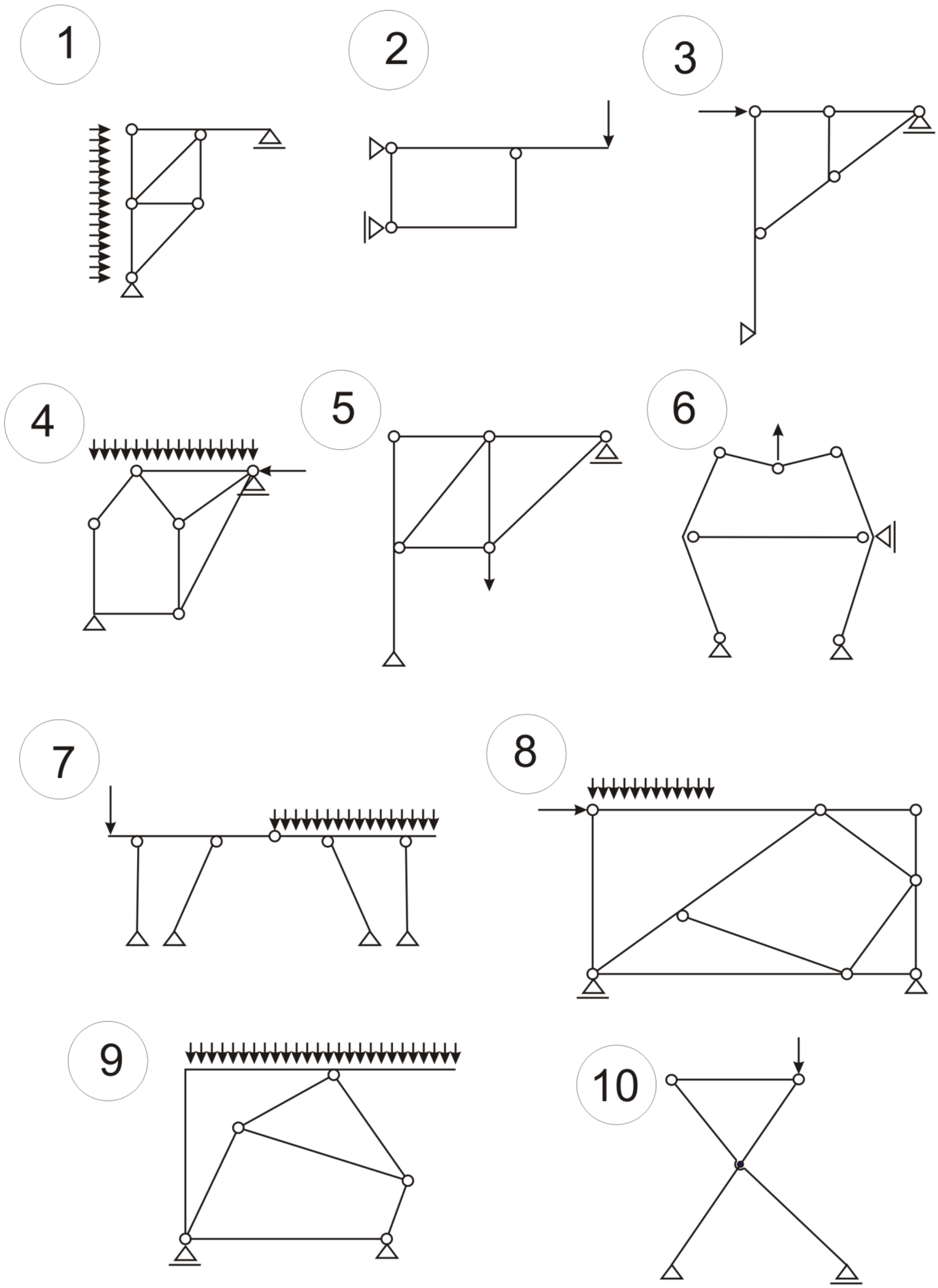

Fig. 12.11 Review problems

Addendum

Hints

The algebra of a solution can be simplified by the choice of a moment axis which eliminates as many unknowns as possible or by the choice of a direction for a force summation which avoids reference to certain unknowns. A few moments of thought to take advantage of these simplifications can save appreciable time and effort.

In extremely difficult cases or when we have no clear idea of the solution procedure, we can break up the structure into separate elements and write the balance equations for them, Fig. 12.12.

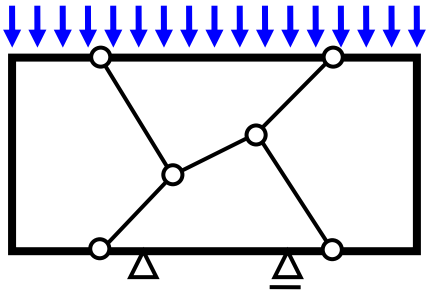

Fig. 12.12 Structure with the truss bars "inside" the beam bars

We get four beam elements and five truss elements, Fig. 12.13.

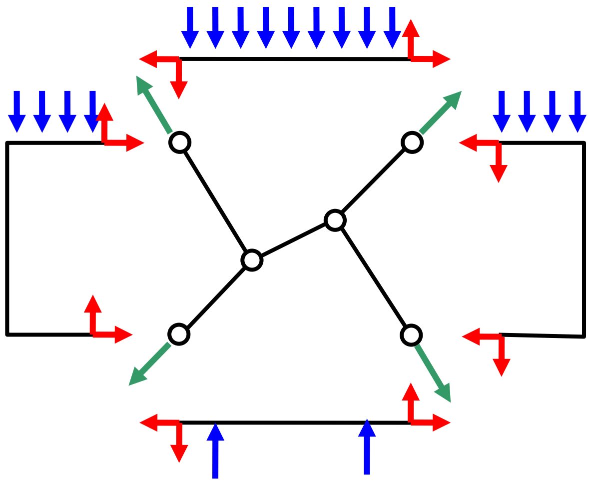

Fig. 12.13 Decomposition of the structure

We have four shields, five bars and three reactions, thus: $4 \cdot 4 + 5 + 3 = 24$ unknowns. We have three equations for each shield and two equations for each hinge, so $4 \cdot 3 + 6 \cdot 2 = 24$ equations.

Glossary

beam element – element belkowy
truss element – element kratowy