Belki na podłożu sprężystym

Podłoże winklerowskie

$$r(x) = bcw(x) = kw(x)$$

$$\frac{d^2 w(x)}{dx^2} = -\frac{M(x)}{EJ}, \quad \frac{dM(x)}{dx} = Q(x), \quad \frac{dQ(x)}{dx} = -q(x) + r(x)$$

$$\frac{d^4 w(x)}{dx^4} = -\frac{d^2}{dx^2}\left(\frac{M(x)}{EJ}\right) = \frac{q(x) - r(x)}{EJ} = \frac{q(x) - kw(x)}{EJ} \qquad \frac{d^4 w(x)}{dx^4} + \frac{bc}{EJ} w(x) = \frac{q(x)}{EJ}$$

$$\alpha \equiv \sqrt[4]{\frac{bc}{4EJ}}, \quad \left[\frac{1}{m}\right], \quad \xi \equiv \alpha x$$

$$\boxed{\frac{d^4 w(\xi)}{d\xi^4} + 4w(\xi) = 4\frac{q(\xi)}{bc}}$$

$$w(\xi) = w_s(\xi) + e^{-\xi}(A \sin\xi + B\cos\xi) + e^{\xi}(C\sin\xi + D\cos\xi)$$

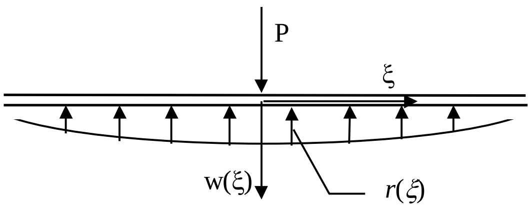
$\sinh(\xi)$ $\cosh(\xi)$

$$M(\xi) = -\alpha^2 EJw''(\xi), \quad Q(\xi) = -\alpha^3 EJw'''(\xi)$$

q/bc

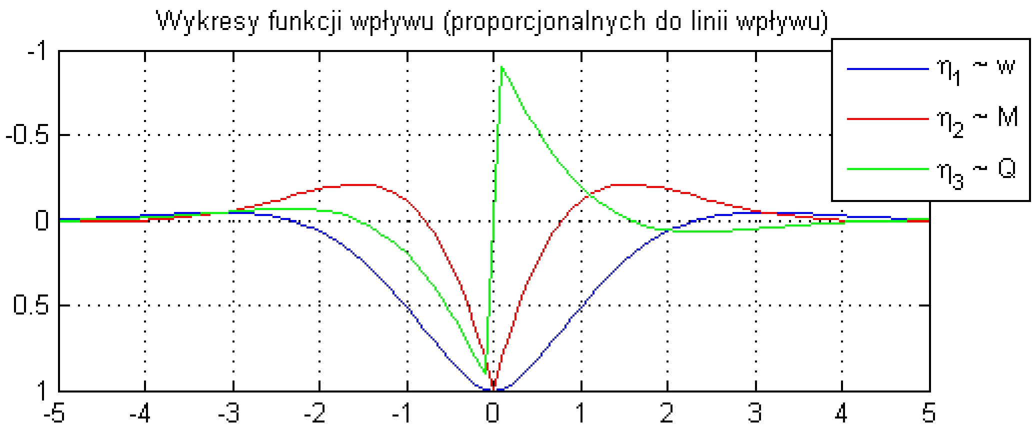
$$\eta_1(\xi) = e^{-\xi}(\sin\xi + \cos\xi),$$
$$\eta_2(\xi) = e^{-\xi}(-\sin\xi + \cos\xi),$$
$$\eta_3(\xi) = e^{-\xi}\cos\xi,$$
$$\eta_4(\xi) = e^{-\xi}\sin\xi,$$

Linie wpływowe M, Q, w



$$\lim_{\xi\to+0} w'(\xi) = 0 \qquad Q(-0) = \tfrac{P}{2} = -Q(+0), \quad EJ\alpha^3 w'''(\xi) = \tfrac{P}{2}$$

$$w(\xi) = \frac{P}{8\alpha^3 EJ}\eta_1(|\xi|),$$
$$M(\xi) = \frac{P}{4\alpha}\eta_2(|\xi|),$$
$$Q(\xi) = \pm\frac{P}{2}\eta_3(|\xi|).$$

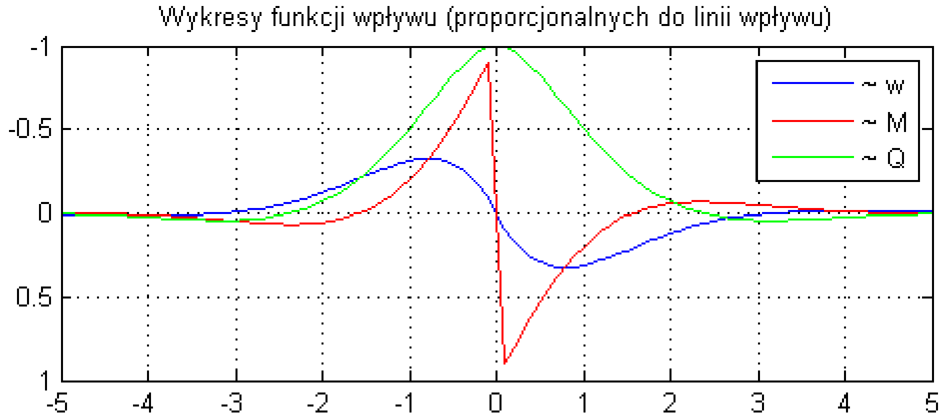


[1]już exp(-4.62) = 0.01

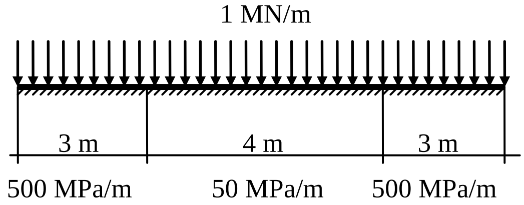
$$w(\xi) = \mp \frac{M_0}{4\alpha^2 EJ}\eta_4(|\xi|),$$

$$M(\xi) = \mp \frac{M_0}{2}\eta_3(|\xi|),$$

$$Q(\xi) = -\frac{M_0 \alpha}{2}\eta_1(|\xi|).$$



Przykład liczbowy

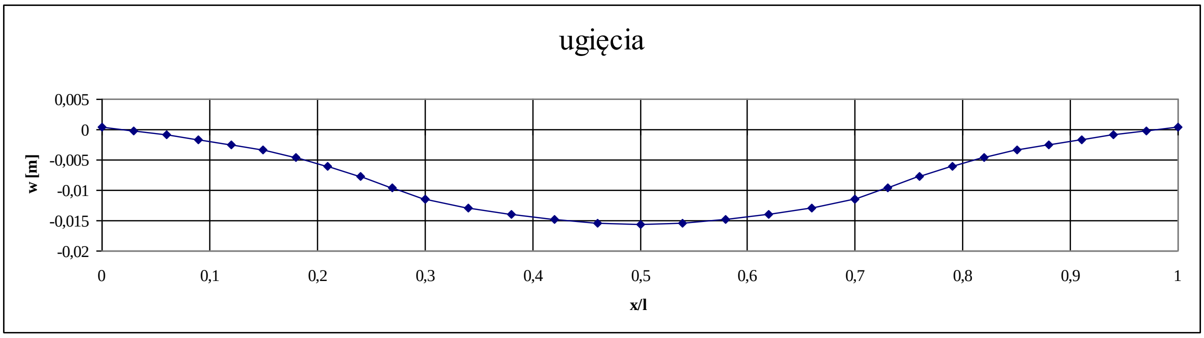


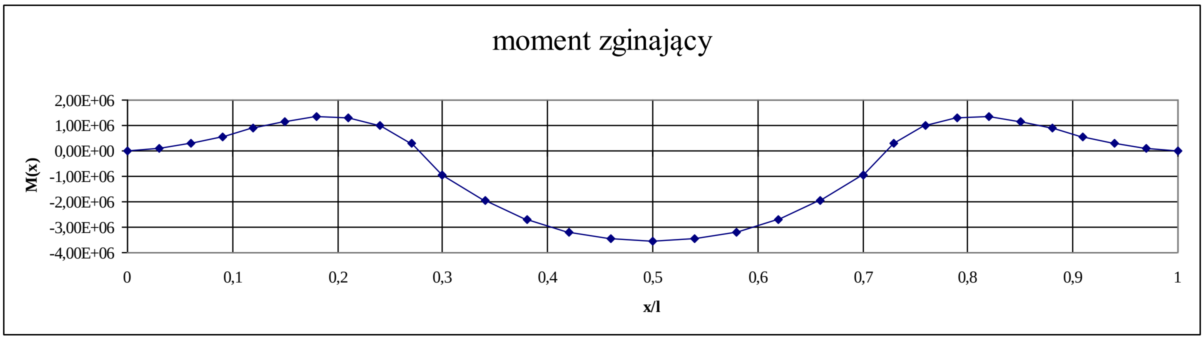
$$w_i(\xi) = \frac{q}{bc}, \quad w_1(\xi) = 0.002, \quad w_2(\xi) = 0.02, \quad w_3(\xi) = 0.002$$

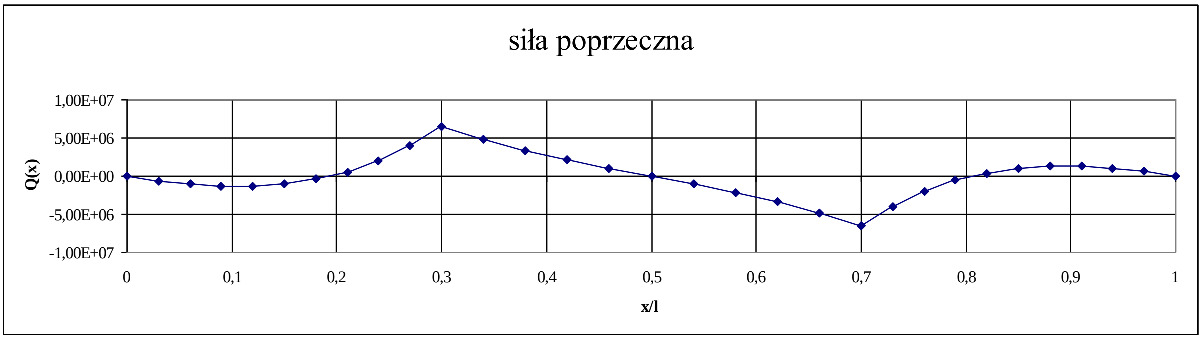
$$w_1(\xi) = e^{-\xi}\left[A_1 \sin(\xi) + B_1 \cos(\xi)\right] + e^{\xi}\left[C_1 \sin(\xi) + D_1 \cos(\xi)\right],$$
$$w_2(\xi) = e^{-\xi}\left[A_2 \sin(\xi) + B_2 \cos(\xi)\right] + e^{\xi}\left[C_2 \sin(\xi) + D_2 \cos(\xi)\right],$$
$$w_3(\xi) = e^{-\xi}\left[A_3 \sin(\xi) + B_3 \cos(\xi)\right] + e^{\xi}\left[C_3 \sin(\xi) + D_3 \cos(\xi)\right].$$

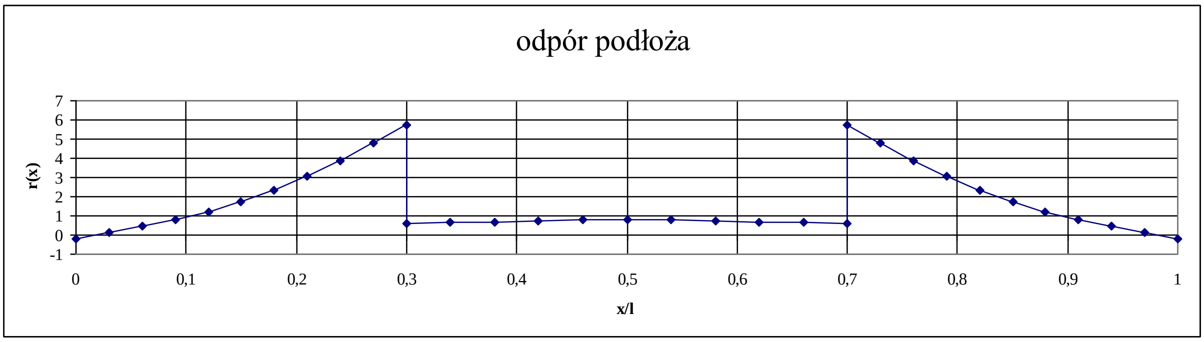
$$M_1(0) = 0 \quad \rightarrow \quad -\alpha_1^2 EJ_y w_1''(0) = 0 \quad \rightarrow \quad -2A_1 + 2C_1 = 0$$
$$Q_1(0) = 0 \quad \rightarrow \quad -\alpha_1^3 EJ_y w_1'''(0) = 0 \quad \rightarrow \quad 2(A_1 + B_1) + 2(C_1 - D_1) = 0$$
$$M_3(2.385) = 0 \quad \rightarrow \quad -\alpha_3^2 EJ_y w_3''(2.385) = 0$$
$$-2e^{-2.385}\left(A_3 \cos 2.385 - B_3 \sin 2.385\right) + 2e^{2.385}\left(C_3 \cos 2.385 - D_3 \sin 2.385\right) = 0$$
$$Q_3(2.385) = 0 \quad \rightarrow \quad -\alpha_3^3 EJ_y w_3'''(2.385) = 0$$
$$-2e^{-2.385}\left[A_3\left(\cos 2.385 + \sin 2.385\right) + B_3\left(\cos 2.385 - \sin 2.385\right)\right] +$$
$$+ 2e^{2.385}\left[C_3\left(\cos 2.385 - \sin 2.385\right) - D_3\left(\cos 2.385 + \sin 2.385\right)\right] = 0$$

$$w_1(2.385) = w_2(0), \quad w_1'(2.385) = w_2'(0),$$
$$\alpha_1^2 w_1''(2.385) = \alpha_2^2 w_2''(0), \quad \alpha_1^3 w_1'''(2.385) = \alpha_2^3 w_2'''(0),$$
$$w_2(1.79) = w_3(0), \quad w_2'(1.79) = w_3'(0),$$
$$\alpha_2^2 w_2''(1.79) = \alpha_3^2 w_3''(0), \quad \alpha_2^3 w_2'''(1.79) = \alpha_3^3 w_3'''(0),$$









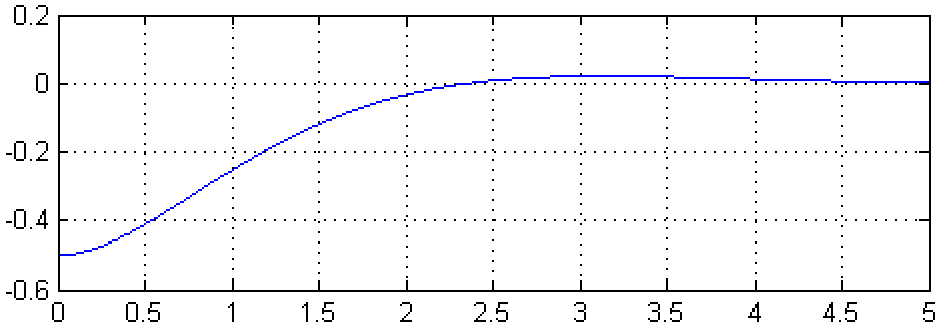
Belki pół-nieskończone

$$\xi \to \infty$$

Przykład 1

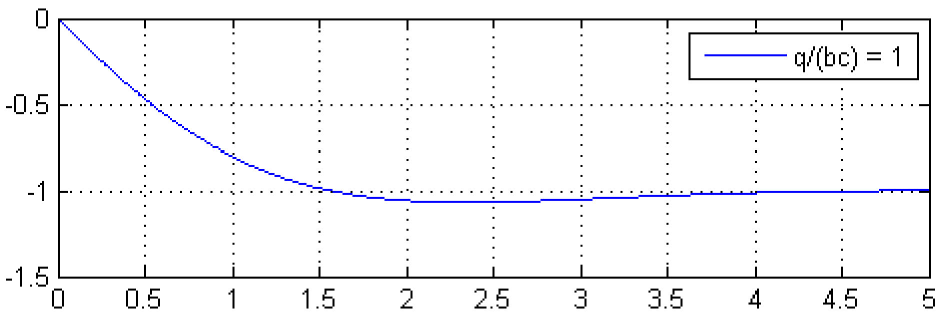
$$w_s(\xi) = 0 \qquad\qquad M(0) = M_0, \quad Q(0^+) = -P$$

$$w(\xi) = \frac{1}{2\alpha^2 EJ} e^{-\xi}\left[M_0 \sin\xi + \left(\tfrac{P}{\alpha} - M_0\right)\cos\xi\right]$$



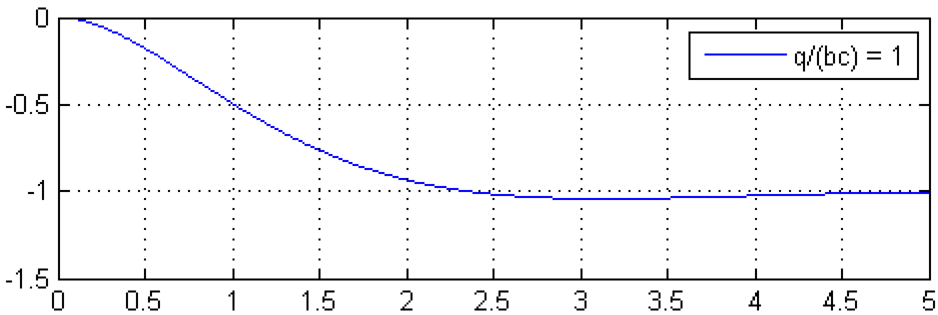
$$w_s(\xi) = \frac{q}{bc} \qquad w(0) = M(0) = 0$$

$$w(\xi) = \frac{q}{bc}\left(1 - e^{-\xi}\cos\xi\right)$$



$$w_s(\xi) = \frac{q}{bc} \qquad w(0) = w'(0) = 0$$

$$w(\xi) = \frac{q}{bc}\left[1 - e^{-\xi}\left(\cos\xi + \sin\xi\right)\right]$$

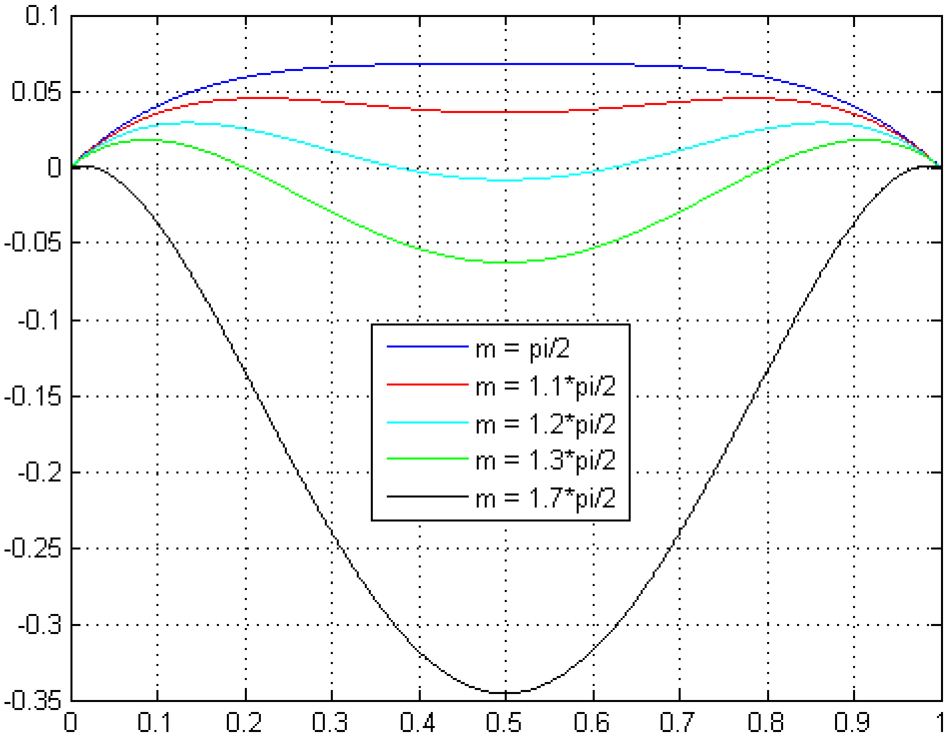


Przykład 4

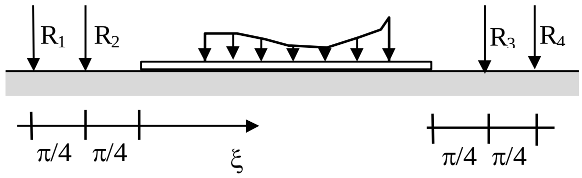
$$w_s(\xi) = \frac{q}{bc}$$

$$\frac{P}{8\alpha^3 EJ}\left[\eta_1(0) + \eta_1(m)\right] + \frac{q}{bc} = 0$$

$$P = \frac{2q}{\alpha} \cdot \frac{1}{1 + e^{-m}(\cos m + \sin m)}$$



Sposób Bleicha

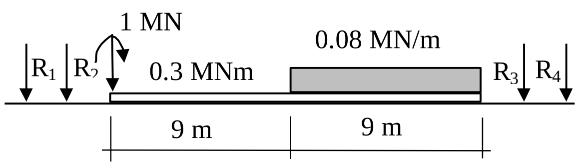


$$M(\xi) = \frac{1}{4\alpha}\sum_i P_i \eta_2(|\xi_i - \xi|) \mp \frac{1}{2}\sum_j M_j \eta_3\begin{pmatrix}\xi_j - \xi \\ \xi - \xi_j\end{pmatrix} + \frac{1}{4\alpha^2} q(\eta_4(\xi_w) - \eta_4(\xi_m))$$

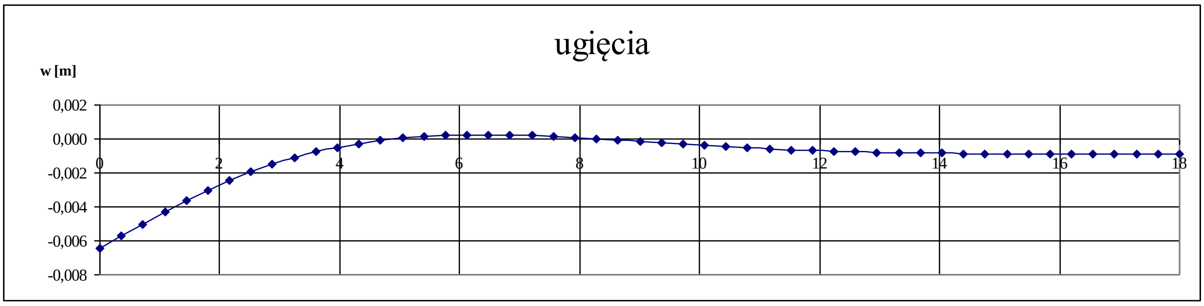
$$Q(\xi) = \pm\frac{1}{2}\sum_i P_i \eta_3\begin{pmatrix}\xi_j - \xi \\ \xi - \xi_j\end{pmatrix} - \frac{\alpha}{2}\sum_j M_j \eta_1(|\xi_i - \xi|) \pm \frac{q}{4\alpha}(\eta_2(\xi_m) - \eta_2(\xi_w))$$

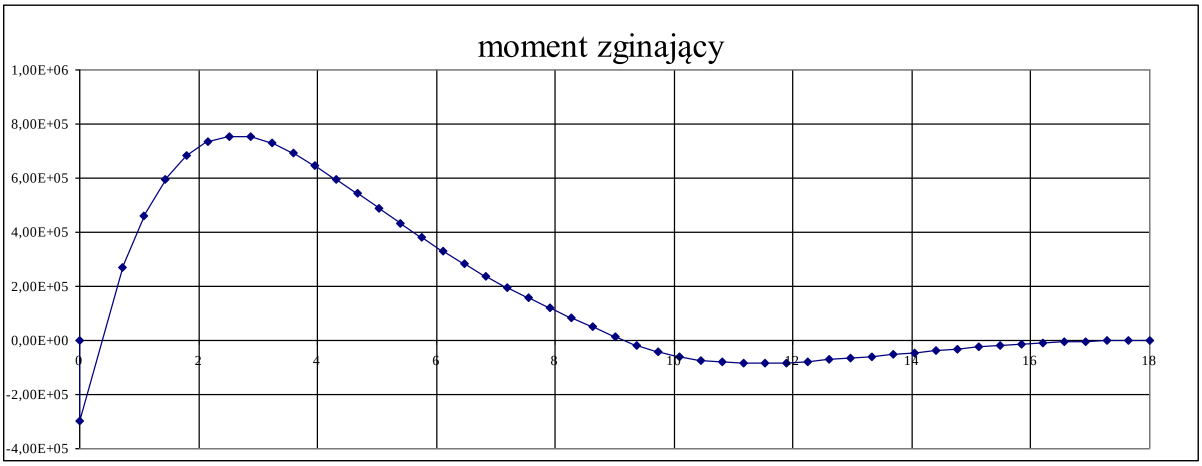
$$r(\xi) = \frac{\alpha}{2}\sum_i P_i \eta_1(|\xi_i - \xi|) \mp \alpha^2 \sum_j M_j \eta_4\begin{pmatrix}\xi_j - \xi \\ \xi - \xi_j\end{pmatrix} + \frac{q}{2}(\eta_3(\xi_m) - \eta_3(\xi_w))$$

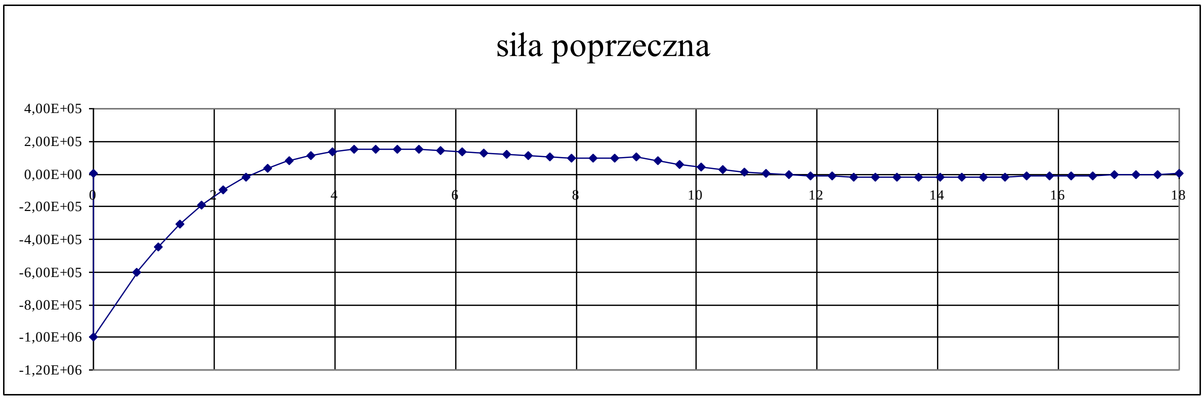
Przykład obliczeniowy



$$\alpha = \sqrt[4]{\frac{bc}{4EJ}} = 0.348\,\tfrac{1}{\mathrm{m}}, k = bc = 96 \cdot 10^6\,\mathrm{Pa}$$






siła poprzeczna
4,00E+05
2,00E+05
0,00E+00
-2,00E+05
-4,00E+05
-6,00E+05
-8,00E+05
-1,00E+06
-1,20E+06
0
2
4
6
8
10
12
14
16
18