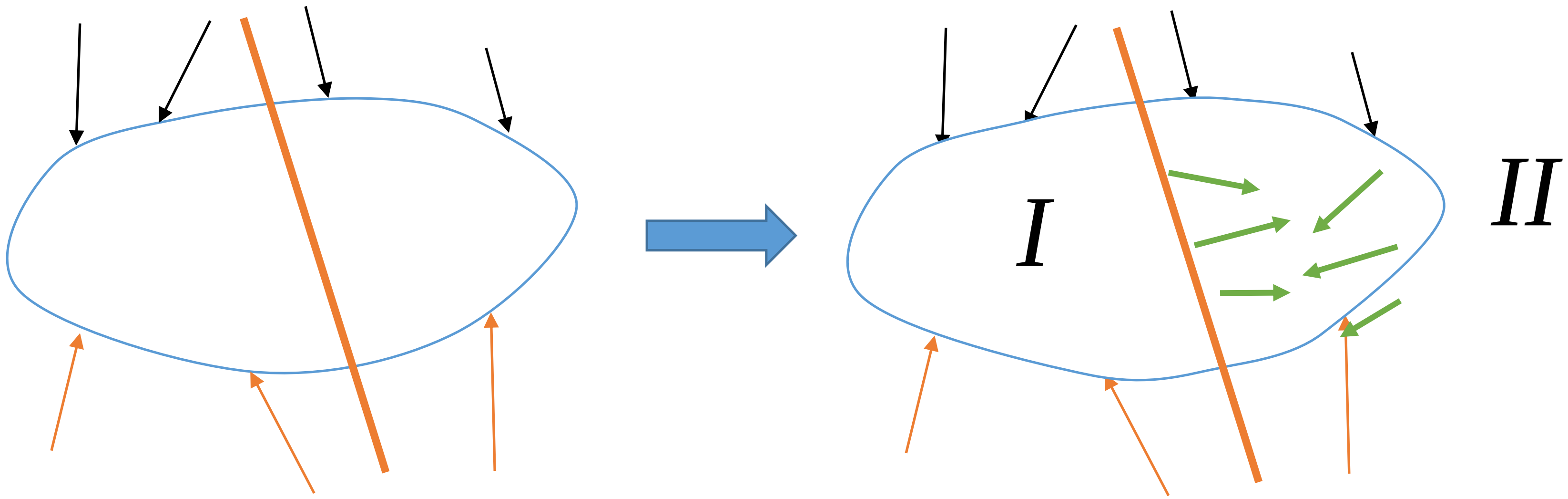
Strength of Materials

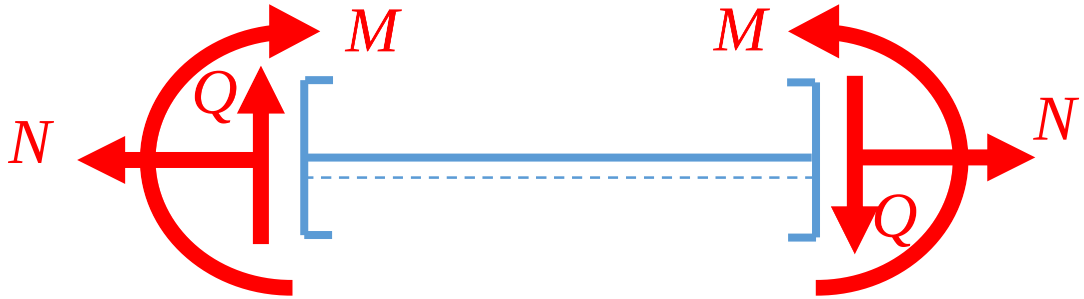
3. Trusses

Cross-sectional forces – a reminder



$$W_{II} = Z_I$$

$$W_I = Z_{II}$$



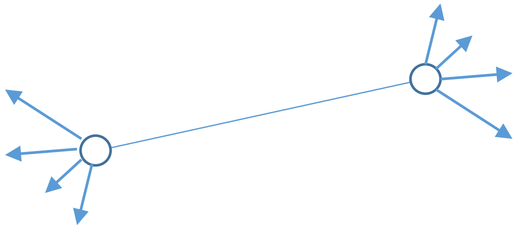
M – bending moment

N – axial force (tension/compression)

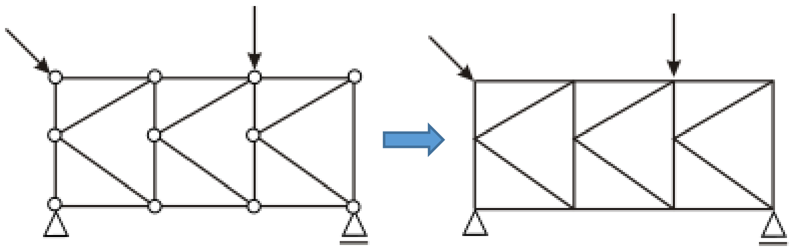
Q – shear (transversal) force

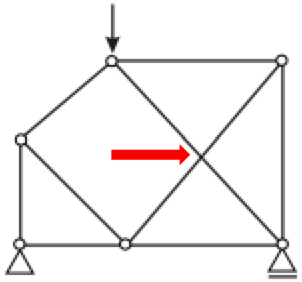
Truss – a definition

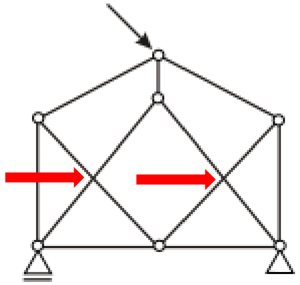
truss element – a straight element, hinge-tipped, loaded solely by point forces at the swivels

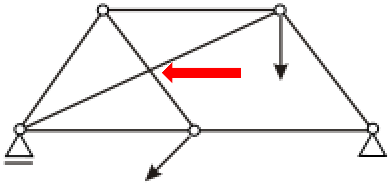


truss – a structure that consists of truss members only

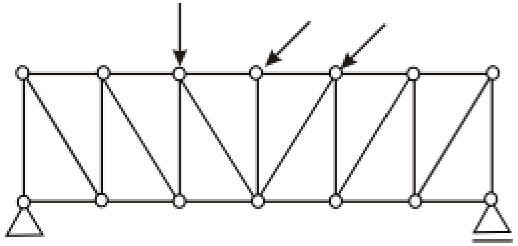


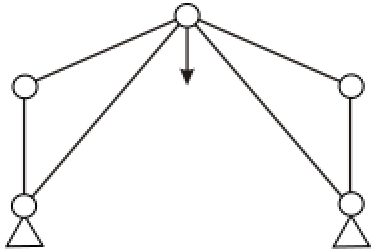






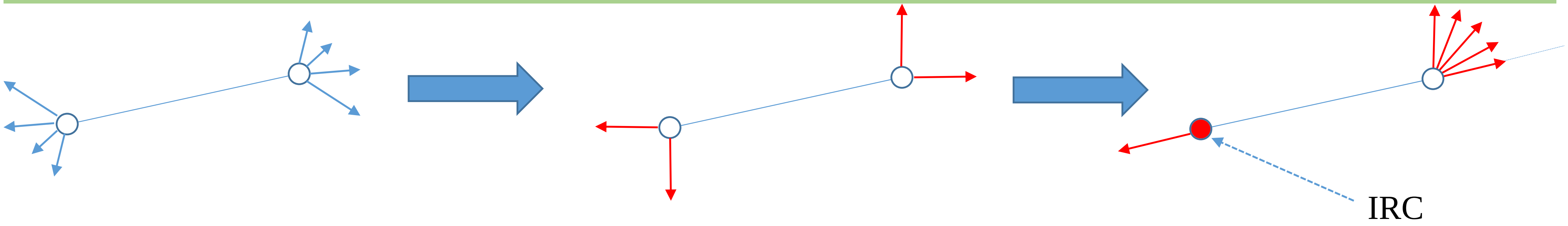
internally stable
externally stable

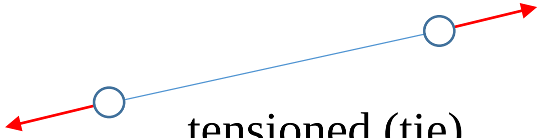




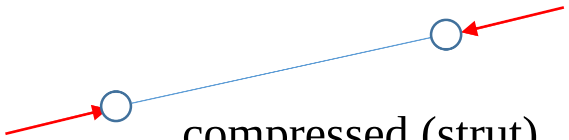
internally unstable
externally stable

Consequences of the truss definition



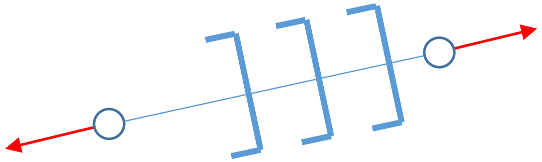


tensioned (tie)



compressed (strut)

truss element = two forces element



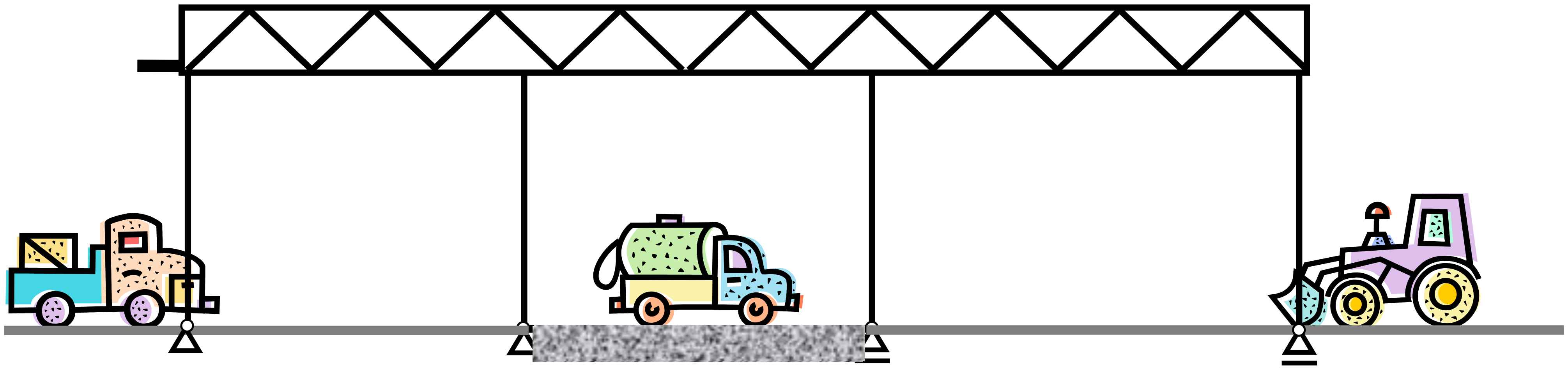
the set of cross-section forces reduces to one constant axial force

position of the section is not relevant

Motivation

Motivation to use trusses is quite specific

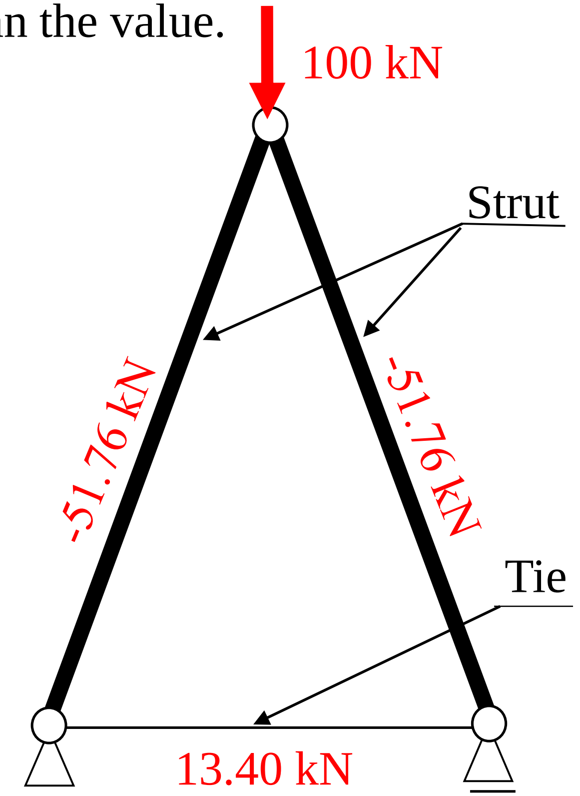
Trusses are aimed to span large areas with a light but durable structure



Signs

The sign of the force is much more important than the value.

The compressed elements are designed in quite different technique, and they are way bigger than the ties.



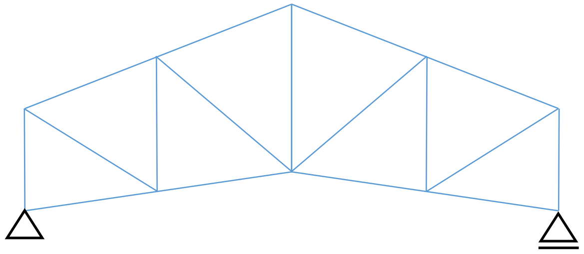
The diagrams of forces are rectangular in shape and are not necessary.
The values with signs will suffice.

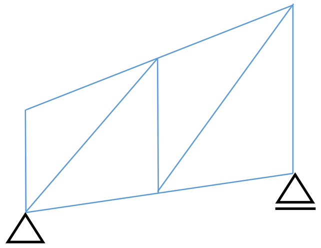
The sign convention:

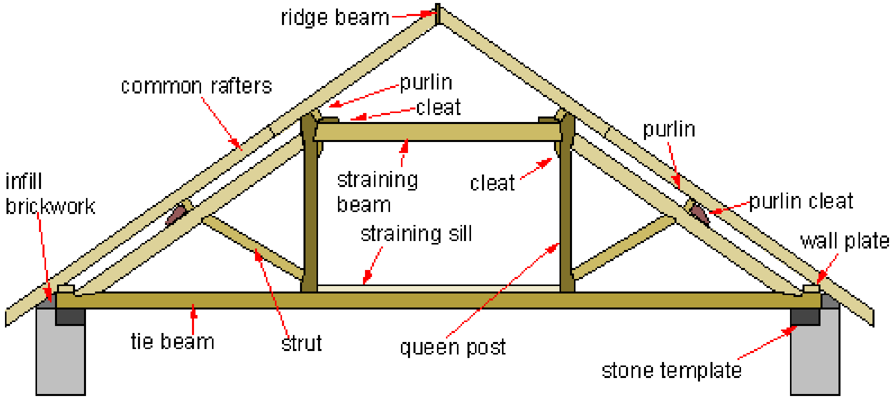
- plus (+) means tension
- minus (-) means compression

Trusses – some examples

Pitched – planar roof truss

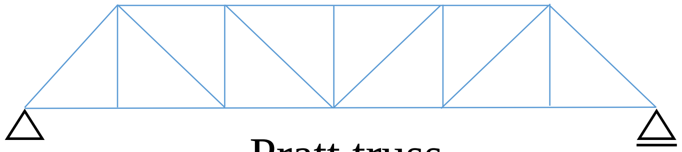




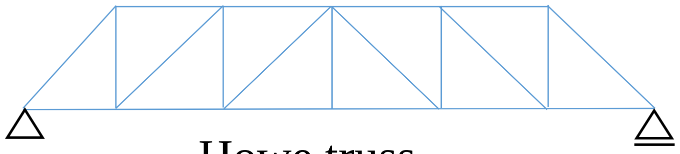


Traditional Queen Post Roof Truss

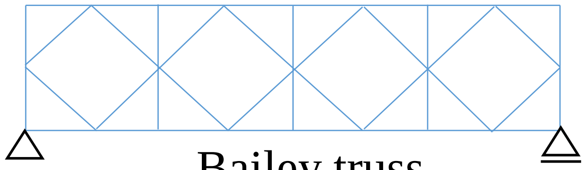
Parallel chord truss



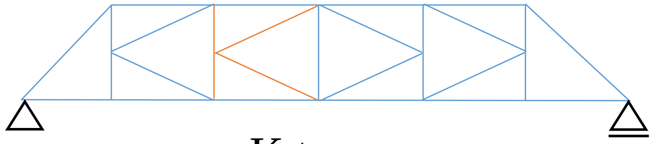
Pratt truss



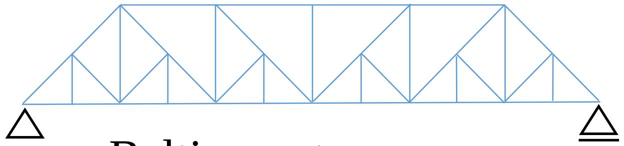
Howe truss



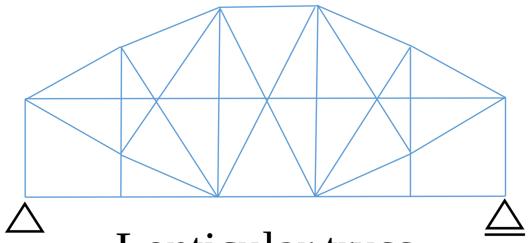
Bailey truss



K truss

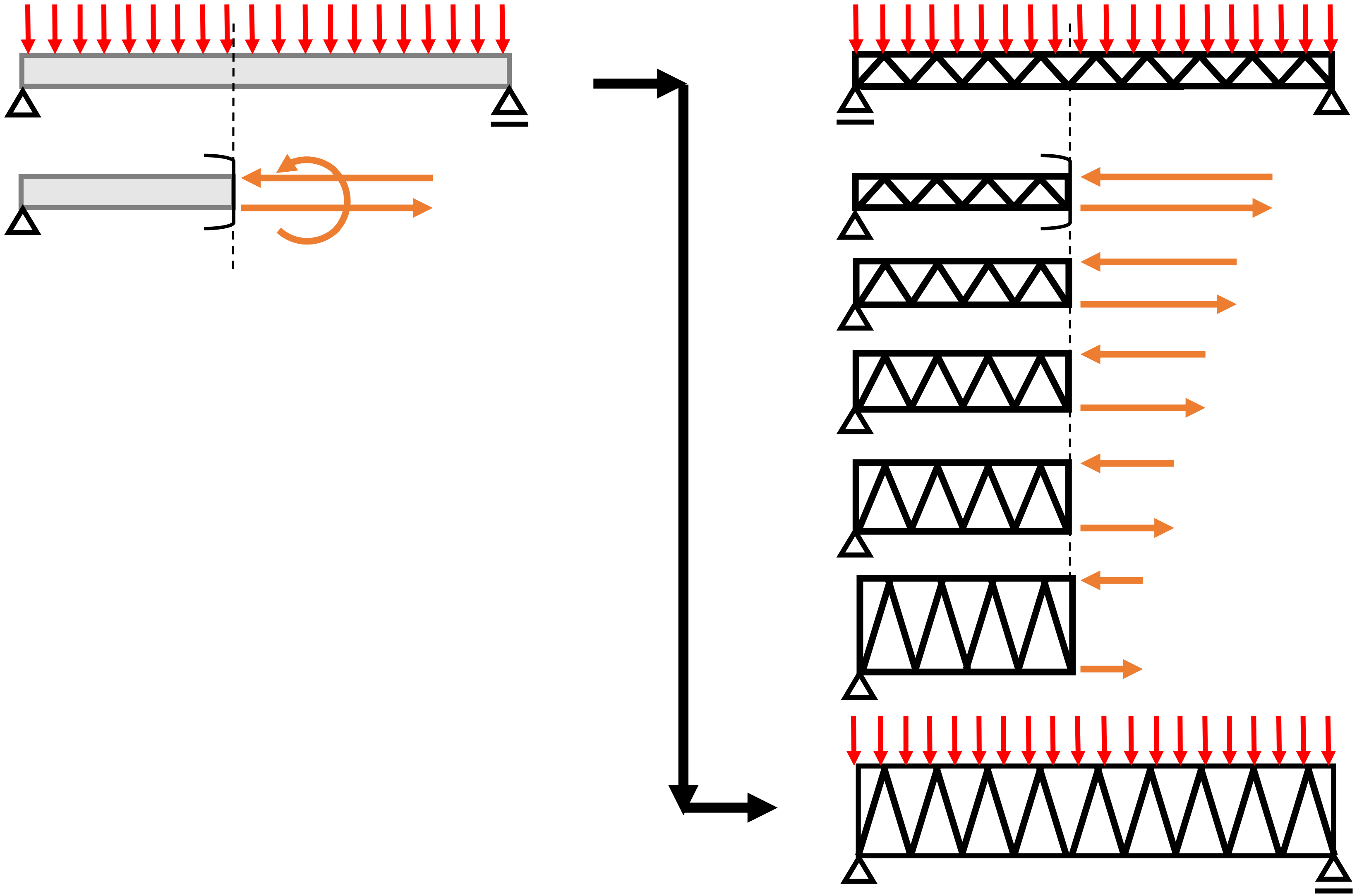


Baltimore truss

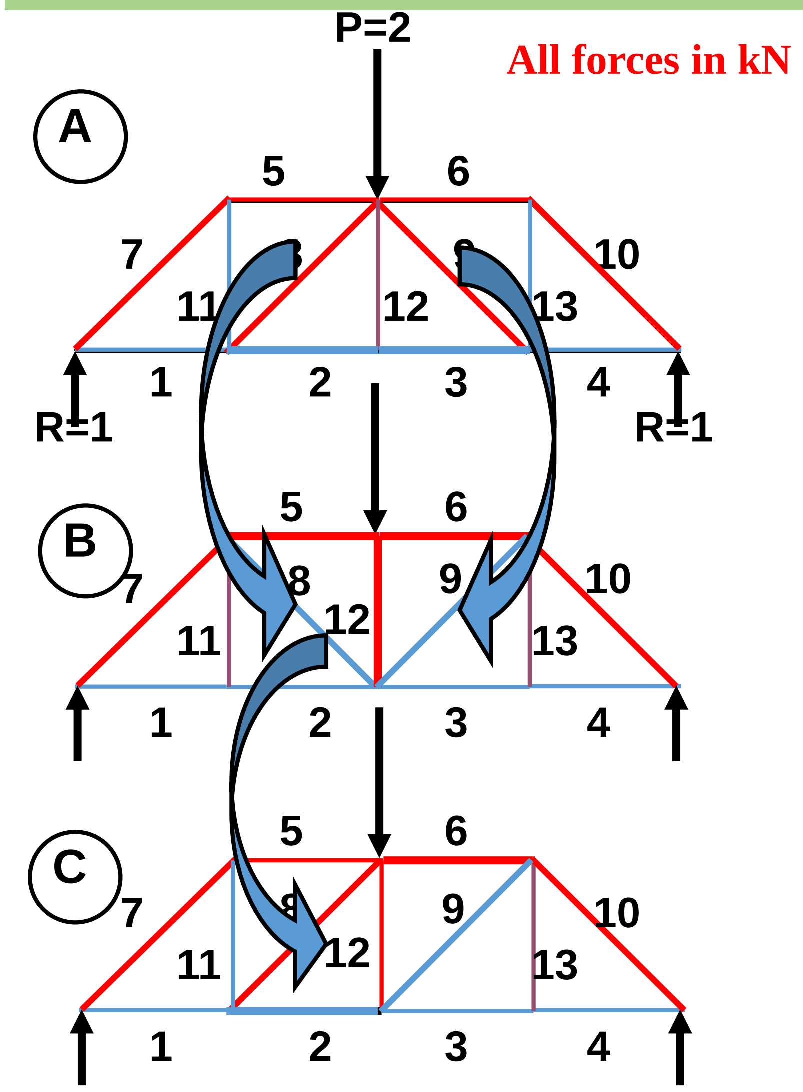


Lenticular truss

How does a truss work?



How does truss work – cont.



Bar	Frame A	frame B	frame C
1	1		
2	2		
3	2		
4	1		
5	-1		
6	-1		
7	-1,41		
8	-1,41		
9	-1,41		
10	-1,41		
11	1		
12	0		
13	1		

Bars under tension

Bars under compression

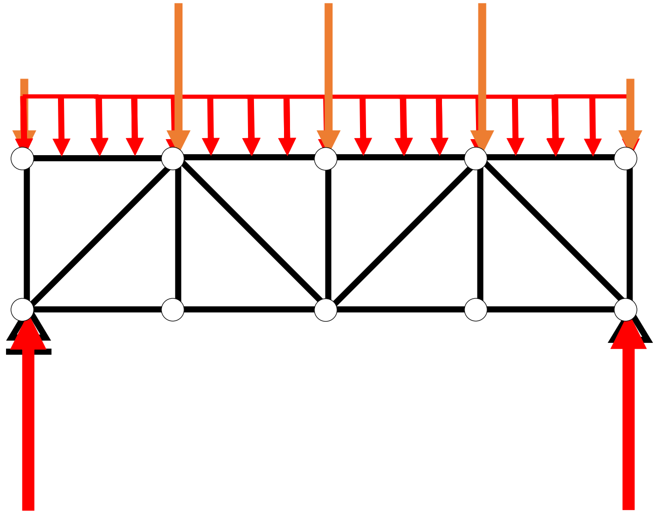
„0-bars”

Geometry – stability

Too many joints, too few bars!

w – number of joints

p – number of bars



Too many bars, too few joints

$\nu = 2w - p - r$

Structure has to be kinematically stable,
but can be statically determined or undetermined!

2w = number of equations

p +r = number of unknowns

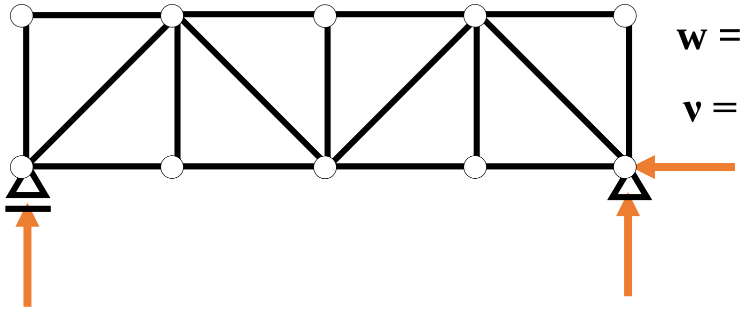
reactions

	Kinematics	Statics
$\nu > 0$	unstable	undefined
$\nu = 0$	stable	determined
$\nu < 0$	stable	undetermined

Trusses – geometry

Examples of kinematic and static determination

$v = 2w - p - r$

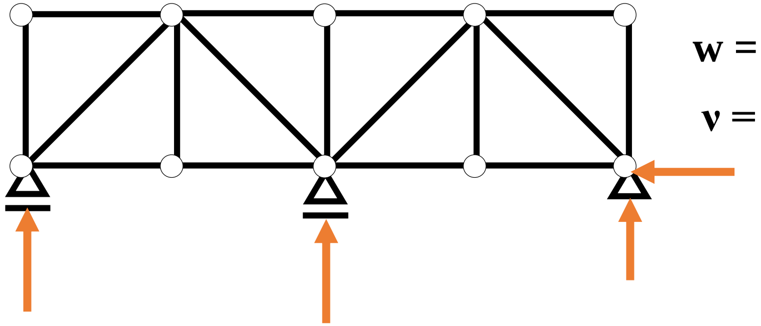


$w = 10,\ p = 17$

$v = 2 \cdot 10 - 17 - 3 = 0$

Internally and externally determined

Kinematically stable



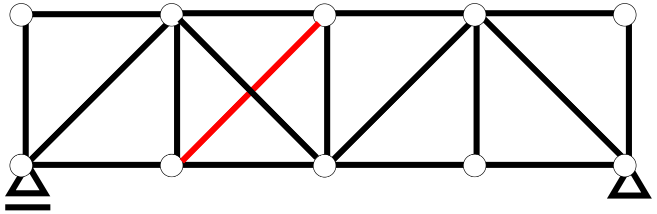
Internally determined

Externally undetermined

$w = 10,\ p = 17$

$v = 2 \cdot 10 - 17 - 4 = -1$

Kinematically stable



Externally determined

$w = 10,\ p = 16$

$v = 2 \cdot 10 - 16 - 3 = 1$

Internally undetermined

Kinematically stable

Geometry: stability check

Two shields theorem: a connection of two shields is rigid iff the shields are connected by three bars not concurrent. (The bars are concurrent when their directions intersect at one point or the bars are parallel. Two bars of the trio can be substituted with a hinge at an intersection point of their directions.)

Three shields theorem: a connection of three shields is rigid iff the shields are connected by three hinges not collinear. (Two hinges of the trio can be at improper points, at infinity.)

Virtual velocities theorem: a structure is unmovable iff there is no consistent field of virtual velocities.

If the structure is free-body stable the constraints reactions calculation can be performer in usual way. In another case, the procedure demands additional steps (like a hinge equation for the three-hinges structure.)

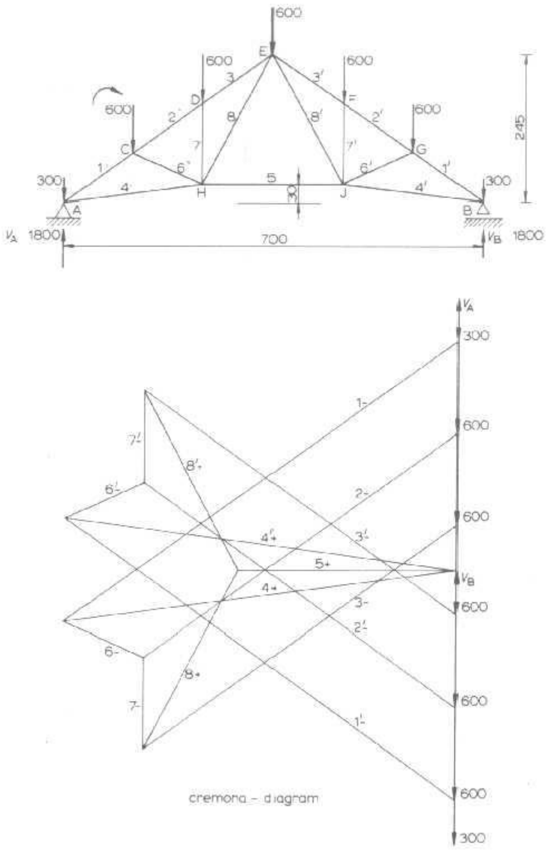
Only stable structures can be analyzed (and calculated) by statics methods. Unstable structures (mechanisms) should be analyzed (calculated) using dynamics (with account of inertia forces).

Calculation methods

Graphical methods (of historical importance): funicular polygon, Cremona-Maxwell diagram, Culmann method

Analytical methods (they are really numerous):

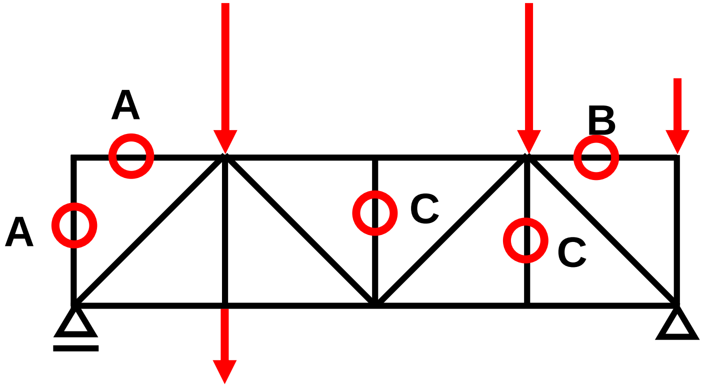
- method of joints
- method of sections
- method of bars conversion
- method of virtual works

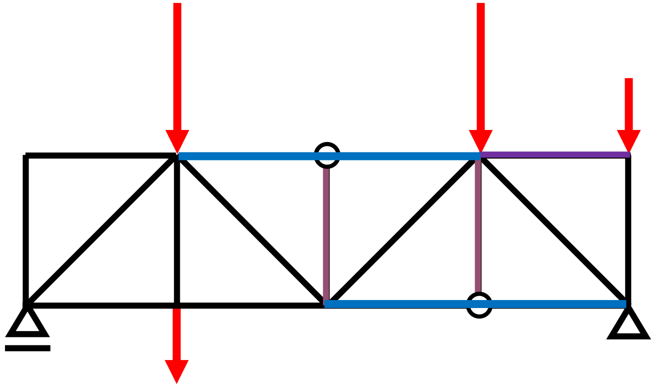


Usually, we start with the zero-force members theorems.

Theorems of zero forces members

Certain bars are required solely for the purpose of keeping a truss kinematically stable. The cross-sectional forces in these bars vanish; one can call them „0-bars".



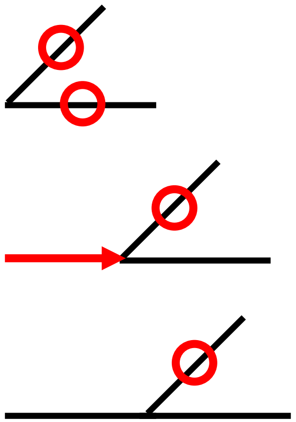


There are three cases in which we can easily spot „0-bars":

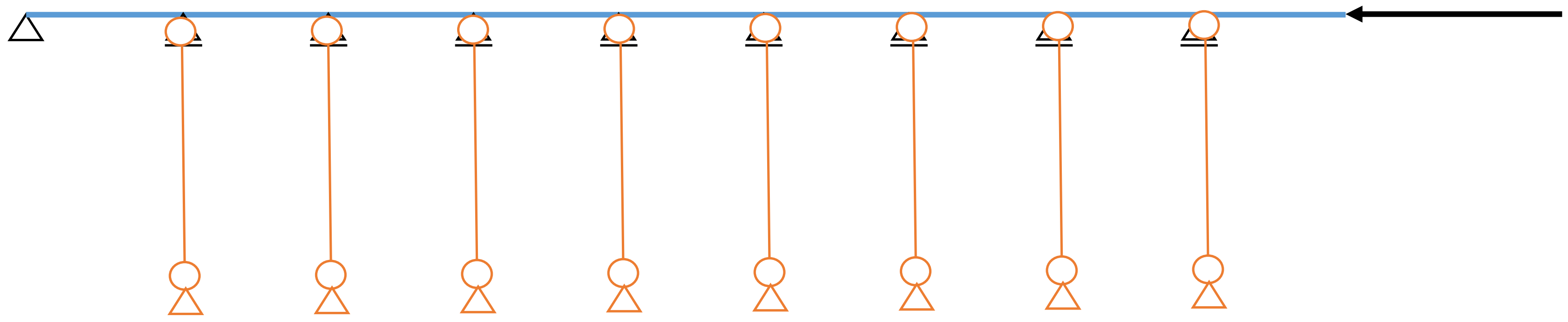
A. When only two bars converge in a node which is free of loading

B. When a node connects two bars but the loading acts along of any of these bars

C. When unloaded node connects three bars, two of them being co-linear



ZFM application

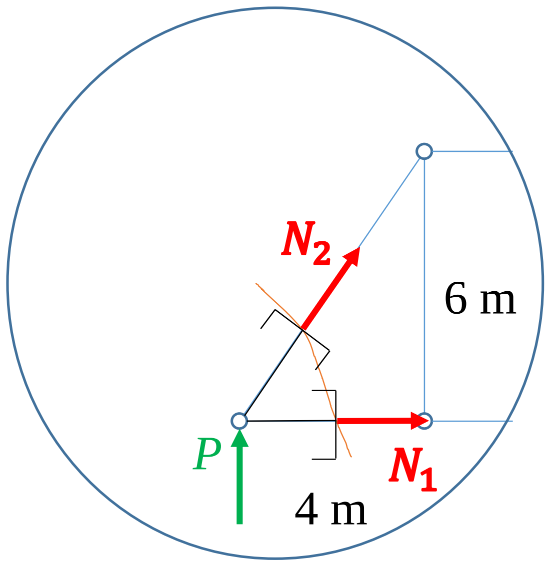


The loading capacity grows inversely to the length squared: the column 10 times shorter has 100 times greater loading capacity.

Method of joints

For a hinge of two concurrent bars with unknown forces we can use the balance equations set (for a concurrent forces set):

$$\Sigma X = 0$$
$$\Sigma Y = 0$$



$$P = 25\ kN$$

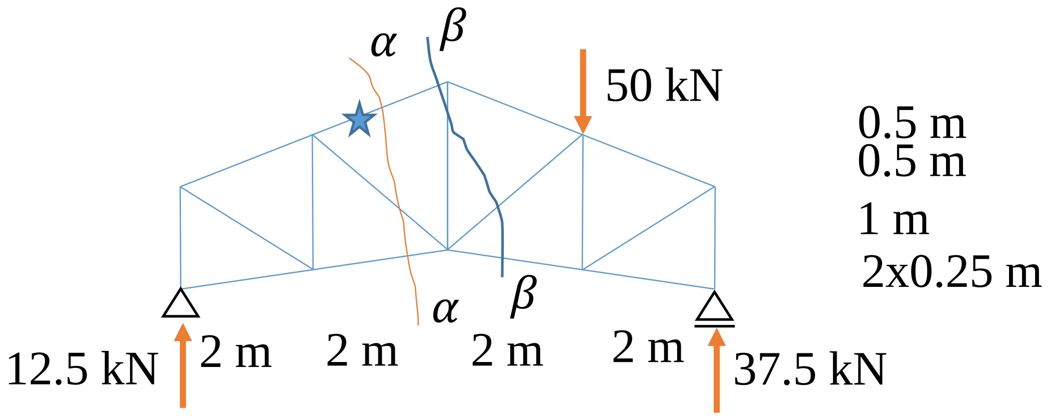
$$\Sigma Y = 0 \rightarrow N_2 \frac{6}{\sqrt{4^2+6^2}} + 25 = 0 \rightarrow N_2 \cdot 0.832 + 25 = 0 \rightarrow N_2 = -30.0\ kN$$

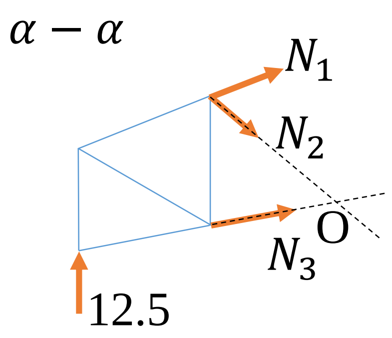
$$\Sigma X = 0 \rightarrow N_2 \frac{4}{\sqrt{4^2+6^2}} + N_1 = 0 \rightarrow N_1 = 16.6\ kN$$

Method features:

- start in the node with two unknown
- routine procedure for each node
- ineffective for a long triangular truss

Method of sections





first: translation of the force to the point above O
next: the force decomposition

$$\Sigma M_0 = N_1 \frac{2}{\sqrt{2^2+0.5^2}} \cdot 2 + 12.5 \cdot 4 = 0 \rightarrow N_1 = -25.8\ kN$$

(The same equation will arise for the section $\beta - \beta$ and the point O.)

Method features:

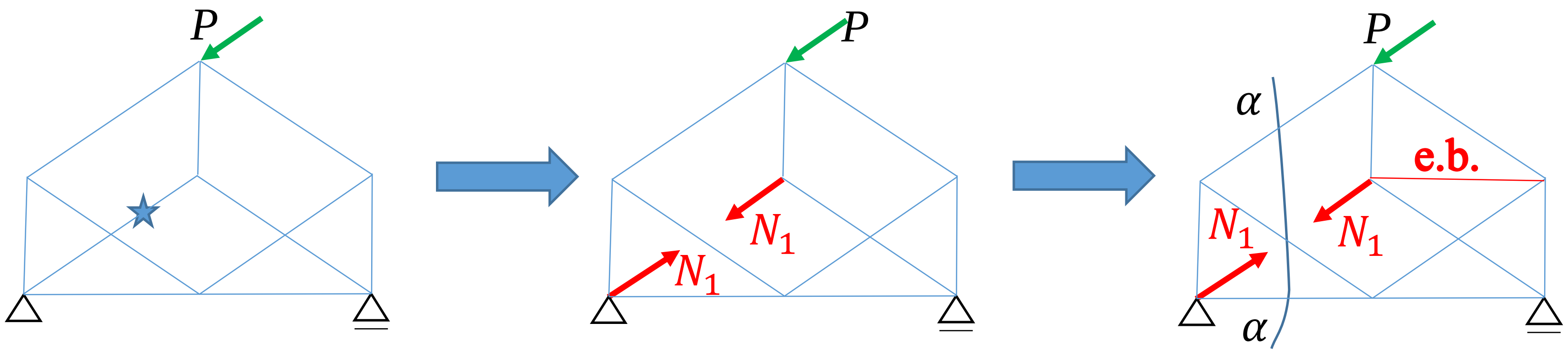
- simple idea of a section through three non-concurrent bars
- quick result in most cases
- uncoupled equations set is advised

Method of bars conversion (Henneberg)

To begin with the previous method we should have a suitable point of start:

- a node with two concurrent bars only (for the joints method)
- a section through three non-concurrent bars (for the sections method)

Sometimes, neither of the previous conditions is fulfilled.



N_1 - self-balanced set of forces

e.b. – extra bar

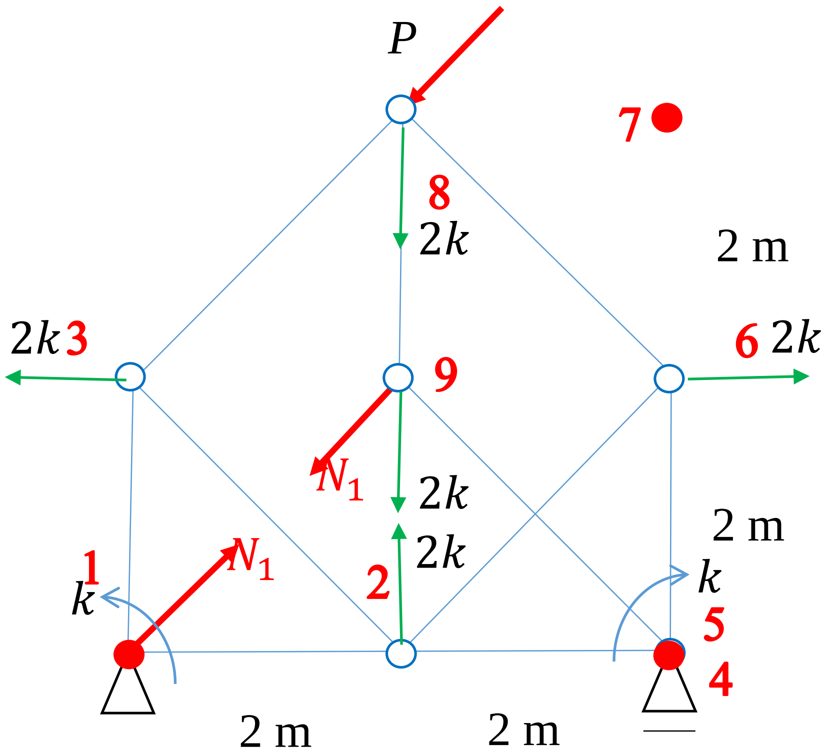
free-body stability: 3ST – stable
overall stability: 2ST – stable
no node with two bars only
no section through three non-concurrent bars
so: Henneberg method

$$N_{eb} = N_{eb}(P_i) + N_{eb}(N_1) = N_{eb}(P_i) + N_1 N_{eb}(1) = 0$$

$$N_1 = -\frac{N_{eb}(P_i)}{N_{eb}(1)}$$

We calculate the force in the extra bar twice: the first time for external forces, the second time for self-balanced internal unit forces

Method of virtual displacements



10

$$\delta L = 0$$

$$\delta L = P\frac{\sqrt{2}}{2}2k + N_1\sqrt{2}\cdot 2k$$

$$N_1 = -0.5P$$

The principle of virtual works:
for a system in static equilibrium, the virtual work of applied forces on virtual displacements is zero,
or, the principle of virtual powers:
the virtual power of the applied forces is zero for all virtual velocities of the system in static equilibrium.

1. there is IRC at the pin support, we assume some angular speed k
2. the velocity at the lower middle hinge is vertical and equal $2k$
3. the velocity at the left upper hinge is horizontal and equal to $2k$
4. the relative velocity at the lower right hinge is vertical, so its total velocity is vertical also, which is excluded by the roller, so it is IRC
5. the angular velocity at the bottom right IRC is k
6. the velocity at upper right hinge is $2k$
7. due to symmetry, the velocity at upper hinge is vertical: there is another IRC
8. the velocity of the actual bar is $2k$, so
9. the vertical component of velocity at the hinge where force N_1 acts should be $2k$, so, total velocity at the point is $2k\sqrt{2}$
10. now, we can calculate the virtual power of the forces P and N_1

Thank you for your attention!