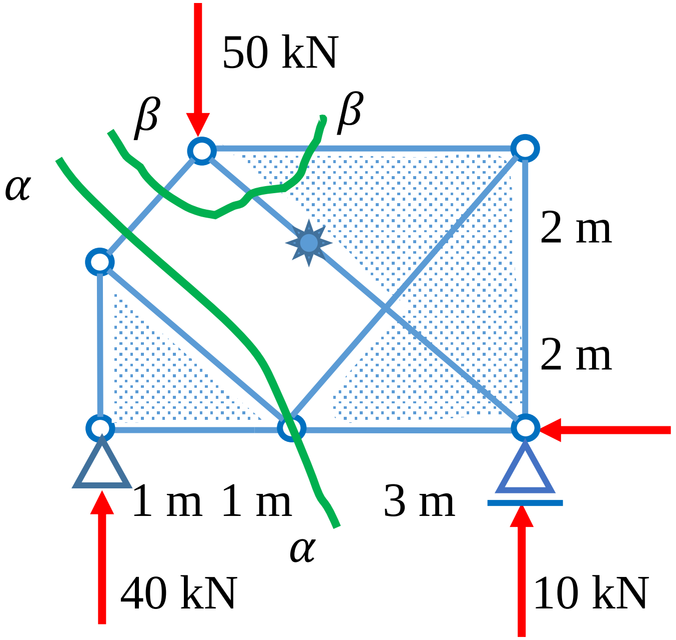
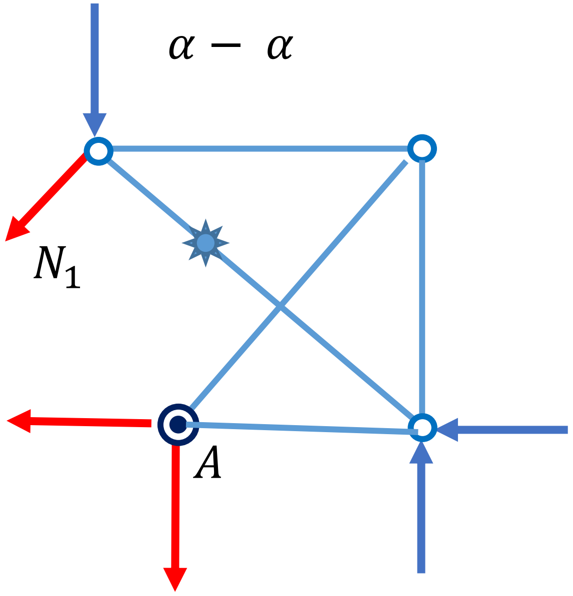
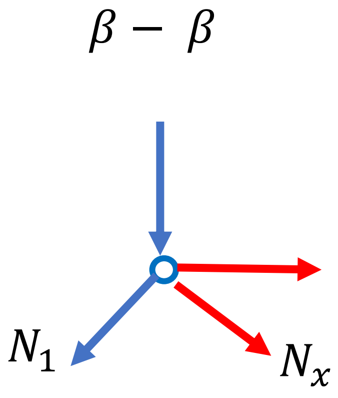
Strength of Materials

6. Trusses & frames

Trusses – example No. 1







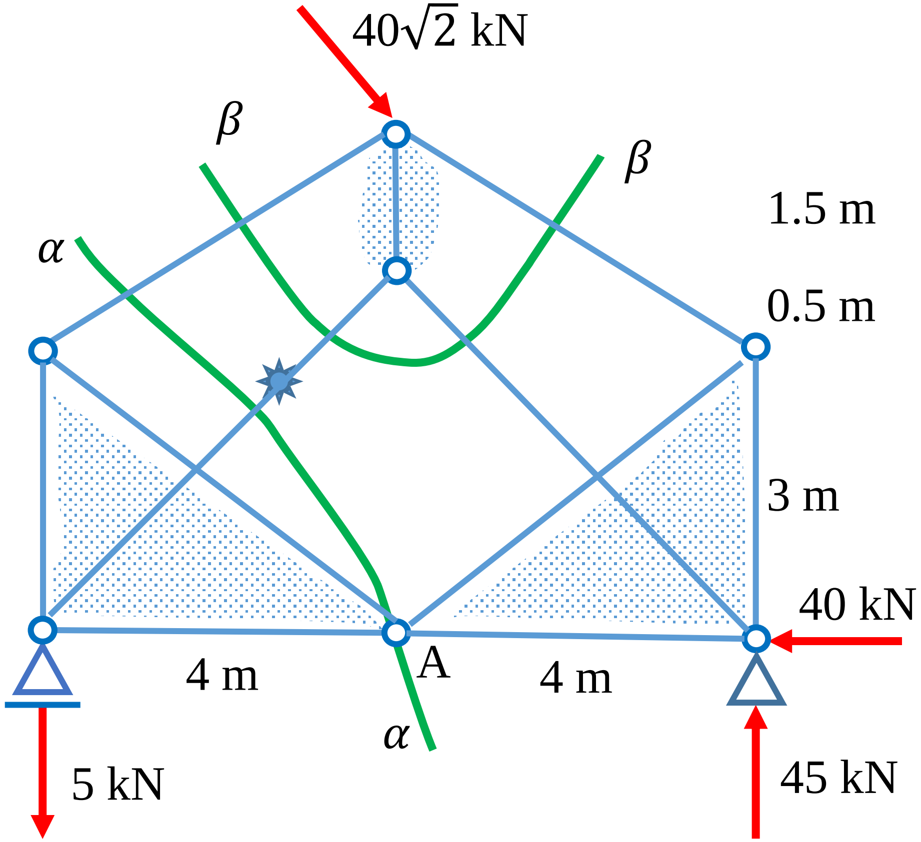
$$\Sigma Y = 0 \rightarrow N_x$$

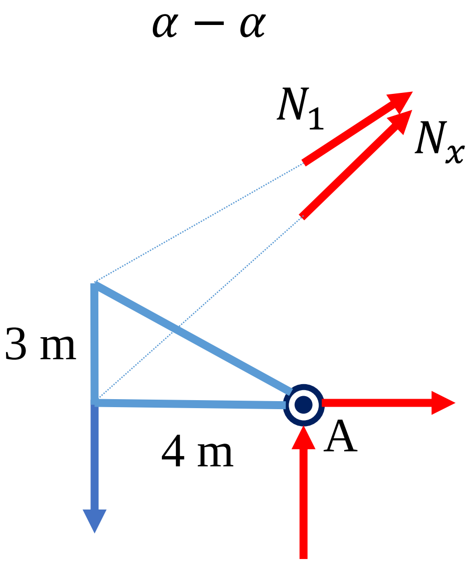
$$\Sigma M_A = 0 \rightarrow N_1$$

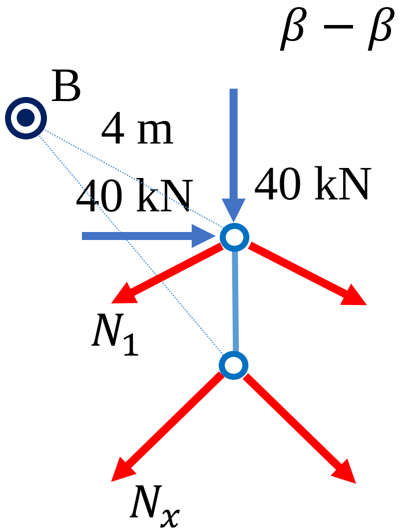
$$\alpha - \alpha: \;\; 50 \cdot 1 + N_1 \cdot \frac{2}{\sqrt{5}} \cdot 1 + N_1 \cdot \frac{1}{\sqrt{5}} \cdot 4 + 10 \cdot 3 = 0 \;\rightarrow\; N_1 = -29.81 \text{ kN}$$

$$\beta - \beta: \; -\frac{2}{\sqrt{5}} N_1 - \frac{\sqrt{2}}{2} N_x - 50 = 0 \;\rightarrow N_x = -33 \text{ kN}$$

Truss – example No. 2





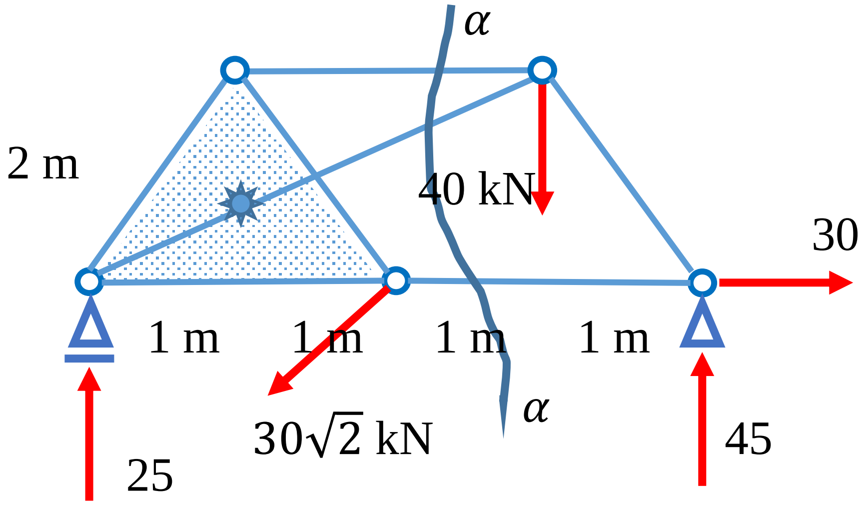


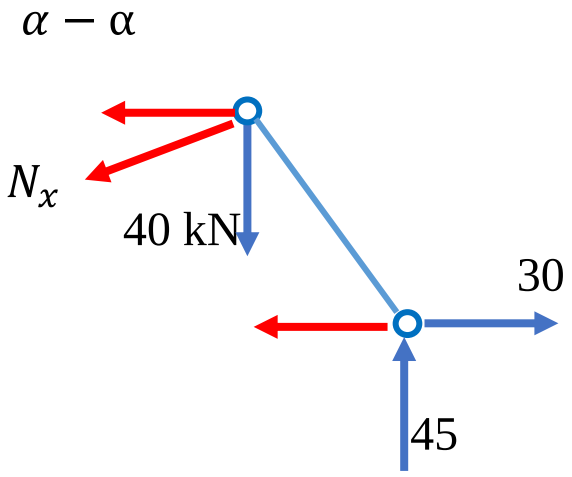
$$\Sigma M_A = N_1 \frac{4}{\sqrt{20}} \cdot 5 + N_x \frac{4}{\sqrt{3.5^2 + 4^2}} \cdot 3.5 - 5 \cdot 4 = 0$$

$$\Sigma M_B = -N_1 \frac{4}{\sqrt{20}} \cdot 4 - N_x \frac{4}{\sqrt{3.5^2 + 4^2}} \cdot 7 + 40 \cdot 2 - 40 \cdot 2 = 0$$

$$N_1 = -22.36 \text{ kN}; N_x = 30.37 \text{ kN}$$

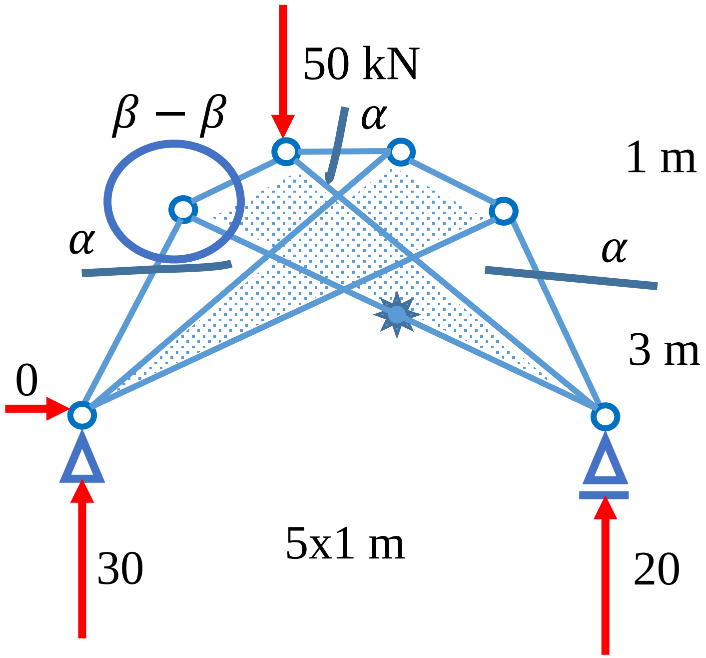
Truss – example No. 3



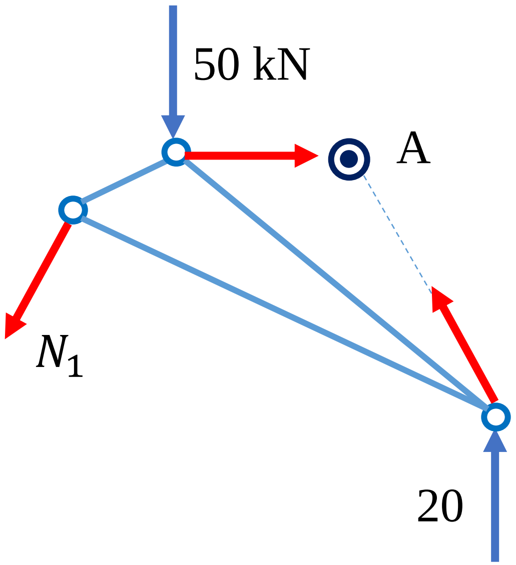


$$\Sigma Y = 0 \rightarrow N_x = 9.014 \text{ kN}$$

Example No. 4

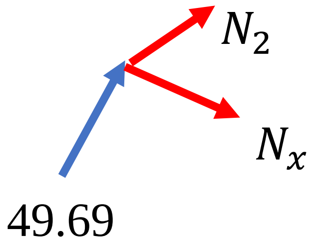


$\alpha - \alpha$



$$\Sigma M_A = 0 \rightarrow N_1 = -49.69 \text{ kN}$$

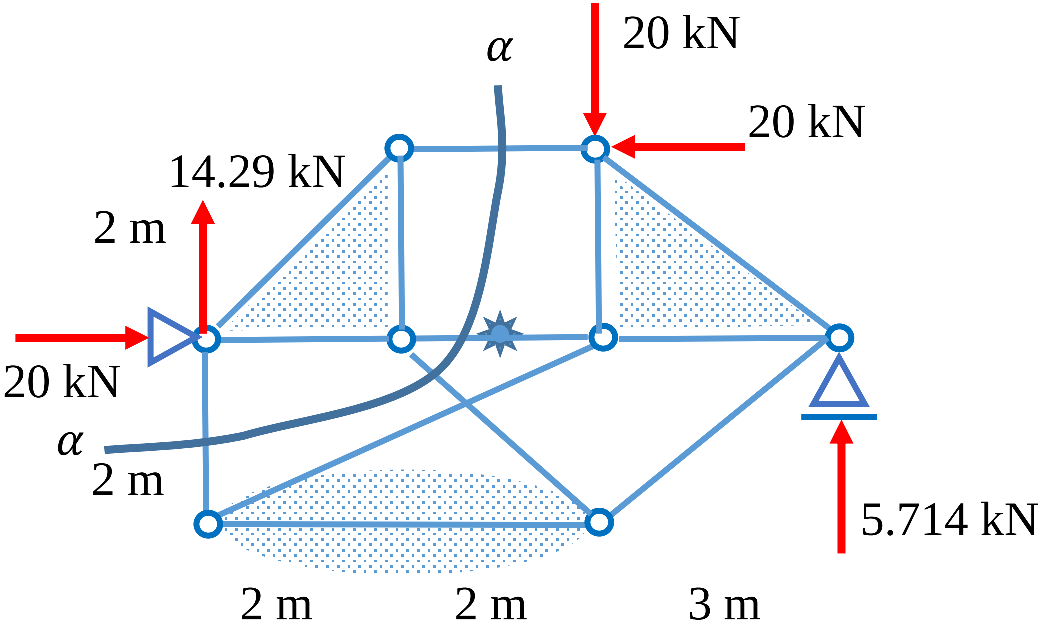
$\beta - \beta$

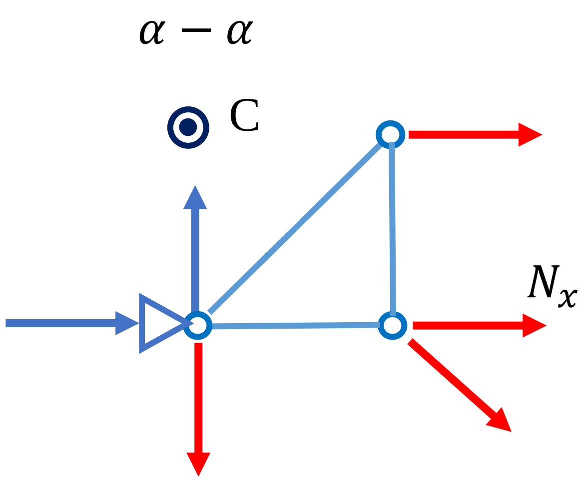


$$\Sigma X = 0$$
$$\Sigma Y = 0$$

$$N_2 = -47.62 \text{ kN}$$
$$N_x = 22.45 \text{ kN}$$

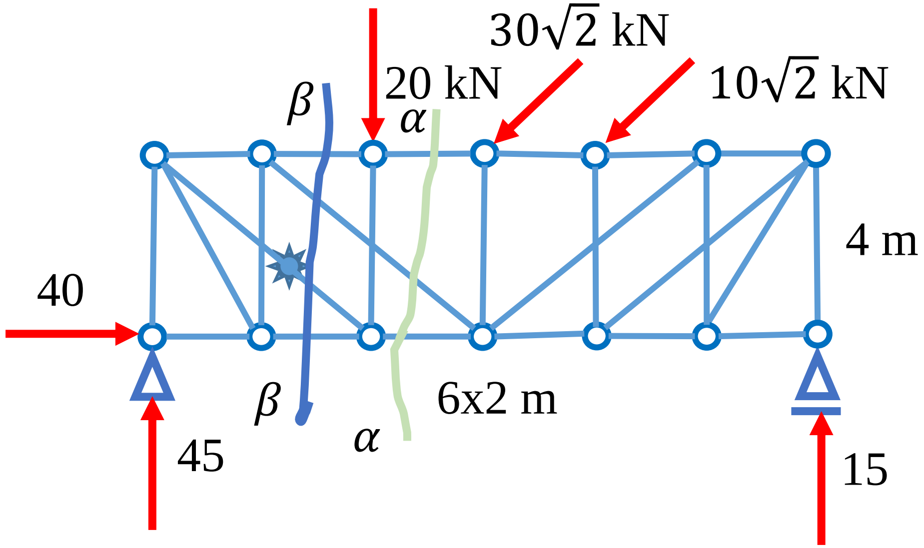
Example No. 5

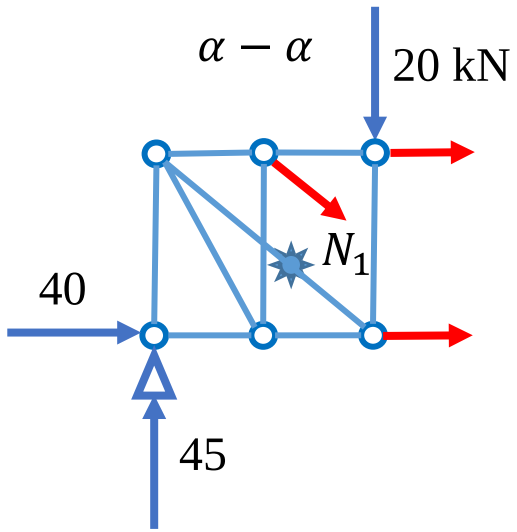




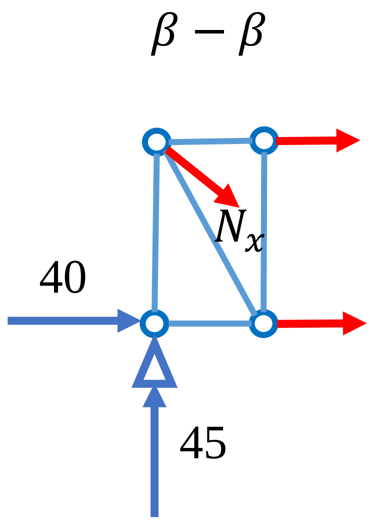
$$\Sigma M_C = 0 \rightarrow 2 \cdot 20 + 2N_x = 0 \rightarrow N_x = -20 \text{ kN}$$

Example No. 6



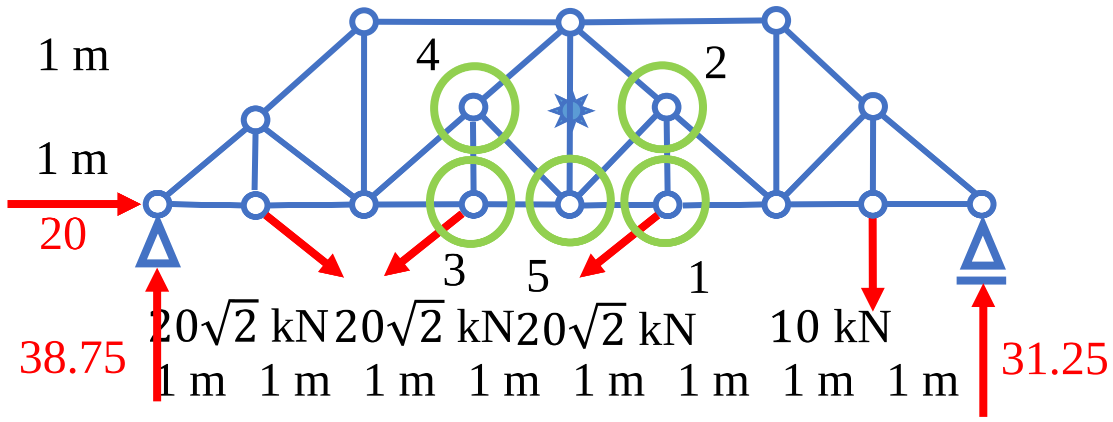


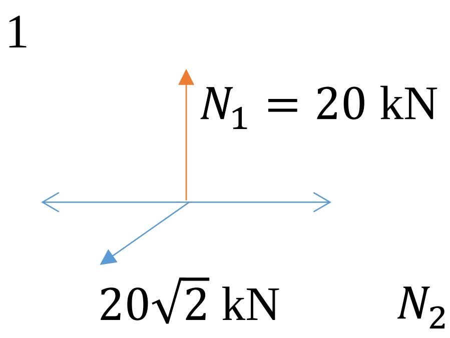
$\Sigma Y = 0 \rightarrow N_1 = 35.36$ kN

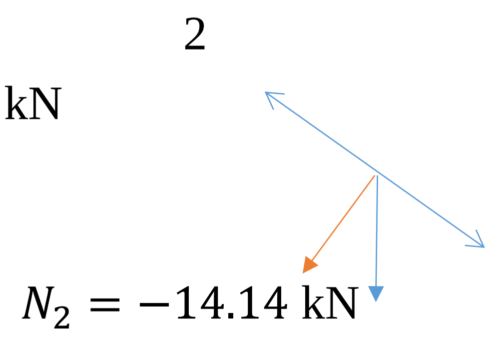


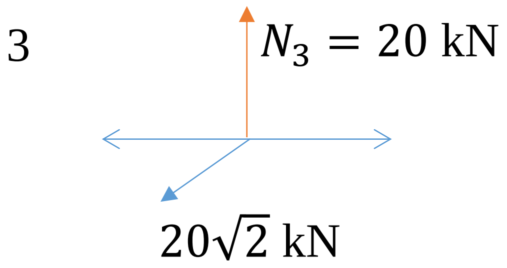
$\Sigma Y = 0 \rightarrow N_x = 28.28$ kN

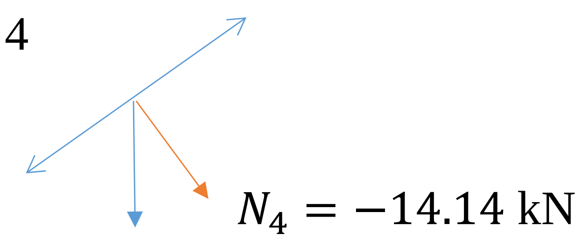
Example No. 7

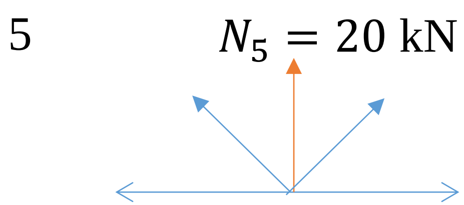

1 m
1 m
4
2
20
3
5
1
38.75
20√2 kN
20√2 kN
20√2 kN
10 kN
31.25
1 m 1 m 1 m 1 m 1 m 1 m 1 m 1 m


1
$N_1 = 20$ kN
20√2 kN

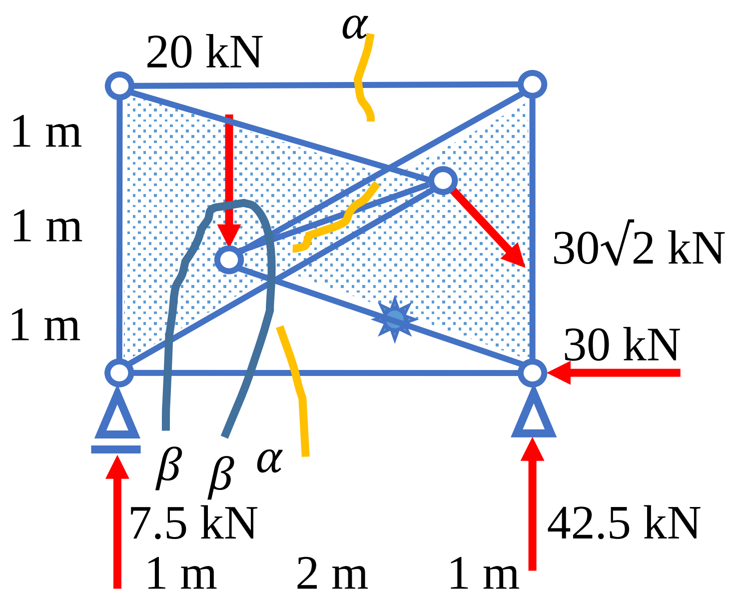

2
$N_2 = -14.14$ kN

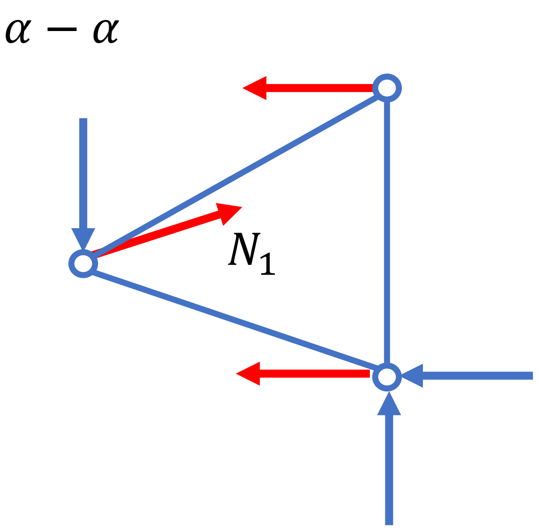

3
$N_3 = 20$ kN
20√2 kN


4
$N_4 = -14.14$ kN

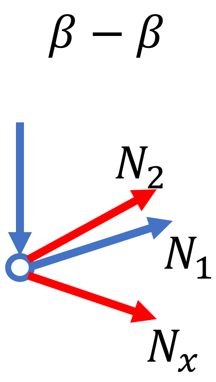

5
$N_5 = 20$ kN

Example No. 8





$\Sigma Y = 0 \rightarrow N_1$



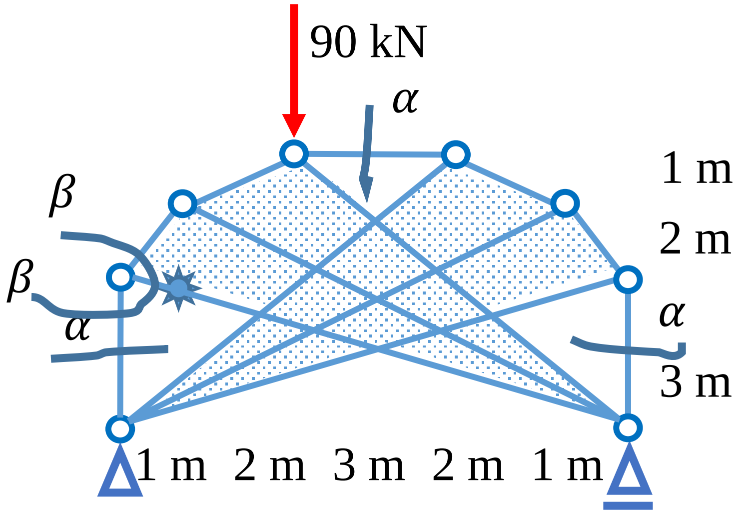
$\Sigma X = 0$

$\Sigma Y = 0$

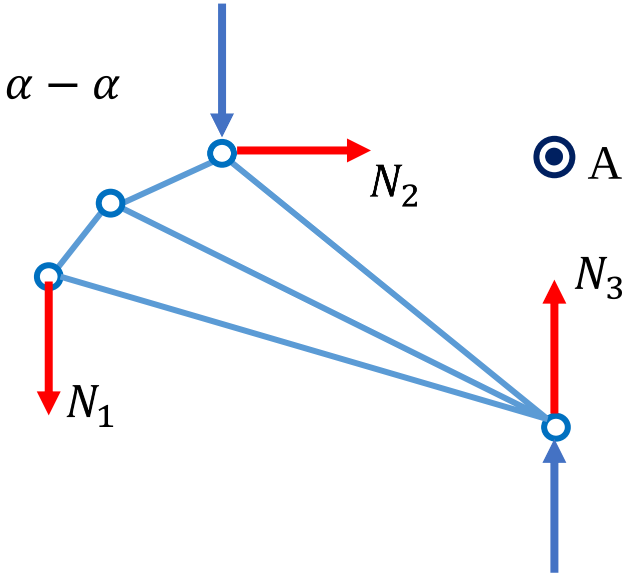
$N_1 = -50.31 \text{ kN}$

$N_x = -13.18 \text{ kN}$

Example No. 9

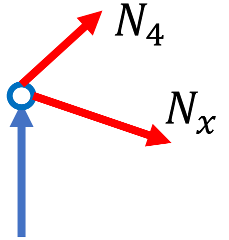


$\alpha - \alpha$



$\Sigma M_A = 0 \rightarrow N_1$

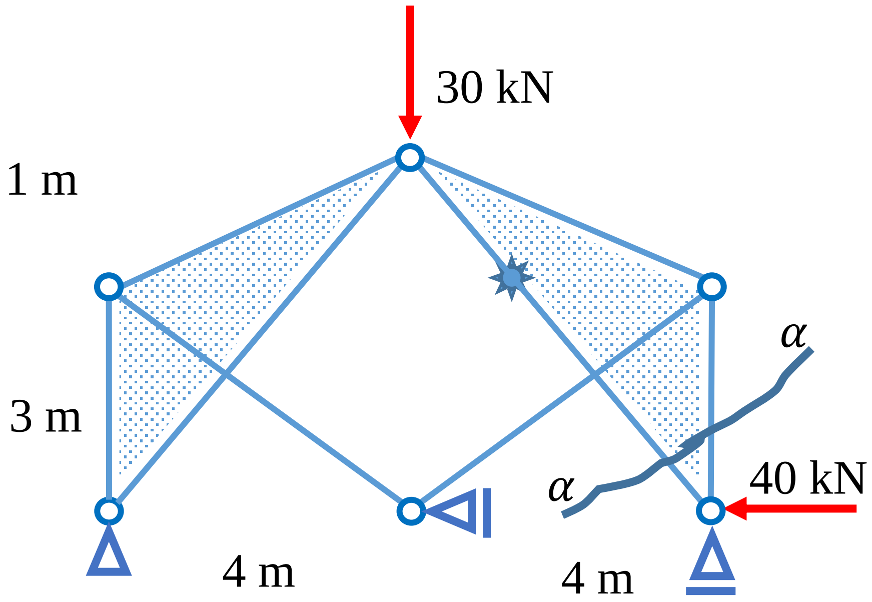
$\beta - \beta$



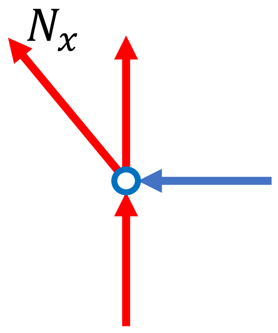
$\Sigma X = 0$
$\Sigma Y = 0$

$N_1 = -60$ kN; $N_4 = -57.5$ kN; $N_x = 27.11$ kN;

Example No. 10

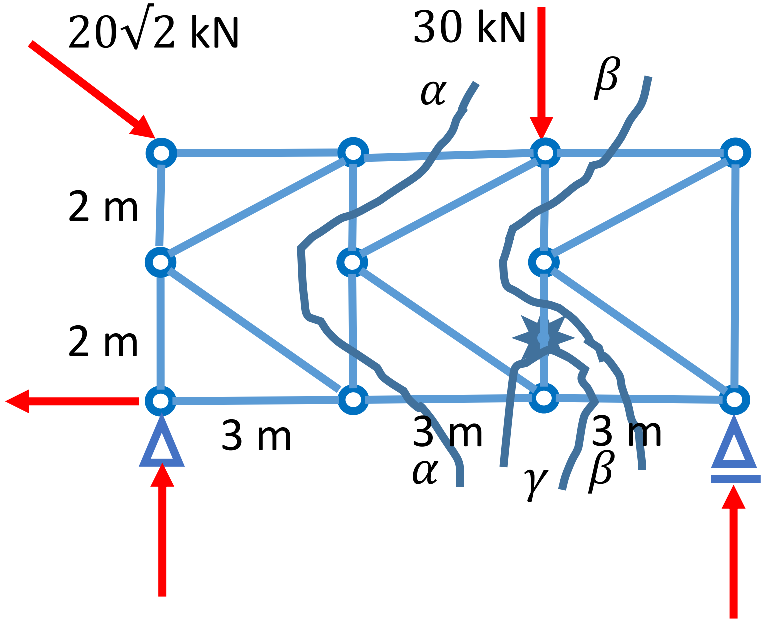


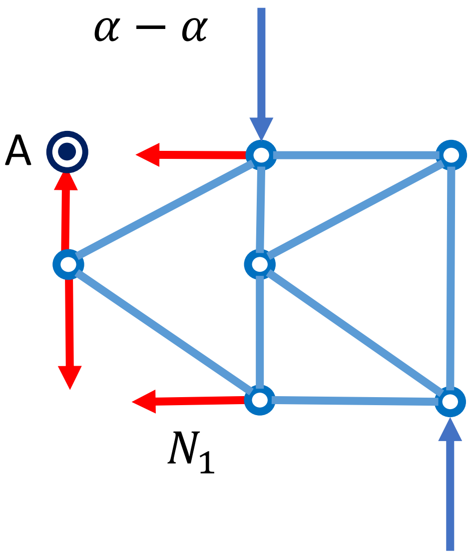
$\alpha - \alpha$

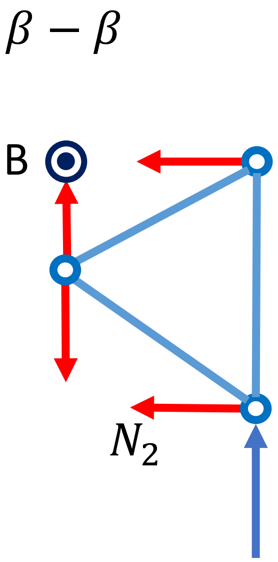


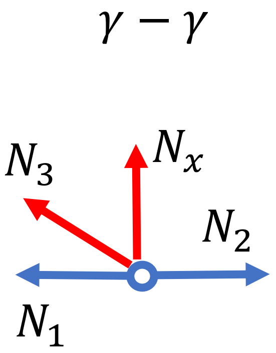
$$\Sigma X = 0 \rightarrow N_x = -56.67 \text{ kN}$$

Example No. 11









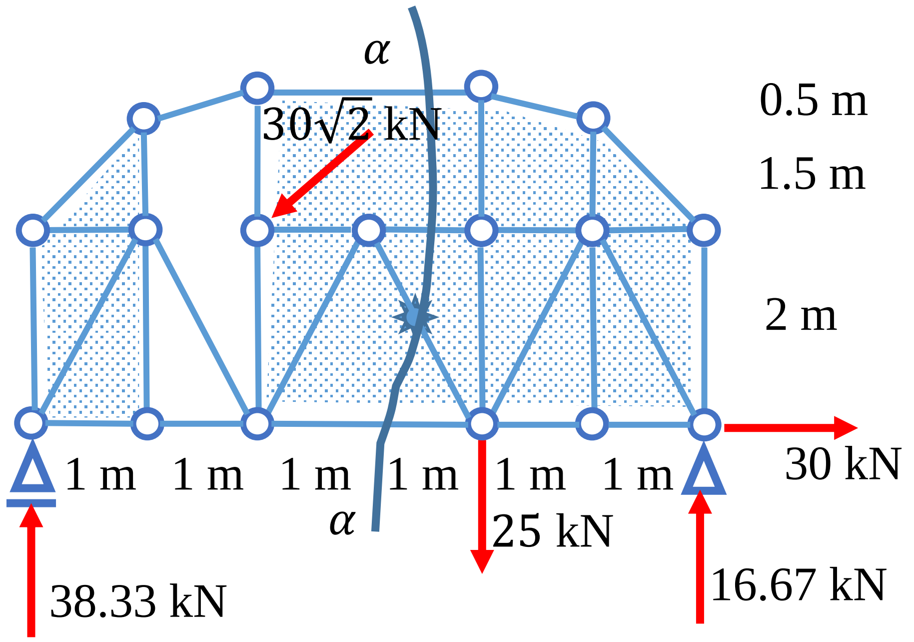
$\Sigma M_A = 0 \rightarrow N_1$

$\Sigma M_B = 0 \rightarrow N_2$

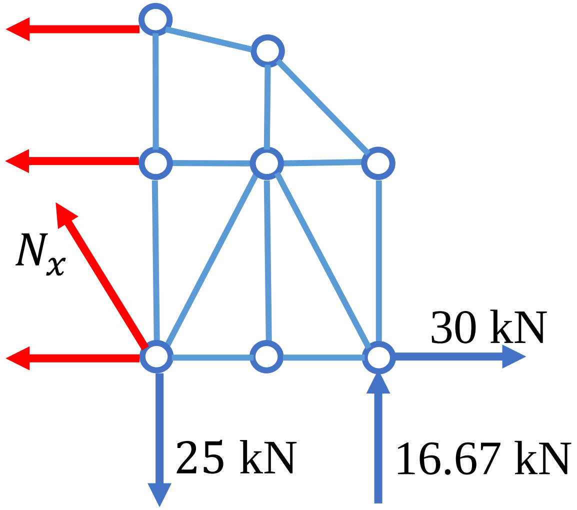
$\Sigma X = 0$
$\Sigma Y = 0$

$N_1 = 20.83$ kN; $\quad N_2 = 21.67$ kN; $\quad N_3 = 0.5556$ kN; $\quad N_x = -0.5556$ kN

Example No. 12



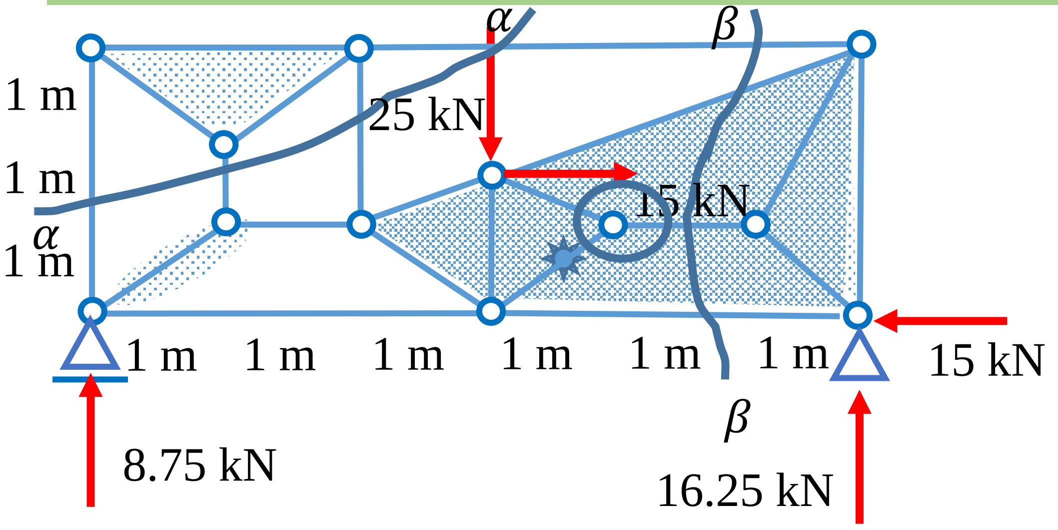
$\alpha - \alpha$

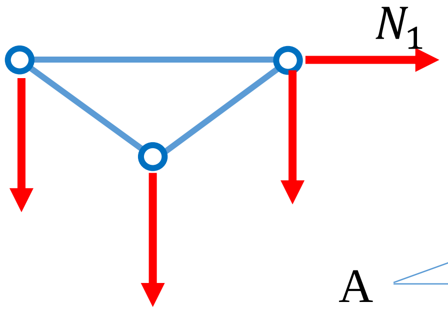


$$\Sigma Y = 0 \rightarrow N_x$$

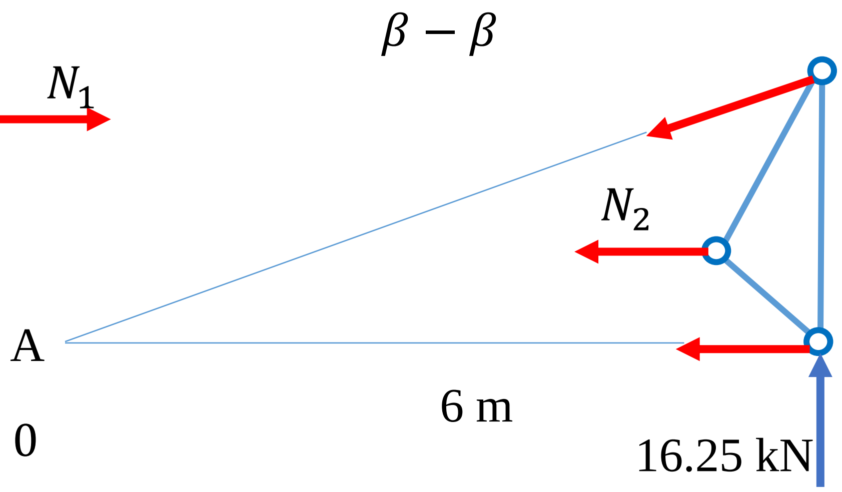
$$N_x \frac{2}{\sqrt{1^2+2^2}} - 25 + 16.67 = 0 \rightarrow N_x = 9.313 \text{ kN}$$

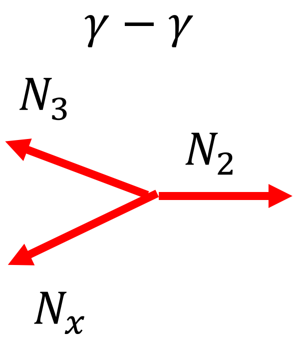
Example No. 13





$\Sigma X = 0 \rightarrow N_1 = 0$





$\Sigma X = 0$
$\Sigma Y = 0$

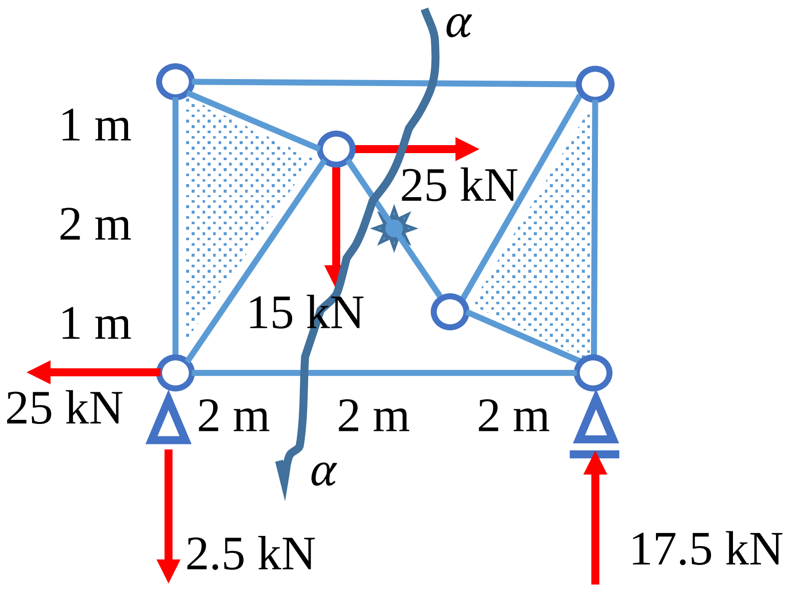
$\Sigma M_A = 0 \rightarrow N_2 = -97.5 \text{ kN}$

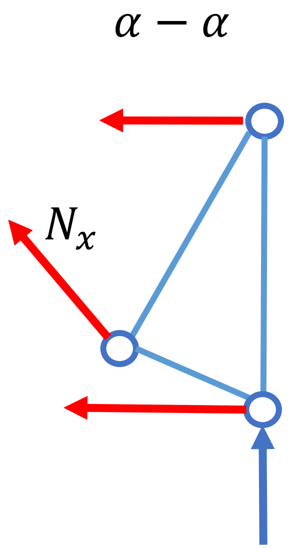
$$-\frac{\sqrt{2}}{2} N_x - \frac{1}{\sqrt{1+0.5^2}} N_3 - 97.5 = 0$$

$$-\frac{\sqrt{2}}{2} N_x + \frac{0.5}{\sqrt{1+0.5^2}} N_3 = 0$$

$N_x = -45.96 \text{ kN}, N_3 = -72.67 \text{ kN}$

Example No. 14

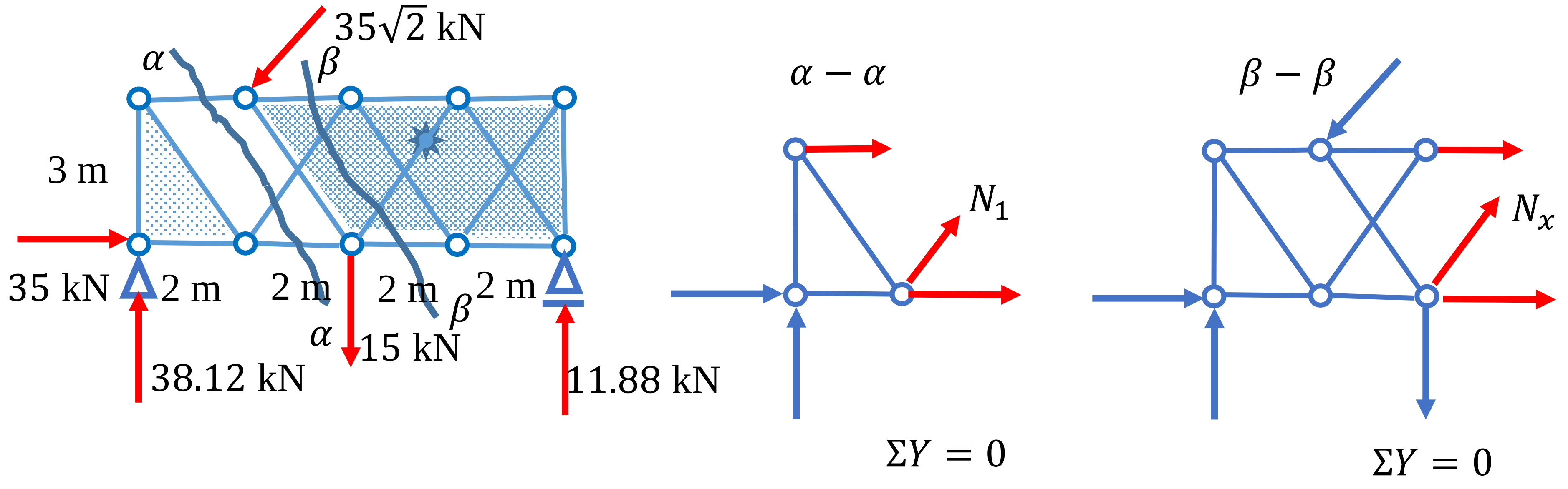




$\Sigma Y = 0 \rightarrow N_x$

$N_x = -24.75$ kN

Example No. 15



$\alpha - \alpha$: $\Sigma Y = 0 \rightarrow N_1 = -45.82$ kN

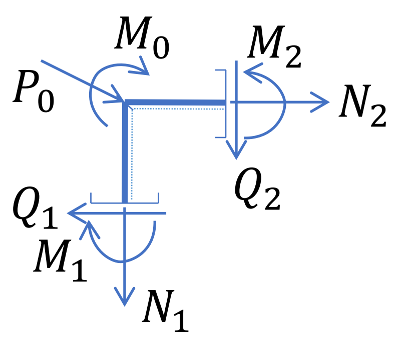
$\beta - \beta$: $\Sigma Y = 0 \rightarrow N_x = 18.03$ kN

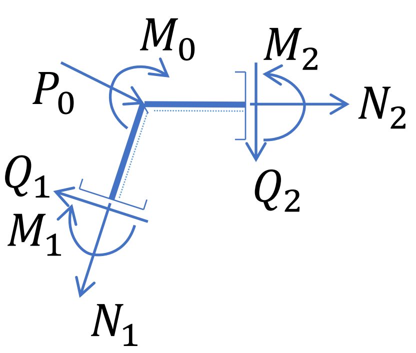
Frame – definitions

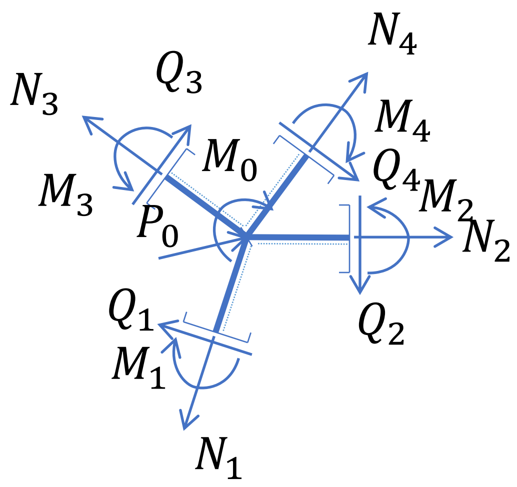
Frame – a structure composed of beams (simple beams, slanted beams and columns) rigidly connected.

The connections are called nodes.

The nodes types:
- simple right
- simple
- complex



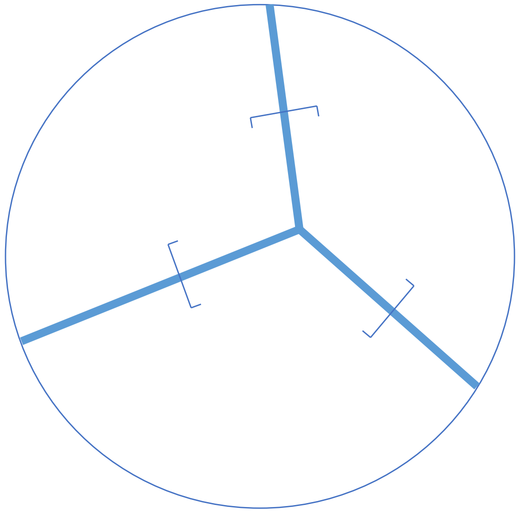




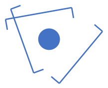
The node balance: $\Sigma X = 0$ $\Sigma Y = 0$ $\Sigma M_0 = 0$

Frame – node balance

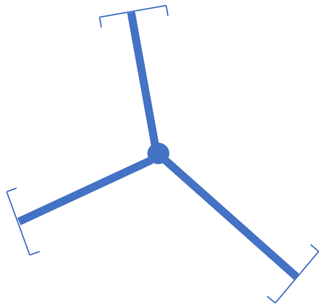
zoom of a node



real situation
(but poor visibility)



more convincing drawing
(but real lengths are zero)

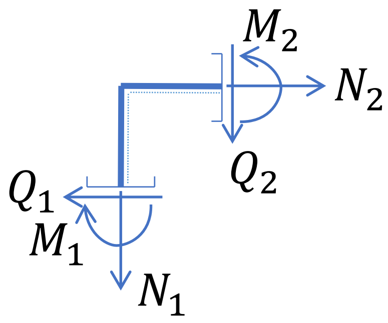


for $\Sigma X = 0$ and $\Sigma Y = 0$ the moments are irrelevant and can be omitted

for $\Sigma M_0 = 0$ the forces are irrelevant (their arms are zero) and can be omitted

For these reasons, usually, two separate drawings are made: one with the moments and another with the forces

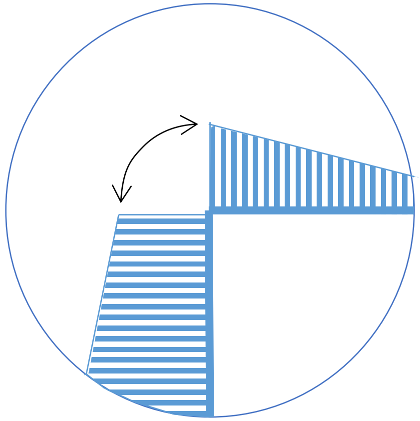
Balance of simple right node without point load



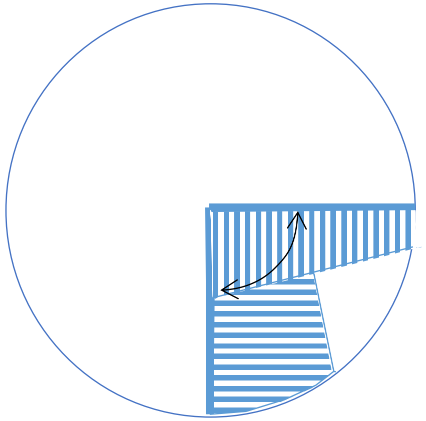
$\Sigma X = 0 \rightarrow Q_1 = N_2$ the shear force in one bar transforms into the axial force in another bar

$\Sigma Y = 0 \rightarrow N_1 = -Q_2$ the axial force in one bar transforms into the shear force in another bar

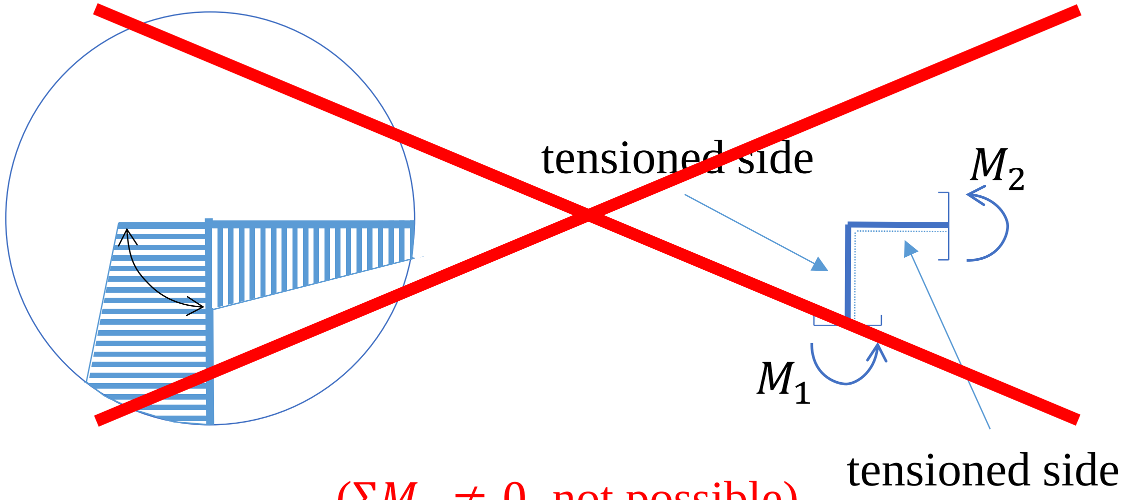
$\Sigma M_0 = 0 \rightarrow M_1 = -M_2$ the bending moments in the two bars are equal but act reversely



(possible)

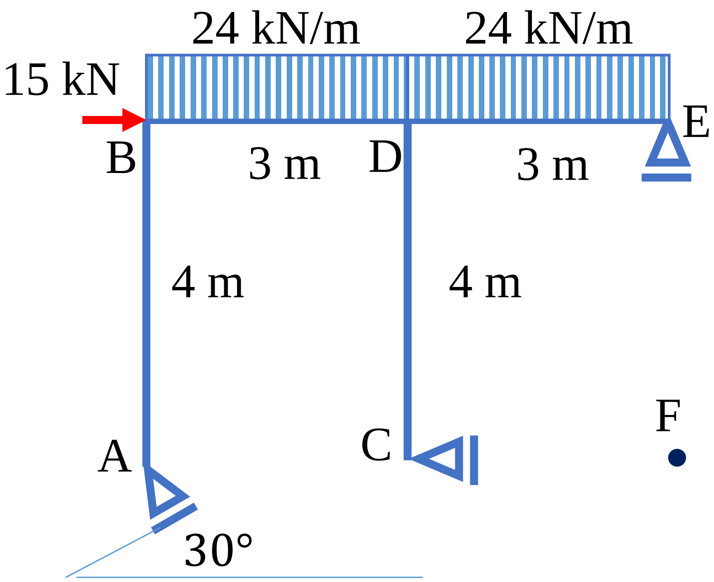


(possible)



($\Sigma M_0 \neq 0$, not possible)

Frame – an example

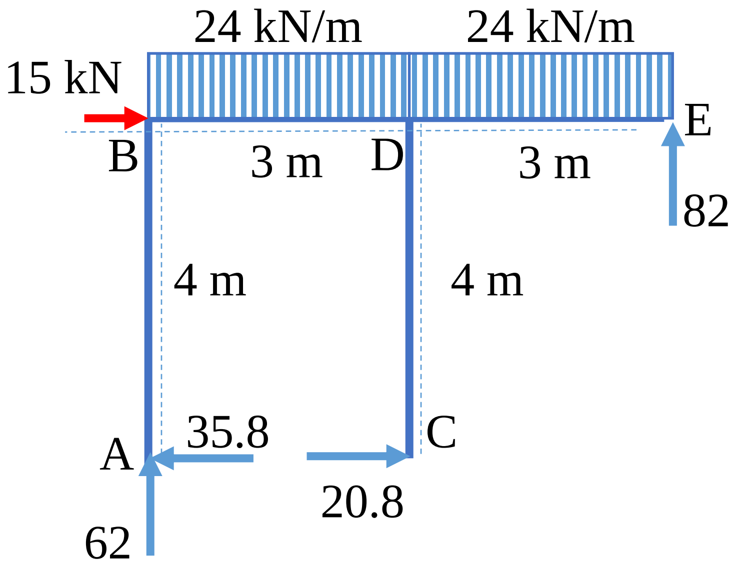


$$\Sigma M_A = 0 \rightarrow R_E = \frac{24 \cdot 6 \cdot 3 + 15 \cdot 4}{6} = 82 \text{ kN}$$

$$\Sigma M_F = 0 \rightarrow R_A = \frac{24 \cdot 6 \cdot 3 - 15 \cdot 4}{6 \cdot \cos 30} = 71.59 \text{ kN} \qquad (35.8, 62) \text{ kN}$$

$$\Sigma X = 0 \rightarrow 15 - 71.59 \cdot \sin 30 - R_C = 0 \rightarrow R_c = -20.8 \text{ kN}$$

For diagrams of cross sectional forces we'll write their functions when needed only



$$M_A = 0; M_B = 143.2 \text{ kNm}; M_C = 0; M_D = 83.2 \text{ kNm}$$

$$M_D^L = 62 \cdot 3 + 35.8 \cdot 4 - 24 \cdot 3 \cdot 1.5 = 221.2 \text{ kNm}$$

$$M_E = 0; M_D^R = 82 \cdot 3 - 24 \cdot 3 \cdot 1.5 = 138 \text{ kNm}$$

$Q_{AB} = 35.8$ kN; $Q_B^R = 62$ kN; $Q_D^L = 62 - 24 \cdot 3 = -10$ kN (sign changes); $Q_{BD} = 0 \rightarrow x_0 = 2.583$ m

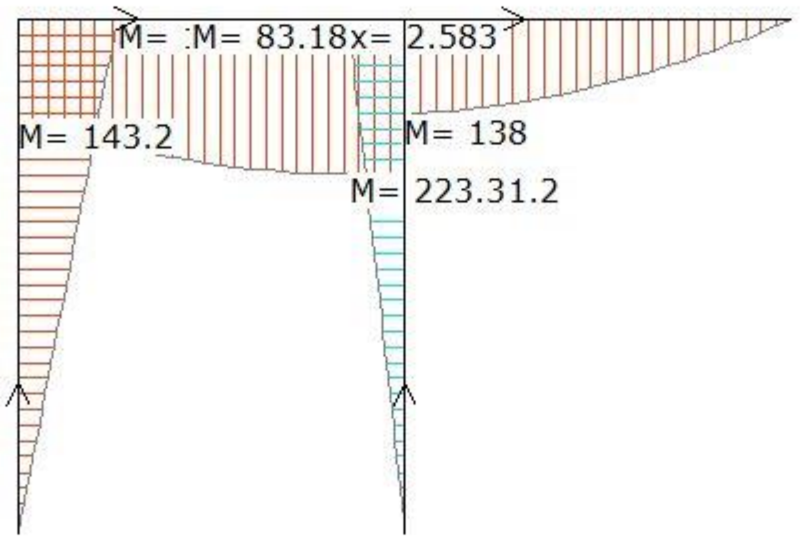
$M_{BD}(2.583) = 62 \cdot 2.583 + 35.8 \cdot 4 - 24 \cdot 0.5 \cdot 2.583^2 = 223.3$ kNm (extremum);

Frame – bending moments

$Q_D^R = -10$ kN; $Q_E = -82$ kN (no extremum)

$N_{AB} = -82$ kN; $N_{BD} = 35.8 - 15 = 20.8$ kN; $N_{DE} = 0$

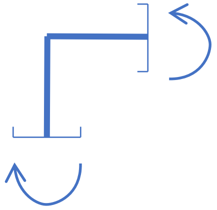
Bending moments diagram



bending moments check:

Node *B*:

143.2

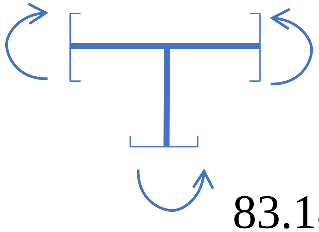


143.2

$$\Sigma M = 143.2 - 143.2 = 0 \text{ , OK}$$

Node *D*:

221.2

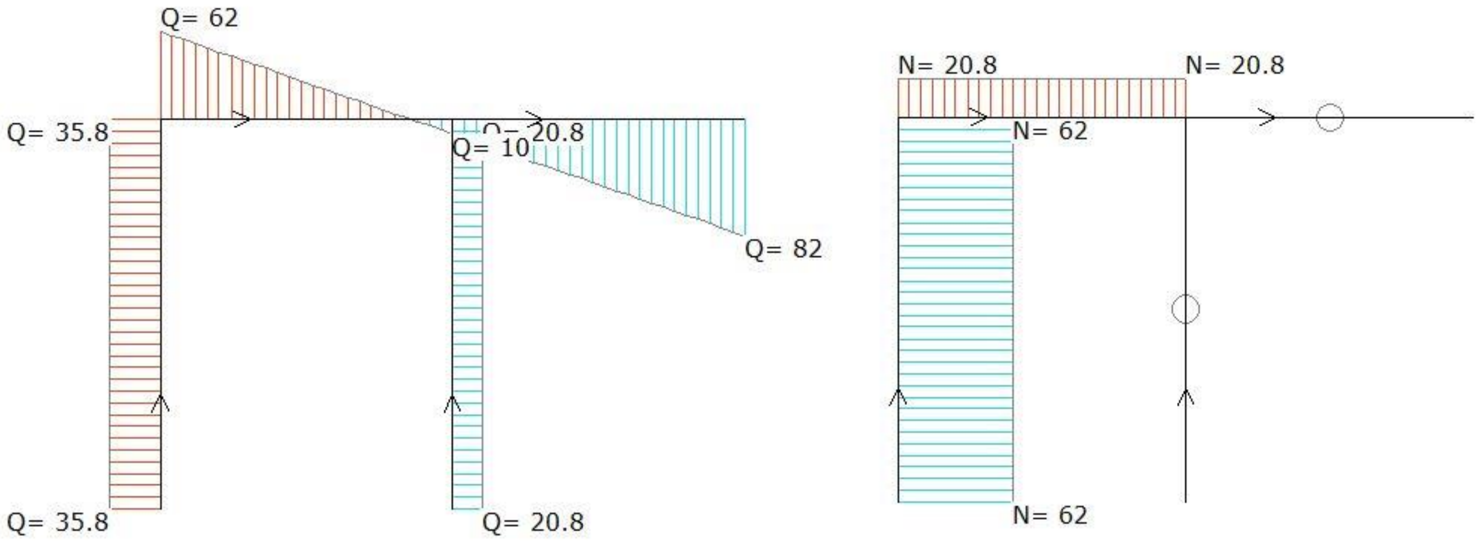


138

83.18

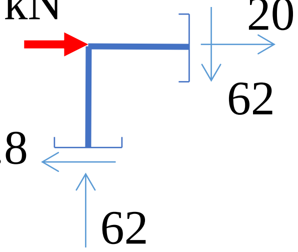
$$\Sigma M = 221.2 - 138 - 83.18 = 0.02 \cong 0 \text{ , OK}$$

Frame – shear and axial forces



shear & axial forces check:

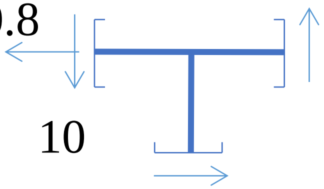
Node *B*:



$$\Sigma X = 15 - 35.8 + 20.8 = 0, \text{ OK}$$

$$\Sigma Y = 62 - 62 = 0, \text{ OK}$$

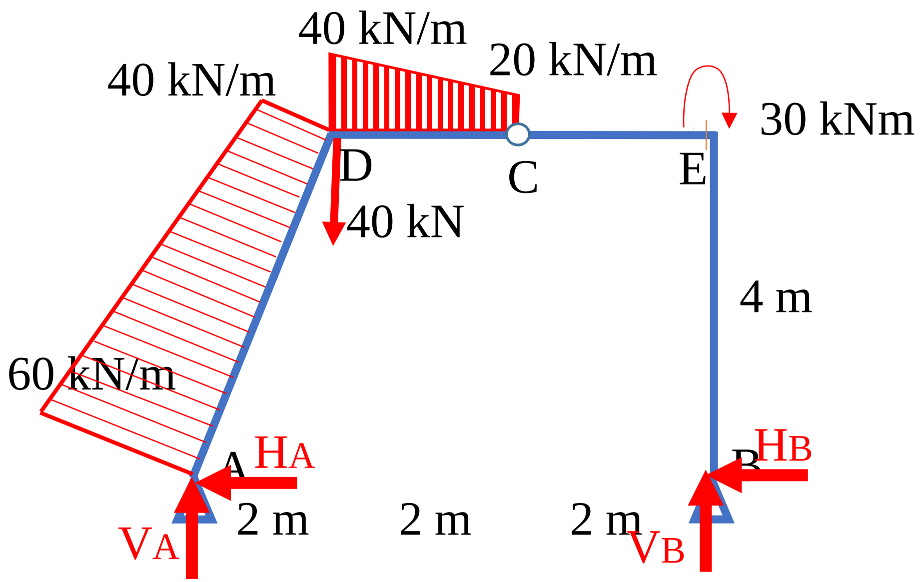
Node *D*:



$$\Sigma X = -20.8 + 20.8 = 0, \text{ OK}$$

$$\Sigma Y = -10 + 10 = 0, \text{ OK}$$

The second example



the structure is free-body unstable but totally stable

the constraints' reactions:

checking:

$$\Sigma X = -145 - 55 + 40 \cdot 4 + \frac{1}{2} \cdot 20 \cdot 4 = 0$$

$$\Sigma Y = 75 + 125 - 40 \cdot 2 - \frac{1}{2} \cdot 20 \cdot 2 - -20 \cdot 2 - \frac{1}{2} \cdot 20 \cdot 2 = 0$$

OK

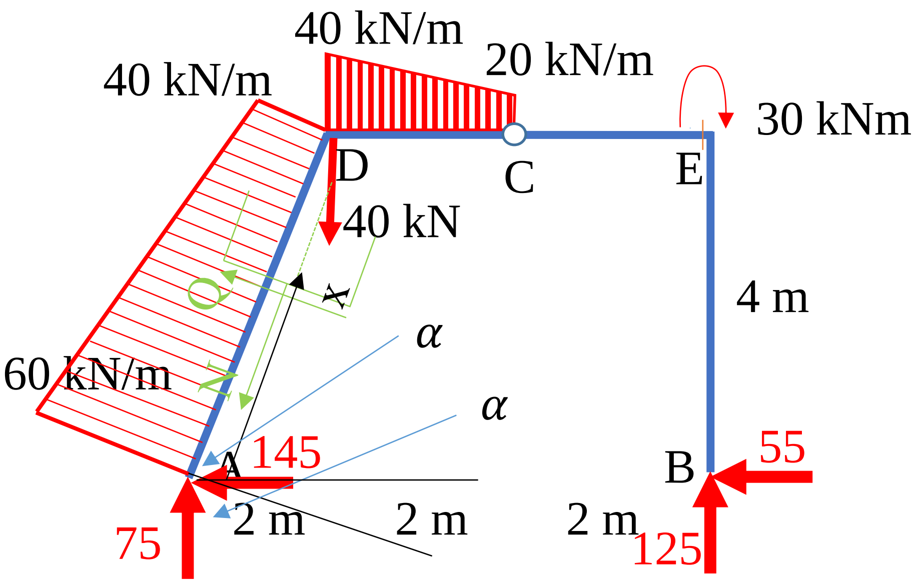
$$\Sigma M_B = 0 \rightarrow \quad V_A = \frac{-40{\cdot}4{\cdot}2 - \frac{1}{2}{\cdot}20{\cdot}4{\cdot}\frac{1}{3}{\cdot}4 + 40{\cdot}2{\cdot}5 + \frac{1}{2}{\cdot}20{\cdot}2{\cdot}\frac{16}{3} + 20{\cdot}2{\cdot}3 + \frac{1}{2}{\cdot}20{\cdot}2{\cdot}\left(\frac{2}{3}{\cdot}2 + 2\right) + 40{\cdot}4 - 30}{6} = 75\text{ kN}$$

$$\Sigma M_A = 0 \rightarrow \quad V_B = \frac{40{\cdot}4{\cdot}2 + \frac{1}{2}{\cdot}20{\cdot}4{\cdot}\frac{4}{3} + 40{\cdot}2{\cdot}1 + \frac{1}{2}{\cdot}20{\cdot}2{\cdot}\frac{2}{3} + 20{\cdot}2{\cdot}3 + \frac{1}{2}{\cdot}20{\cdot}2}{6} = 125\text{ kN}$$

$$\Sigma M_C^L = 0 \rightarrow \quad H_A = \frac{40{\cdot}4{\cdot}2 + \frac{1}{2}{\cdot}20{\cdot}4{\cdot}\frac{8}{3} + 40{\cdot}2{\cdot}3 + \frac{1}{2}{\cdot}20{\cdot}2{\cdot}\frac{10}{3} + 20{\cdot}2{\cdot}1 + \frac{1}{2}{\cdot}20{\cdot}2{\cdot}\frac{4}{3} - 75{\cdot}4}{4} = 145\text{ kN}$$

$$\Sigma M_C^R = 0 \rightarrow \quad H_B = \frac{-30 + 125{\cdot}2}{4} = 55\text{ kN}$$

the first interval AD



$$0 < x < \sqrt{20} = 4.472$$

$$\cos\alpha = \frac{2}{\sqrt{20}} \quad (= 0.4472)$$

$$\sin\alpha = \frac{4}{\sqrt{20}} \quad (= 0.8944)$$

$$M(x) = 75 \cdot \frac{2}{\sqrt{20}} \cdot x + 145 \cdot \frac{4}{\sqrt{20}} \cdot x - 60\frac{x^2}{2} + \frac{20}{\sqrt{20}} \cdot \frac{x^3}{6}$$

$M(0) = 0 \qquad M(4.472) = 196.7$ kNm

$$Q(x) = 75 \cdot \frac{2}{\sqrt{20}} + 145 \cdot \frac{4}{\sqrt{20}} - 60 \cdot x + \frac{20}{\sqrt{20}} \cdot \frac{x^2}{2}$$

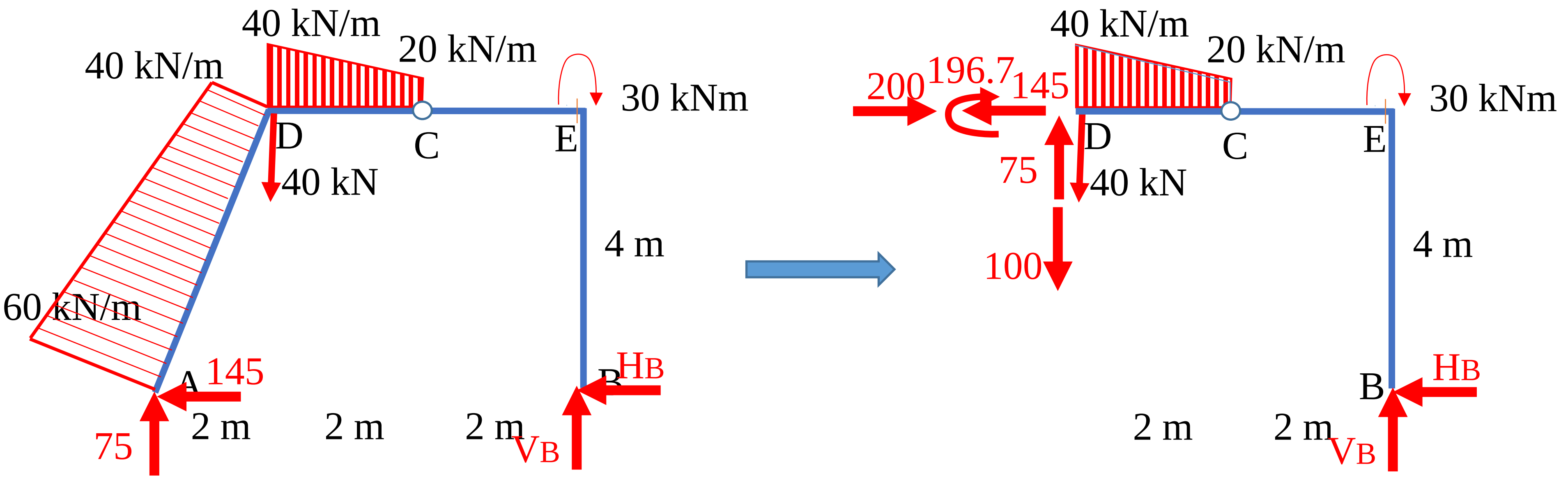
$Q(0) = 163.2$ kN $\qquad Q(\sqrt{20}) = -60.37$ kN $\qquad$ (ch. s.)

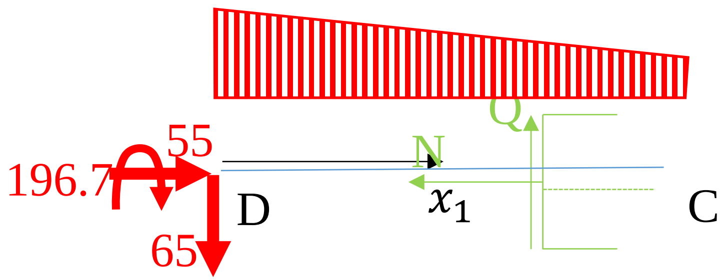
$Q(x_0) = 0 \rightarrow 2.236x^2 - 60x + 163.2 = 0 \rightarrow x_1 = 23.76$ (outside), $x_2 = 3.072$ m

$M(3.072) = 239.9$ kNm

$$N(x) = -75 \cdot \frac{4}{\sqrt{20}} + 145 \cdot \frac{2}{\sqrt{20}} = -2.236 \text{ kN}$$

the second interval DC





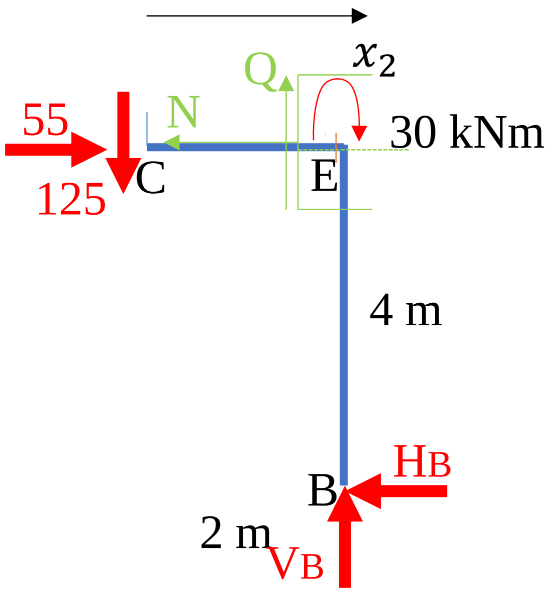
$0 < x_1 < 2$

$$M(x_1) = 196.7 - 65 \cdot x_1 - 40\frac{x_1^2}{2} + \frac{20}{2} \cdot \frac{x_1^3}{6} \qquad M(0) = 196.7 \text{ kNm}$$

$M(2) = 0.033 \cong 0$ (at the hinge)

$$Q(x_1) = -65 - 40 \cdot x_1 + \frac{20}{2}\frac{x_1^2}{2} \qquad Q(0) = -65 \text{ kN} \qquad Q(2) = -125 \text{ kN} \qquad N(x_1) = -55 \text{ kN}$$

the third interval CE



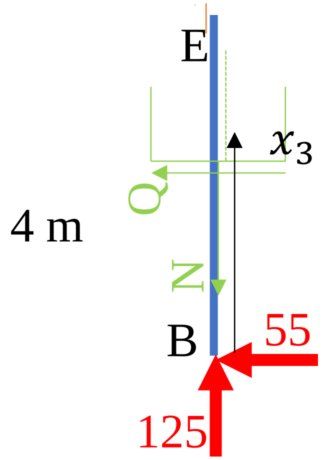
$0 < x_2 < 2$

$M(x_2) = -125 \cdot x_2 \quad M(0) = 0 \quad M(2) = -250$ kNm

$Q(x_2) = -125$ kN

$N(x_2) = -55$ kN

the fourth interval BE



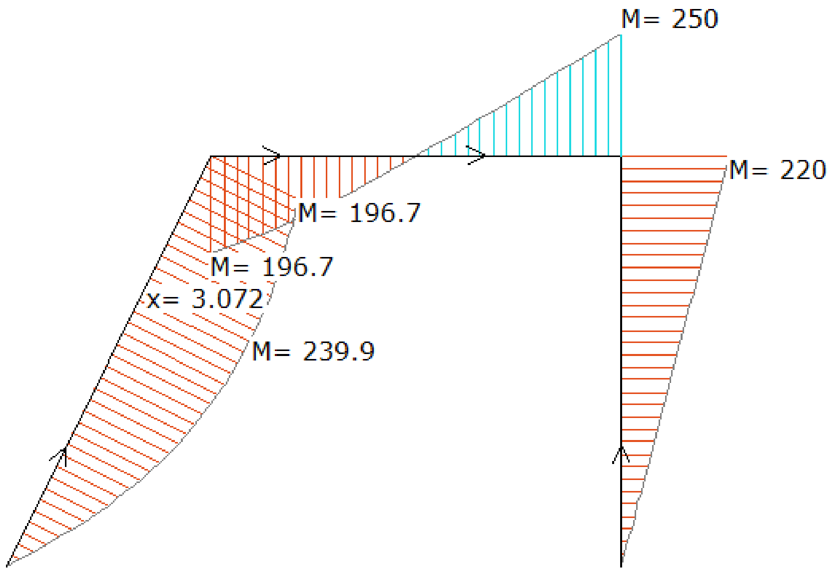
$$0 < x_3 < 4$$

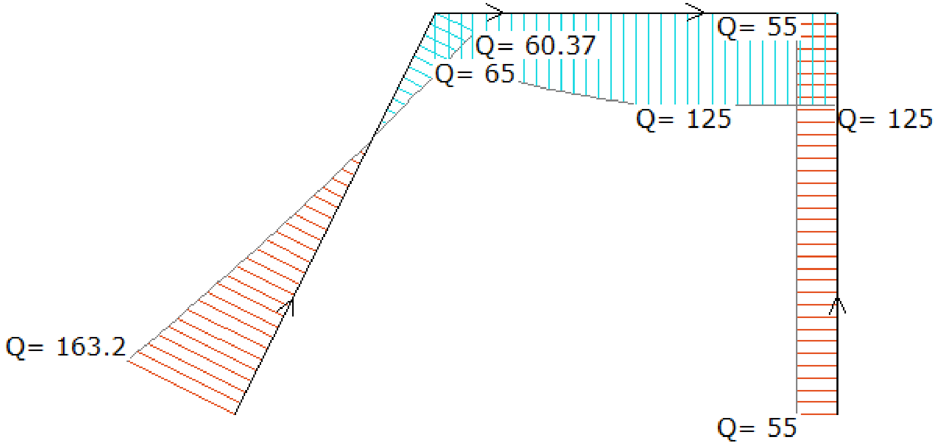
$$M(x_3) = 55 \cdot x_3, \quad M(0) = 0, \quad M(4) = 220 \ \text{kNm}$$

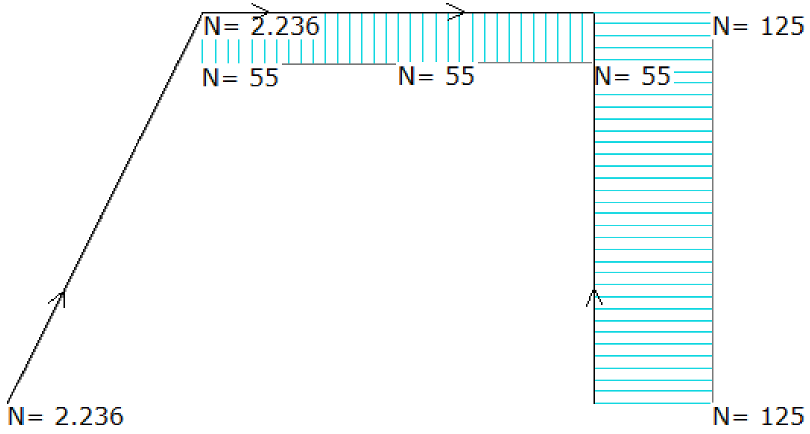
$$Q(x_3) = 55 \text{ kN}$$

$$N(x_3) = -125 \text{ kN}$$

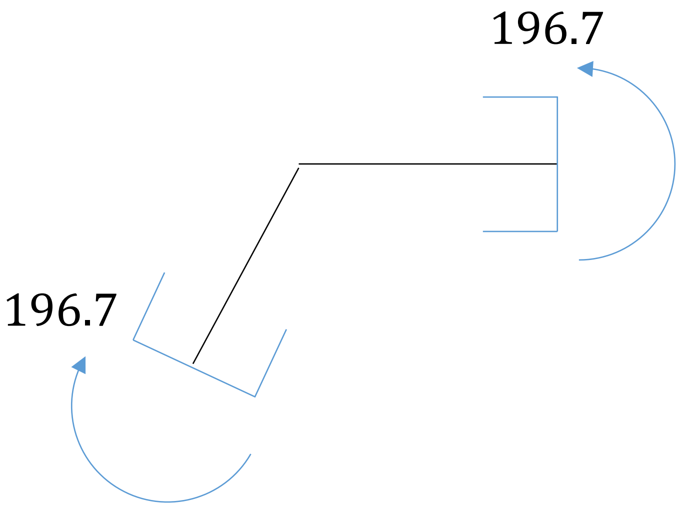
diagrams of the c-s forces

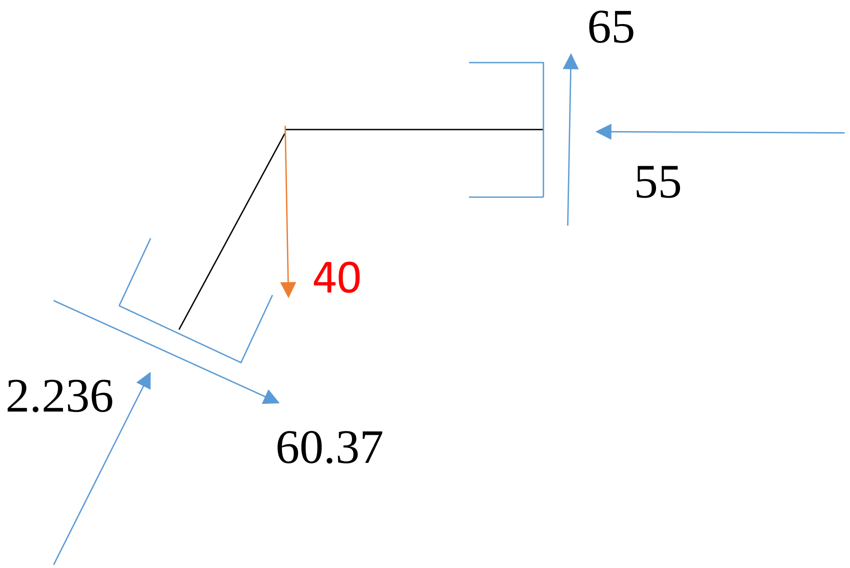






checking of the diagrams, node D



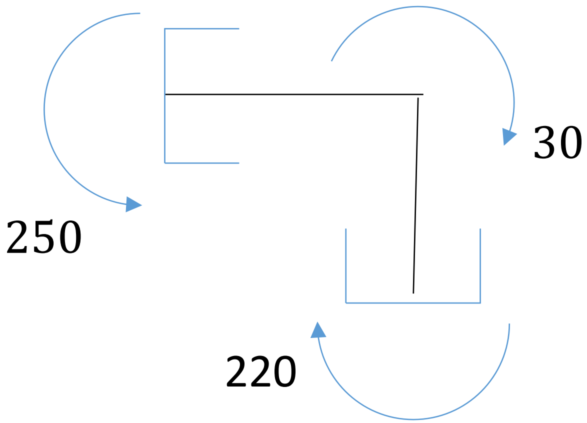


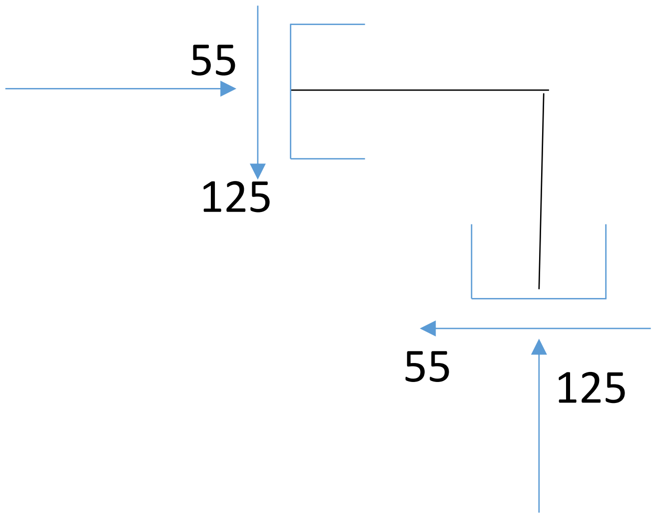
$$\Sigma M = 196.7 - 196.7 = 0$$

$$\Sigma X = 60.37 \cdot \frac{4}{\sqrt{20}} + 2.236 \cdot \frac{2}{\sqrt{20}} - 55 = -0.003 \cong 0$$

$$\Sigma Y = -60.37 \cdot \frac{2}{\sqrt{20}} + 2.236 \cdot \frac{4}{\sqrt{20}} - 40 + 65 = 0.002 \cong 0$$

checking node E





$$\Sigma M = 250 - 30 - 220 = 0$$

$$\Sigma X = 55 - 55 = 0$$

$$\Sigma Y = -125 + 125 = 0$$

Thank you for your attention!