Strength of Materials

8. Statics - summary

Complex structures – definitions

Complex (combined) structure - a structure with parts of different types: beam, frame, truss, arch, etc.

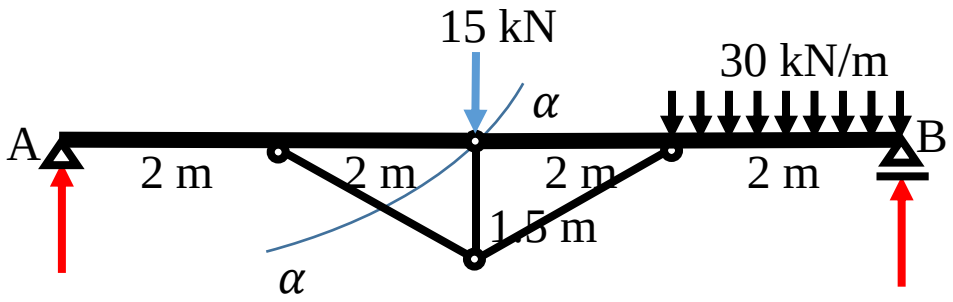
Algorithm of the solution:

- analysis of free-body and external stability
- calculation of the constraints reactions
  - directly from the balance equations
  - hinge equation
  - additional sections
- **distinction between beams and truss bars**
- sections trough the truss bars
- determination of forces in truss bars
- sections isolating beam bars
- determination of cross-sectional forces in beam bars
- final verifications and checking

Three types of complex/combined structures:

- free body stable
- free body unstable
  - three-hinges
  - analogous to three-hinges

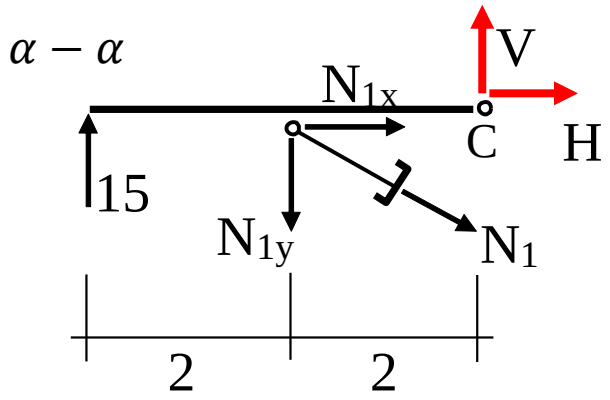
Complex structures free body stable



$$\Sigma X = 0 \rightarrow H_A = 0$$

$$\Sigma M_B = 0 \rightarrow V_A = \frac{15 \cdot 4 + 30 \cdot 2 \cdot 1}{8} = 15 \text{ kN}$$

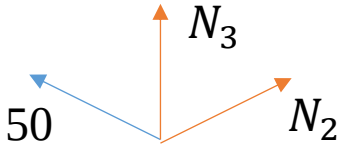
$$\Sigma M_A = 0 \rightarrow R_B = \frac{15 \cdot 4 + 30 \cdot 2 \cdot 7}{8} = 60 \text{ kN}$$



$\alpha - \alpha$

$$\Sigma M_C = 0 \rightarrow 4 \cdot 15 - 2 \cdot \frac{1.5}{\sqrt{2^2 + 1.5^2}} N_1 = 0 \rightarrow N_1 = 50 \text{ kN}$$

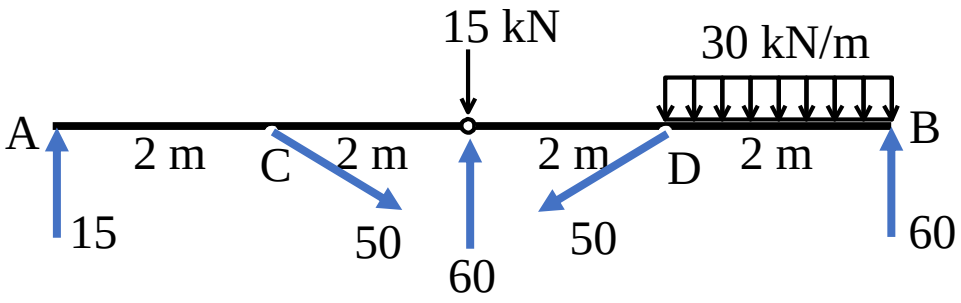
C. s. free body stable – continued



node balance

$$\Sigma X = 0 \rightarrow N_2 = 50 \text{ kN}$$

$$\Sigma Y = 0 \rightarrow N_3 = -2 \cdot \cos\alpha \cdot 50 = -100 \cdot \frac{1.5}{\sqrt{2^2+1.5^2}} = -60 \text{ kN}$$



$$M_C = 15 \cdot 2 = 30 \text{ kNm}$$

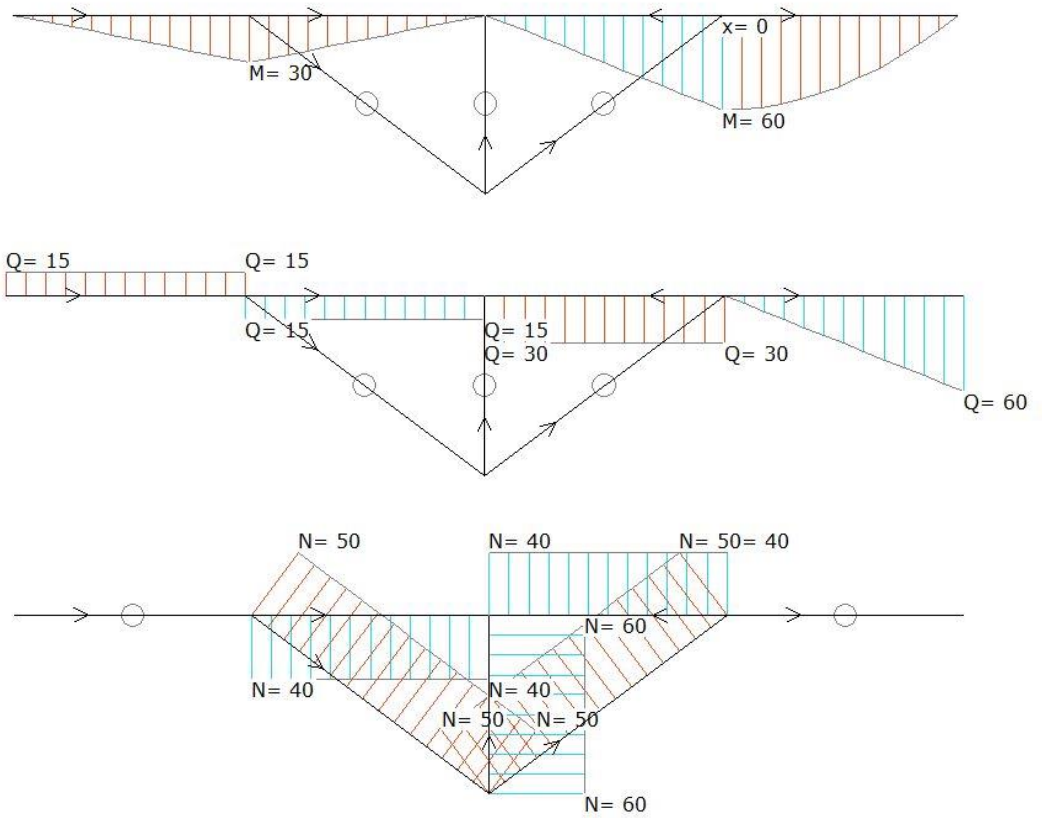
$$M_D = 60 \cdot 2 - 30 \cdot 2 \cdot 1 = 60 \text{ kNm}$$

$$Q_{CE} = 15 - 50 \cdot \frac{1.5}{\sqrt{2^2+1.5^2}} = -15 \text{ kN}$$

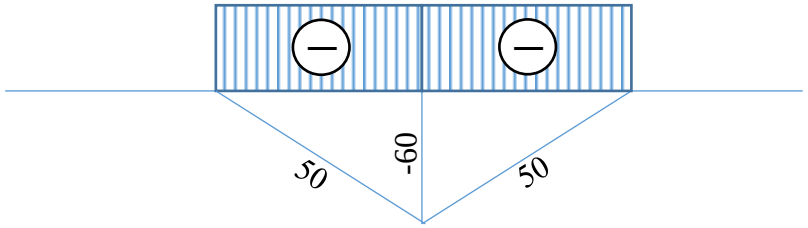
$$Q_{BD} = -60 + 30 \cdot x; Q_B = -60 \text{ kN}$$

$Q_D = 0$ (extremum of bending moment)

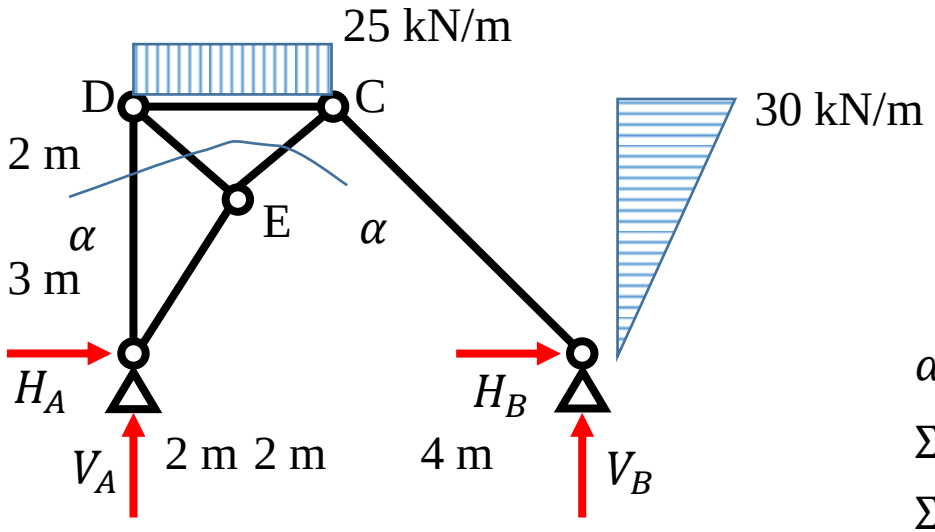
C. s. free body stable – diagrams



but usually, axial forces diagram is drawn in such a way:



Three hinges structures



$$\Sigma M_B = 0 \rightarrow V_A = \frac{25\cdot4\cdot6+0.5\cdot30\cdot5\cdot2/3\cdot5}{8} = 106.2 \text{ kN}$$

$$\Sigma M_A = 0 \rightarrow V_B = \frac{25\cdot4\cdot2-0.5\cdot30\cdot5\cdot2/3\cdot5}{8} = -6.26 \text{ kN}$$

$$\Sigma M_C^L = 0 \rightarrow 4V_A - 5H_A - 25\cdot4\cdot2 = 0 \rightarrow H_A = 45 \text{ kN}$$

$$\Sigma M_C^R = 0 \rightarrow 4V_B + 45 - 0.5\cdot30\cdot5\cdot\frac{5}{3} = 0 \rightarrow H_B = 30 \text{ kN}$$

$\alpha - \alpha$

$$\Sigma M_D = 0 \rightarrow N_{CE}\cdot\frac{\sqrt{2}}{2}\cdot4 + 45\cdot5 = 0 \rightarrow N_{CE} = -79.55 \text{ kN}$$

$$\Sigma M_E = 0 \rightarrow 2N_{AD} + 106.2\cdot2 - 45\cdot3 = 0 \rightarrow N_{AD} = -38.7 \text{ kN}$$

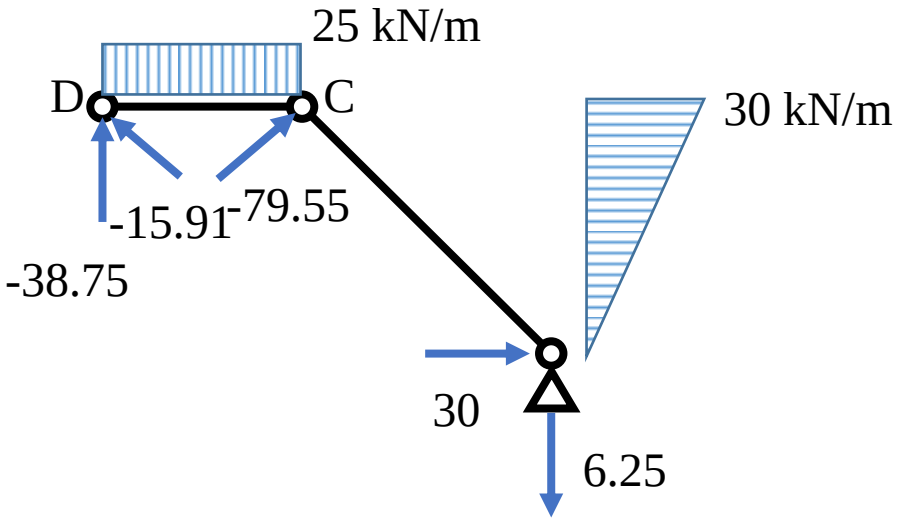
node E

$$\Sigma X = 0 \rightarrow -\frac{\sqrt{2}}{2}N_{DE} + \frac{\sqrt{2}}{2}(-79.55) - \frac{2}{\sqrt{2^2+3^2}}N_{AE} = 0$$

$$\Sigma Y = 0 \rightarrow \frac{\sqrt{2}}{2}N_{DE} + \frac{\sqrt{2}}{2}(-79.55) + \frac{3}{\sqrt{2^2+3^2}}N_{AE} = 0$$

$$N_{DE} = -15.91 \text{ kN};\ N_{AE} = -81.12 \text{ kN}$$

Three hinges structures – continued



$Q_{CD} = 50 - 25x;\ \ Q_D = 50$ kN; $Q_C = -50$ kN

$Q(x_0) = 0 \rightarrow x_0 = 2$ m; $M(2) = 50$ kNm (extr.)

$$Q_{BC} = 6.25 \cdot \frac{4}{\sqrt{4^2+5^2}} + \left(-30 + \frac{30}{5} \cdot \frac{x^2}{2}\right) \cdot \frac{5}{\sqrt{4^2+5^2}};$$

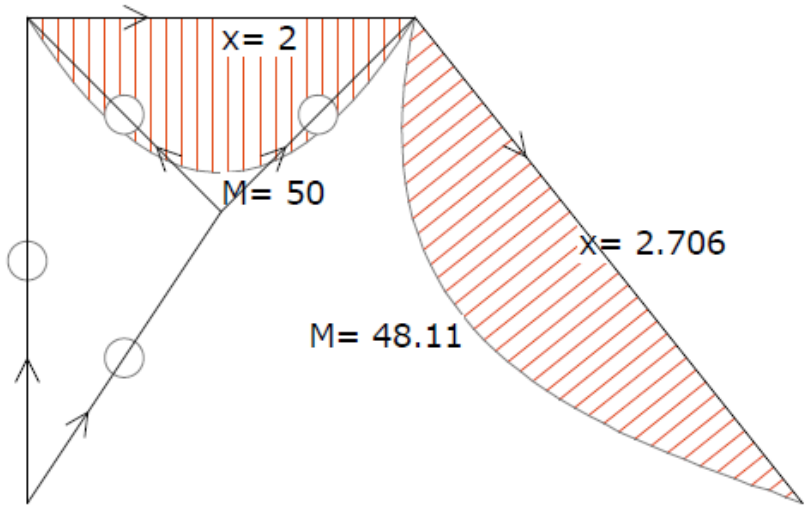
$Q_B = -19.52$ kN; $Q_C = 39.04$ kN (sign changes)

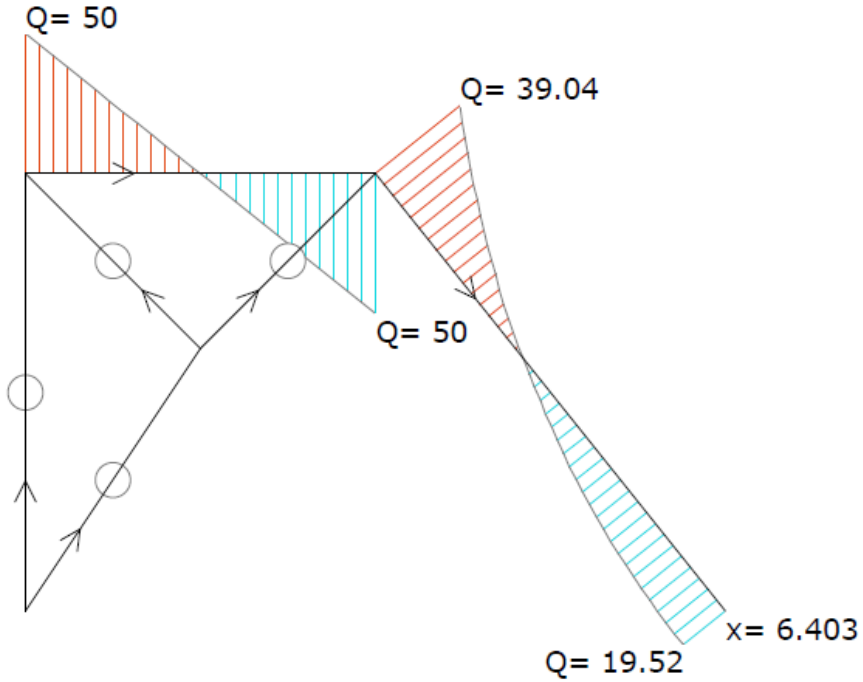
$x_0 = 2.887$ m, $M(2.887) = 48.11$ kNm

$$N_{BC} = 6.25 \cdot \frac{5}{\sqrt{4^2+5^2}} + \left(30 - \frac{30}{5} \cdot \frac{x^2}{2}\right) \cdot \frac{4}{\sqrt{4^2+5^2}};$$

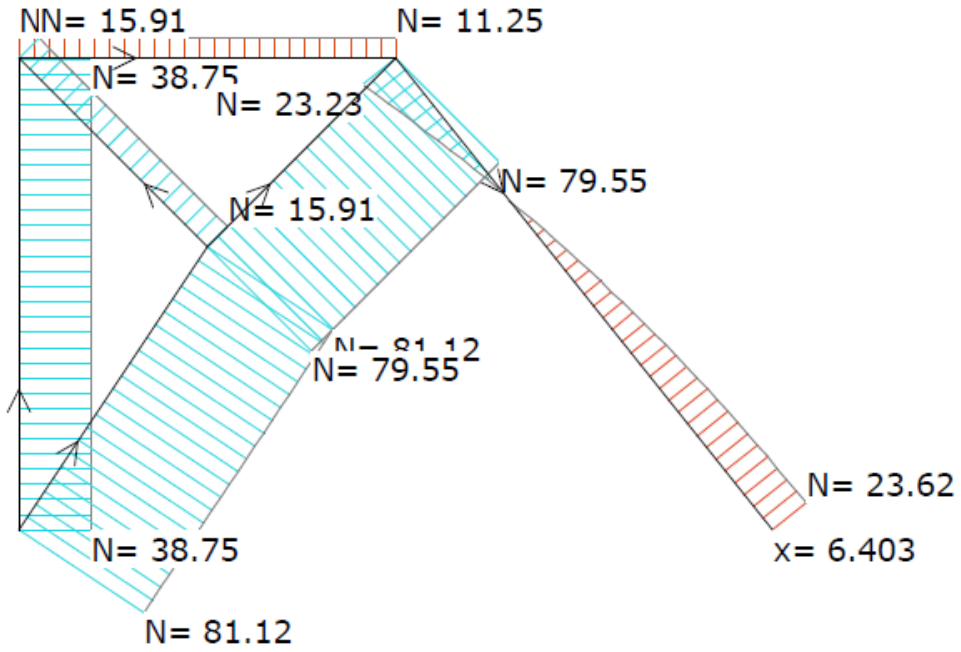
$N_B = 23.62$ kN; $N_C = 23.23$ kN

Three hinges structures – diagrams

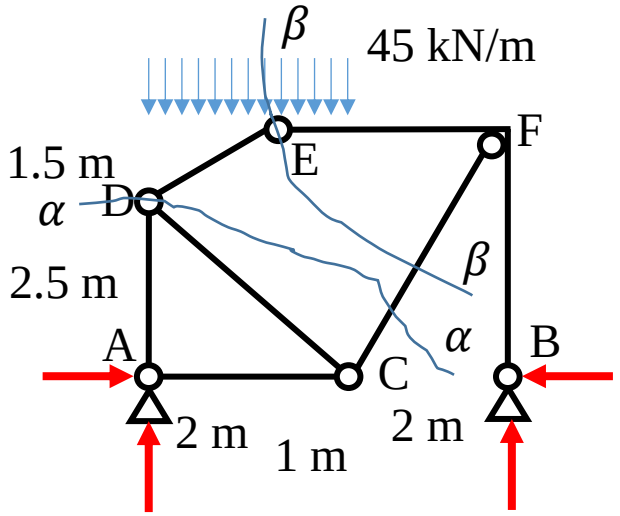




Diagrams – continued

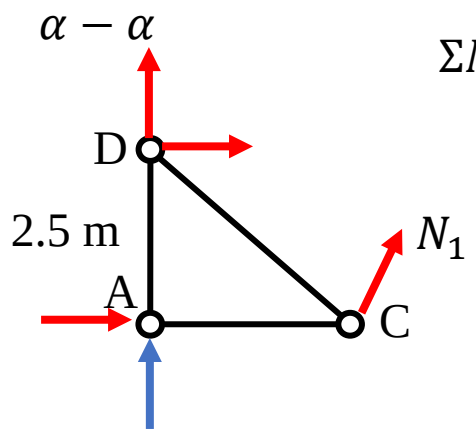


Complex structures free body unstable



$$\Sigma M_B = 0 \rightarrow V_A = \frac{45 \cdot 3 \cdot 3.5}{5} = 94.5 \text{ kN}$$

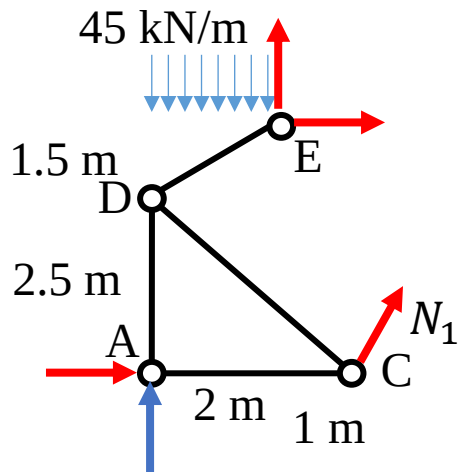
$$\Sigma M_A = 0 \rightarrow V_B = \frac{45 \cdot 3 \cdot 1.5}{5} = 40.5 \text{ kN}$$



$$\Sigma M_D^L = 0 \rightarrow 2.5H_A + N_1 \frac{2}{\sqrt{2^2 + 4^2}} \cdot 2.5 + N_1 \frac{4}{\sqrt{2^2 + 4^2}} \cdot 3 = 0 \rightarrow 2.5H_A + 3.801N_1 = 0$$

C. s. free body unstable – continued

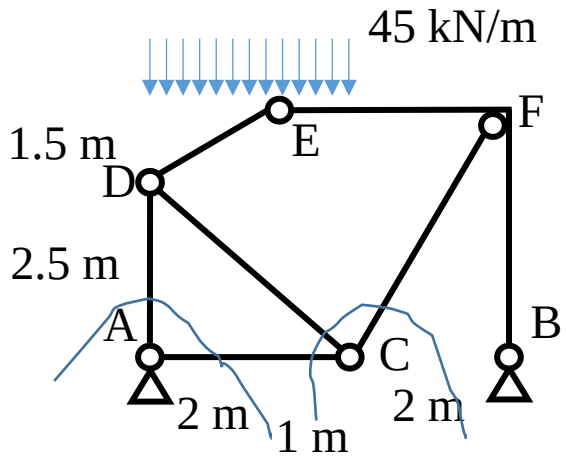
$\beta - \beta$

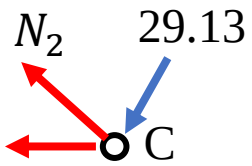


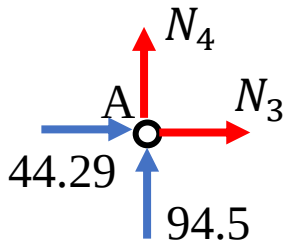
$$\Sigma M_E^L = 0 \rightarrow 4H_A + 45 \cdot 2 \cdot 1 + N_1 \frac{2}{\sqrt{2^2+4^2}} \cdot 4 + N_1 \frac{4}{\sqrt{2^2+4^2}} \cdot 1 - 94.5 \cdot 2 = 0 \rightarrow$$

$$4H_A + 2.683N_1 - 99 = 0$$

$$H_A = 44.29 \text{ kN}, N_1 = -29.13 \text{ kN}$$







node C:

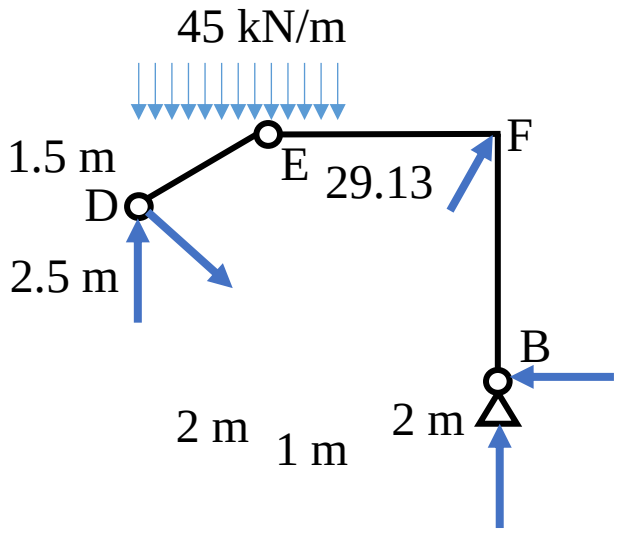
$$\Sigma Y = 0 \rightarrow \frac{2.5}{\sqrt{3^2+2.5^2}} N_2 = \frac{4}{\sqrt{2^2+4^2}} \rightarrow N_2 = 40.7 \text{ kN}$$

node A:

$N_3 = -44.29$ kN

$N_4 = -94.5$ kN

C. s. free body unstable – continued



$$\max M_{DE} = \frac{ql_x^2}{8} = \frac{45 \cdot 2^2}{8} = 22.5 \text{ kNm}$$

$$\Sigma M_E^L = 0 \rightarrow Q_D = \frac{45 \cdot 2 \cdot 1}{\sqrt{2^2+1.5^2}} = 36 \text{ kN}$$

$$Q_E^L = 36 - 45 \cdot 2 \cdot \frac{2}{\sqrt{2^2+1.5^2}} = -36 \text{ kN}$$

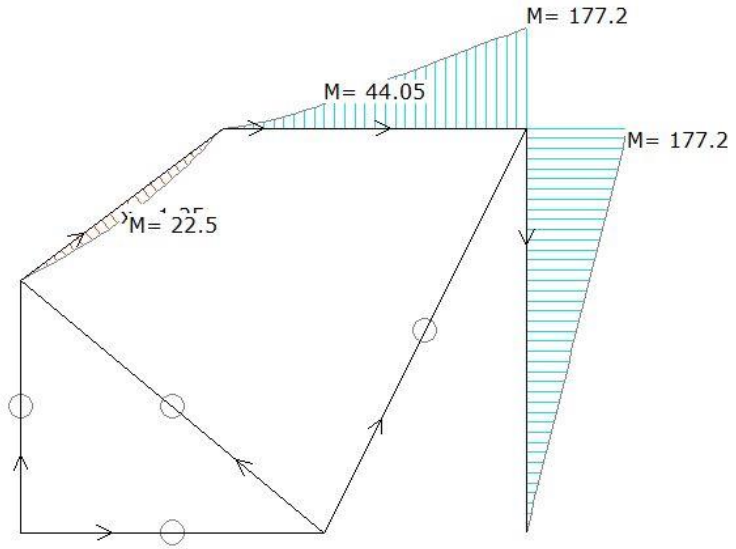
$$Q_E^R = 94.5 - 90 - 40.7 \cdot \frac{2.5}{\sqrt{2.5^2+3^2}} = -21.56 \text{ kN}$$

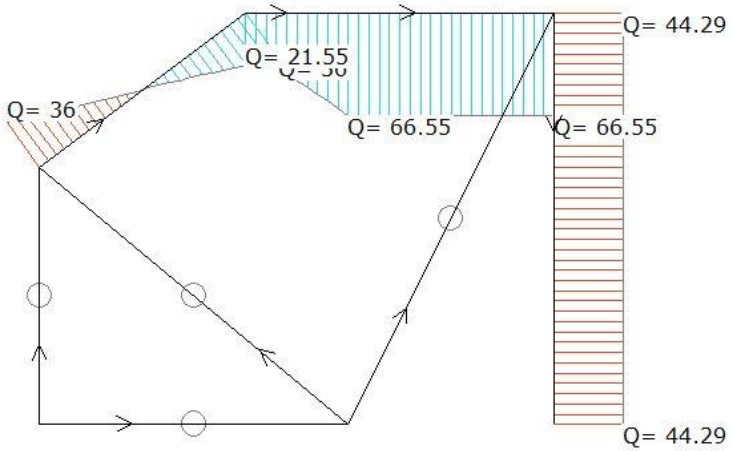
$$Q_{EF}(x = 1) = -21.56 - 45 = -66.56 \text{ kN}$$

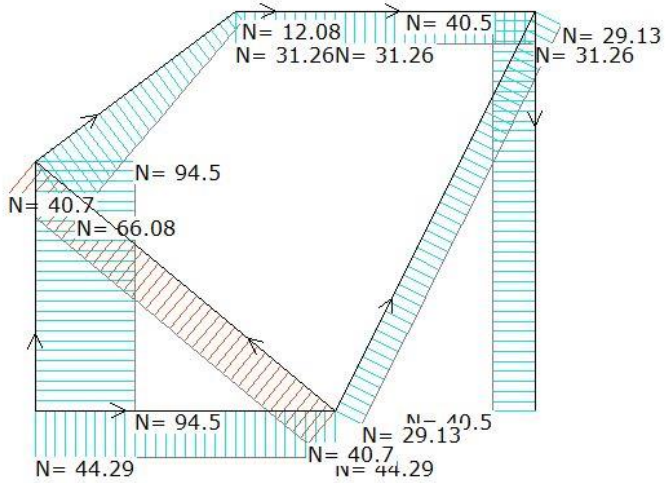
(no extremum in EF)

$$M_F = 44.29 \cdot 4 = 177.2 \text{ kNm}$$

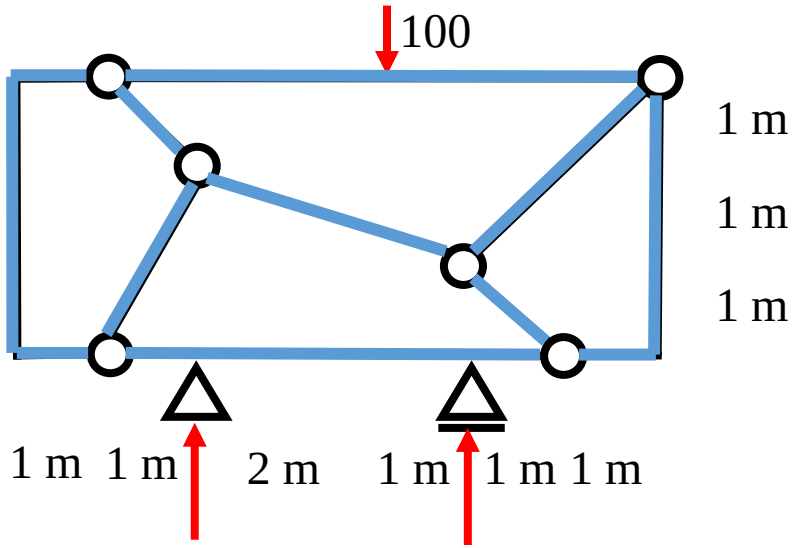
C. s. free body unstable – diagrams

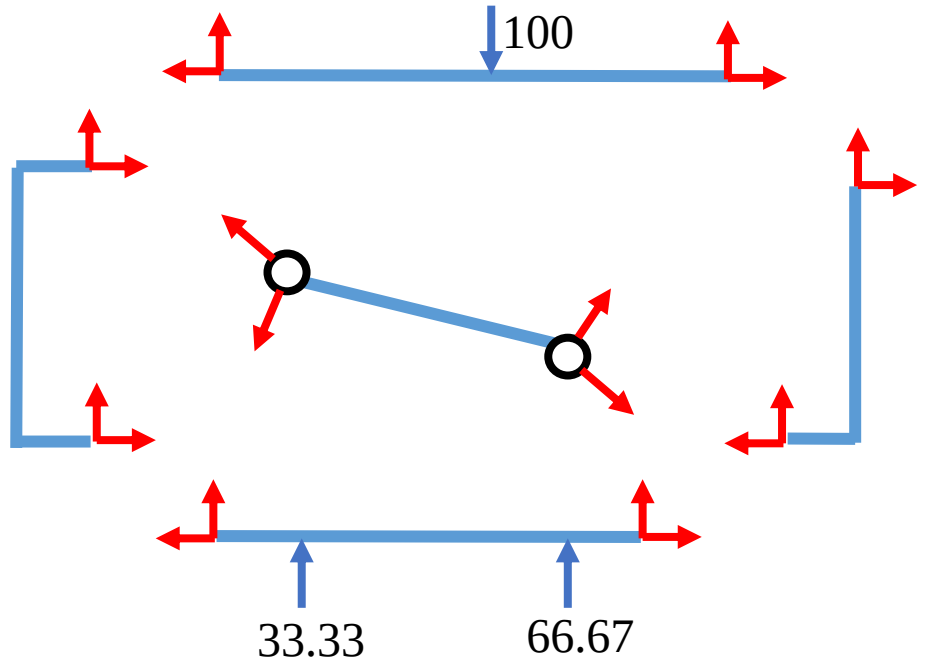






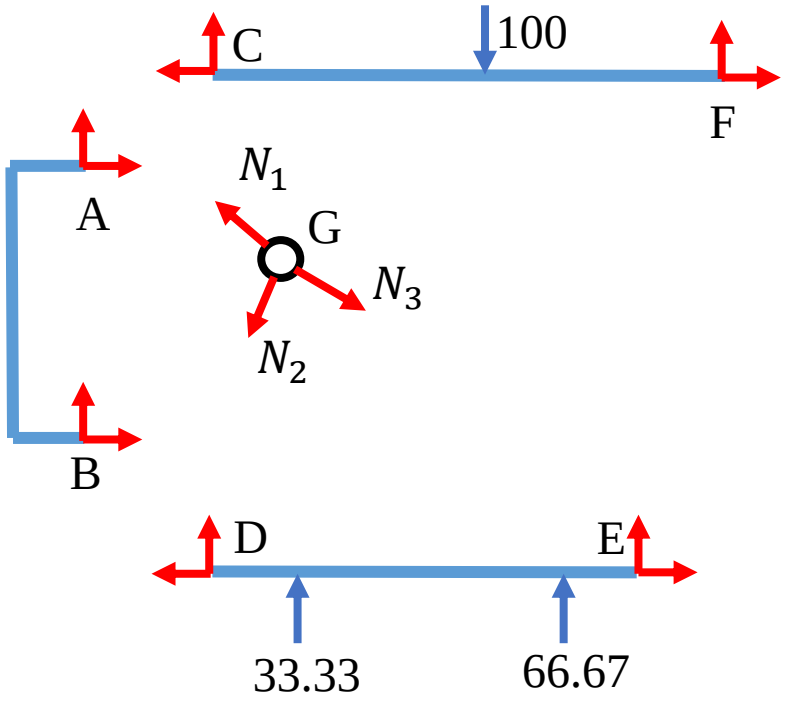
Disaggregation into simple subsets





There are: 4 shields with 4 unknows each, and 5 truss bars, so 17 unknwns in total.
For each shield we have 3 equations and 2 equations for each truss bar, so 17 equations.

Disaggregation – cont.



$\Sigma M_F = 0 \rightarrow V_C = 50$ kN

node AC:

$\Sigma Y = 0 \rightarrow V_A = -\frac{\sqrt{2}}{2} N_1 - 50$

$\Sigma Y_{AB} = 0 \rightarrow V_B = -V_A = \frac{\sqrt{2}}{2} N_1 + 50$

$\Sigma M_E = 0 \rightarrow V_D = -40$ kN

node BD:

$\Sigma Y = 0 \rightarrow -\frac{\sqrt{2}}{2} N_1 + \frac{2}{\sqrt{5}} N_2 - 10 = 0$

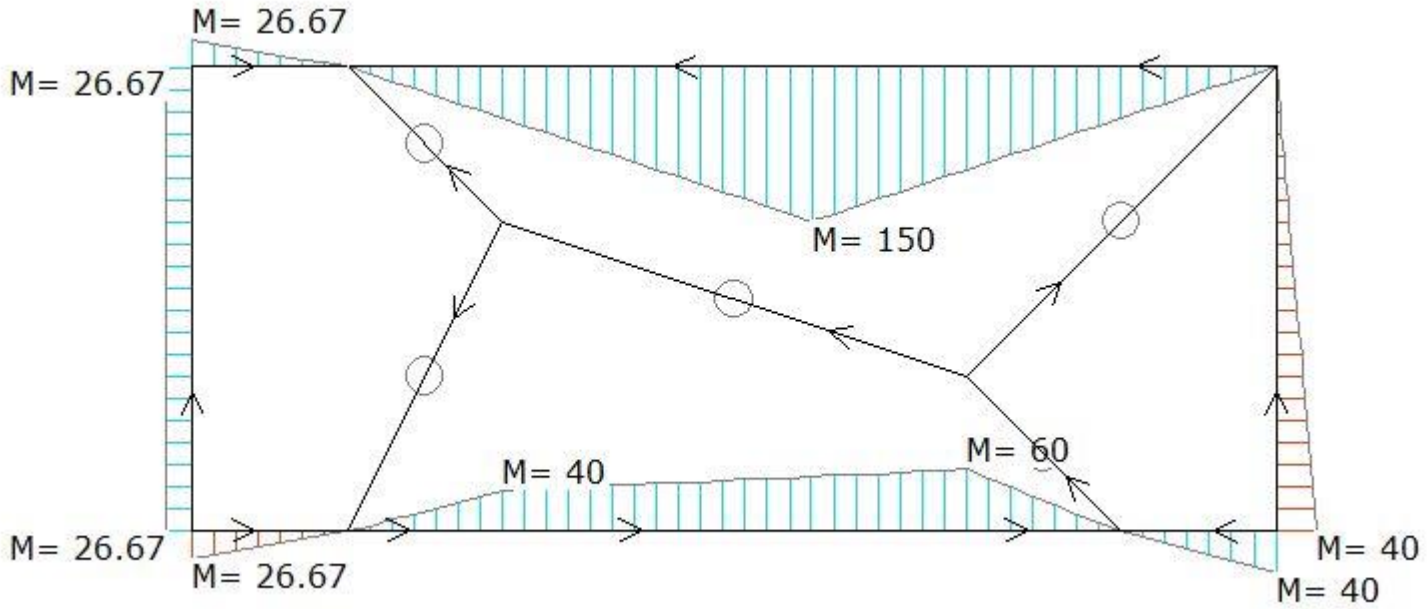
node G:

$\Sigma X = 0 \rightarrow -\frac{\sqrt{2}}{2} N_1 - \frac{1}{\sqrt{5}} N_2 + \frac{3}{\sqrt{10}} N_3 = 0$

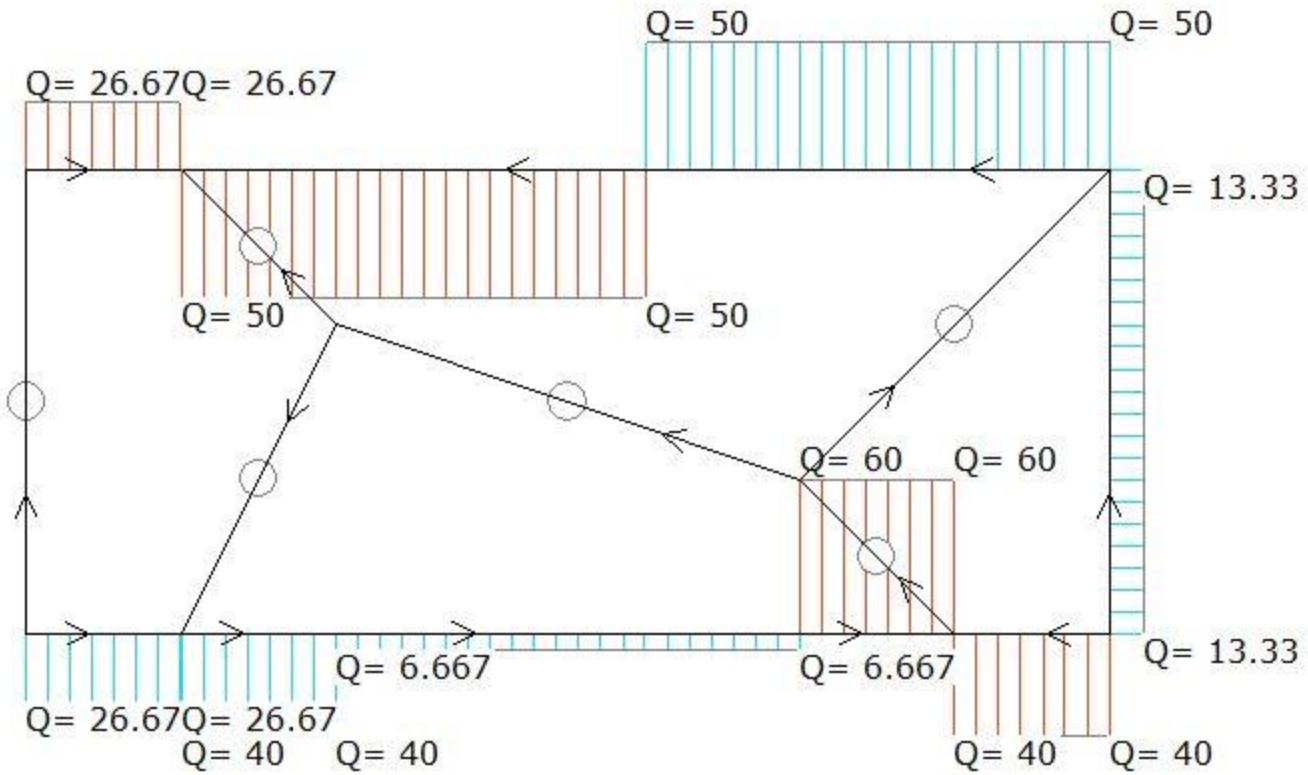
$\Sigma Y = 0 \rightarrow \frac{\sqrt{2}}{2} N_1 - \frac{2}{\sqrt{5}} N_2 - \frac{1}{\sqrt{10}} N_3 = 0$

$N_1 = -33.0$ kN, $N_2 = -14.91$ kN, $N_3 = -31.62$ kN

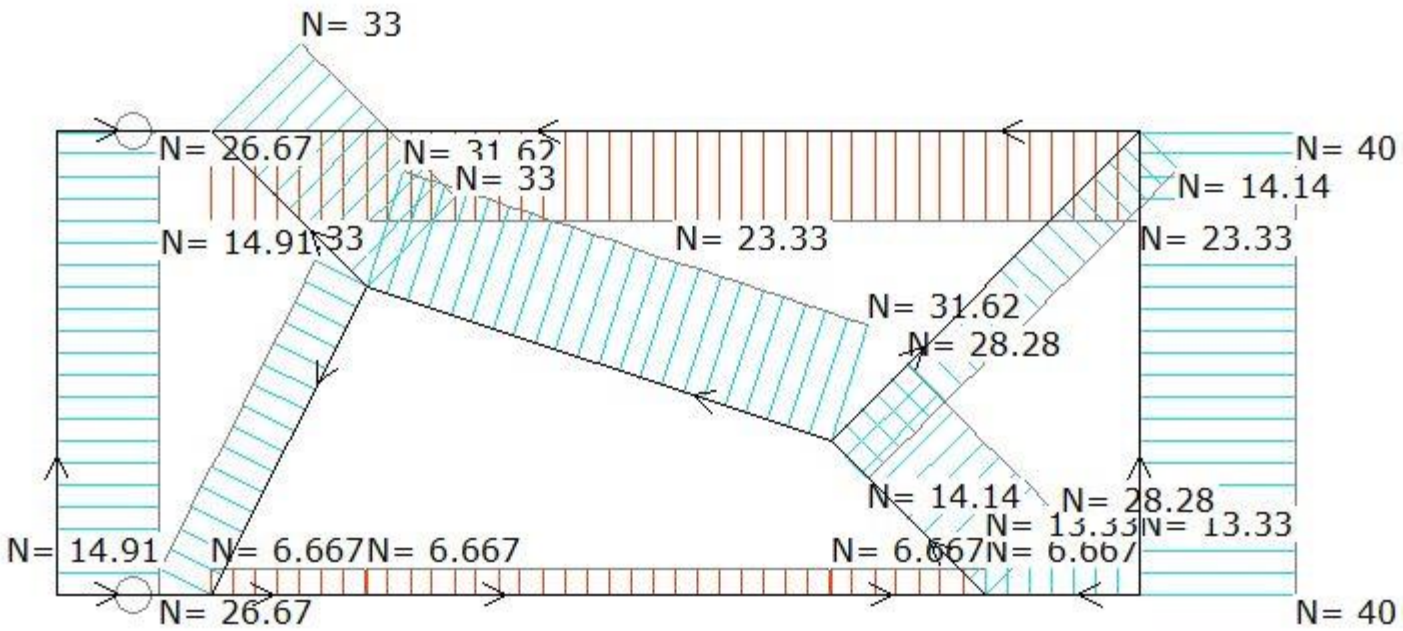
Disaggregation – moment diagram



Disaggregation – shear forces diagram



Disaggregation – axial forces diagram



Statics – summary

internal forces
cross-section
proper cross-sectional coordinate set
cross-sectional forces
truss element
theorem of zero-forces truss elements
trusses
- joints method
- section method
- exchange method

beam element
slant beams
continued beams
frames: node balance
arches
complex structures:
- analysis of free body stability
- distinction between truss and beam elements

Thank you for your attention!