Strength of Materials

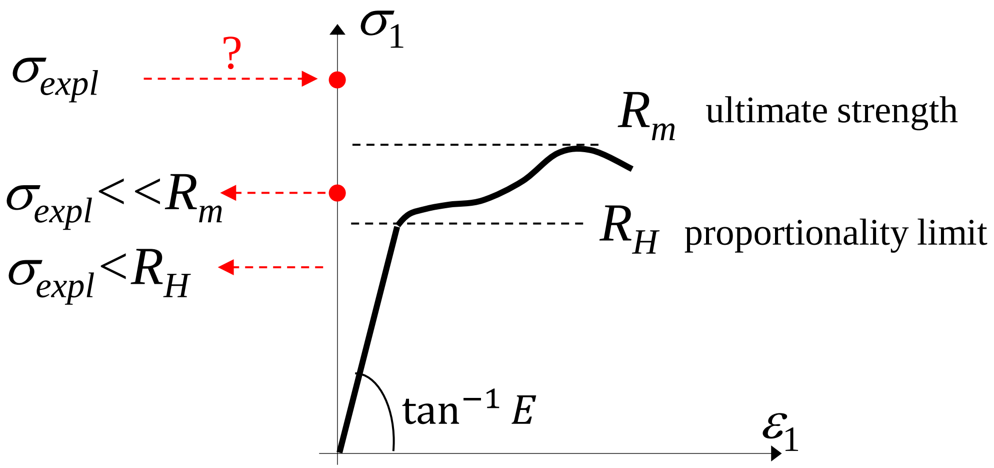
10. Exertion criteria

Uniaxial stress state

The cult diagram of uniaxial tensile testing of mild steel

- homogeneous state of stress and strain
- easy and accurate measurement
- overall clear picture of the ongoing processes
- easy interpretation of actual mechanical state
    - proportionality range,
    - elasticity stage,
    - elastic-plastic range,
    - work hardening stage,
    - necking phase
- every stage can be precisely determined
- all possible mechanical states can be experimented in one test
- the consequences of the stress/strain level are obvious
- experimental data use is clear and straightforward
- easy and clear use of safety coefficients

show me the point and I will explain where we are



Multiaxial stress state

None of the previous statements is true in the multiaxial state:

- the state of stress and strain may be nonhomogeneous and nonlinear
- measurement is neither easy nor accurate
- overall picture of the ongoing processes is not clear
- interpretation of actual mechanical state is very difficult and not precise
- multiple tests are required to catch the features of different mechanical states
- the consequences of the stress/strain level are not obvious
- experimental data use is very complicated and ambiguous
- use of safety coefficients is complex

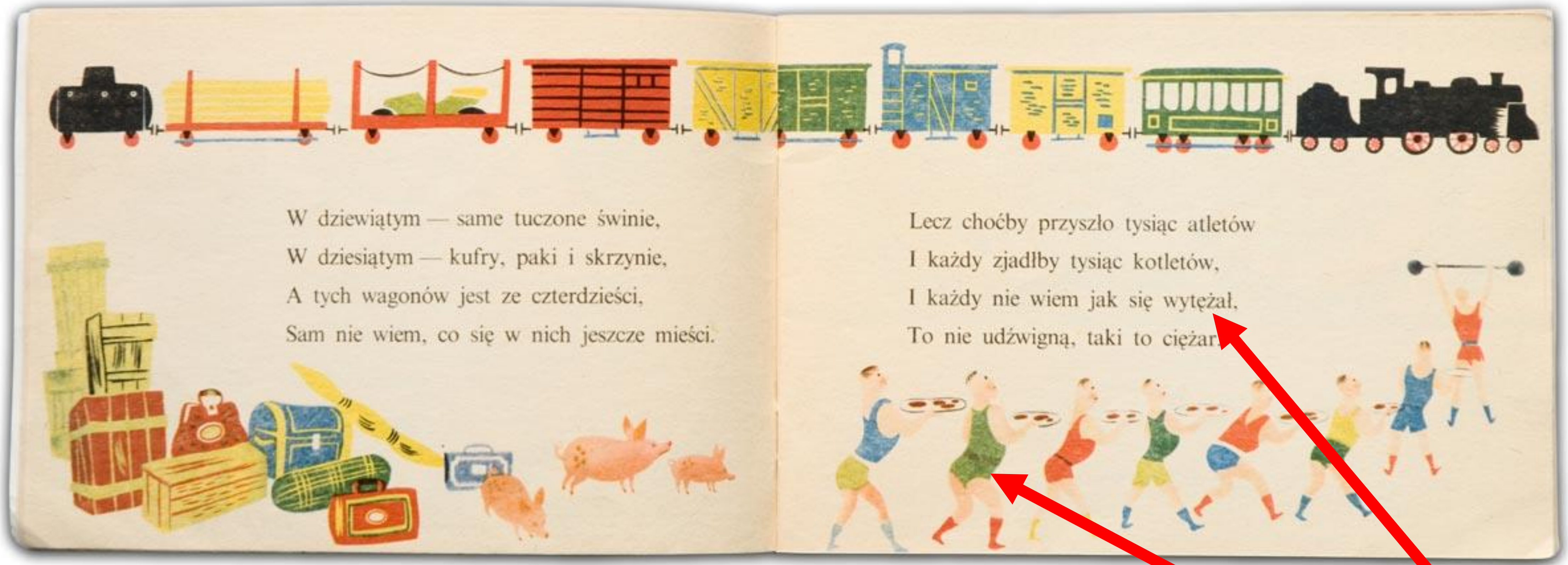
Which of the stress matrix below would be the most favorable?

$$T_\sigma = \begin{pmatrix} 350 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, T_\sigma = \begin{pmatrix} 350 & 0 & 0 \\ 0 & -100 & 0 \\ 0 & 0 & 0 \end{pmatrix}, T_\sigma = \begin{pmatrix} 350 & 0 & 0 \\ 0 & -100 & 0 \\ 0 & 0 & -100 \end{pmatrix}, T_\sigma = \begin{pmatrix} 300 & 0 & 0 \\ 0 & -100 & 0 \\ 0 & 0 & -100 \end{pmatrix}$$

Despite the diagonal form of the matrices, it is difficult to rank them growingly

We decidedly need some magic formula that might transfer 3-dimenstional stress state into the uniaxial

Locomotive



The exertion or effort means an approaching degree to the chosen limit state.

The crucial question is: what is the actual bearing capacity of a material in a particular mechanical state?

The answer to the question

The answer to the question doesn’t exist, really, and for many reasons:

- there is a huge amount of materials, possibly used in construction (including manufactured ones)
- structural materials have quite different mechanical properties
- but the same material can be used for various purposes and at different stress/strain level (different mechanical state)
- a detailed material description demands the use of many material parameters
- extensive (and very expensive) investigations of every structural material should be performed to determine all needed material parameters
- obviously a general mathematical description of such various properties is not possible

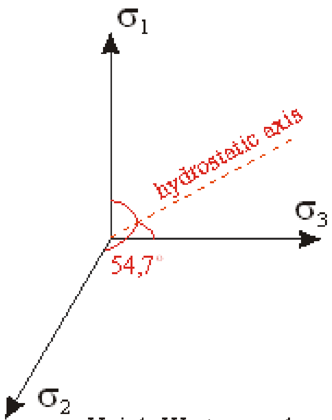
Is there any way out?

Yes:

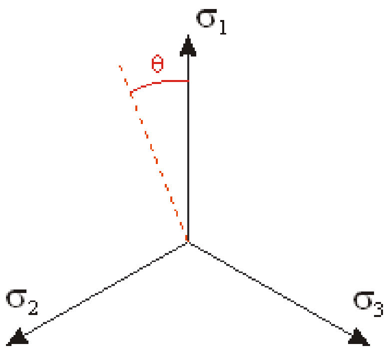
- for standard structural materials (steel, concrete, timber, aluminum)
- in standard situations (e.g. codes’ prescriptions)
- standard exertion hypotheses are used

Visualization ways

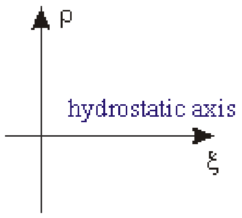
1) The *Haigh-Westergaard space* is the space with the set of principal stress coordinates. The *hydrostatic axis* (the mean stress axis) is an axis equally inclined to the principal stress axes.
2) The *Meldahl surface* (the *deviatoric surface*) is the surface perpendicular to the hydrostatic axis. This is the front view along the hydrostatic axis.
3) The surface passing through the hydrostatic axis is the *meridian plane*, where its angle describes angle between the meridian plane and the first principal axis.
4) Another cross-section is the section by the plane of $\sigma_2 = 0$, for the plane state of stress.
5) Sometimes, the same set of coordinates $\sigma - |\tau|$, like for Mohr's circles.



Haigh-Westergaard

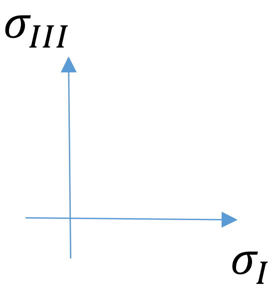


deviatoric plane (Meldahl)

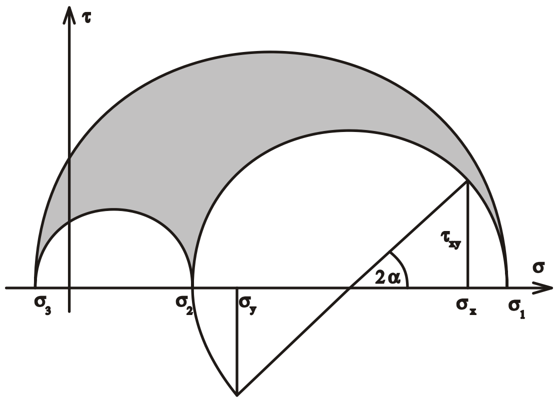


meridian plane

$\sigma_I \geq \sigma_{II} \geq \sigma_{III}$



plane stress state



Mohr's circles

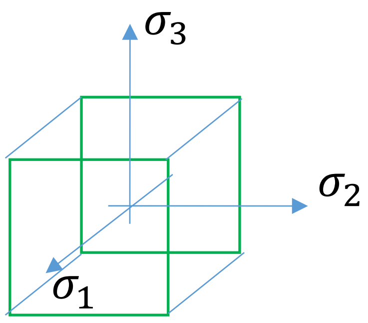
Galileo criterion

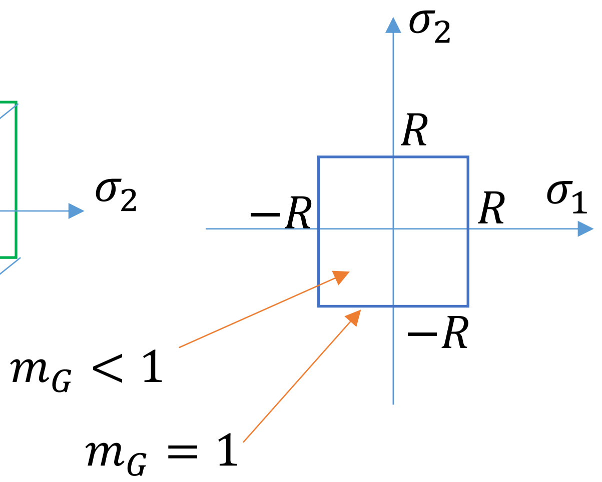
Galileo criterion: the effort measure is the greatest absolute value of the principal stresses

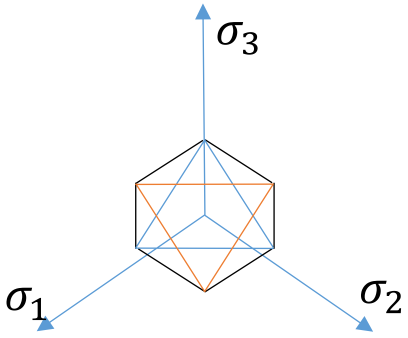
$$\sigma_1, \sigma_2, \sigma_3 \rightarrow \ \sigma_I \geq \sigma_{II} \geq \sigma_{III} \rightarrow m_G = \frac{\max(|\sigma_I|, |\sigma_{III}|)}{R}$$

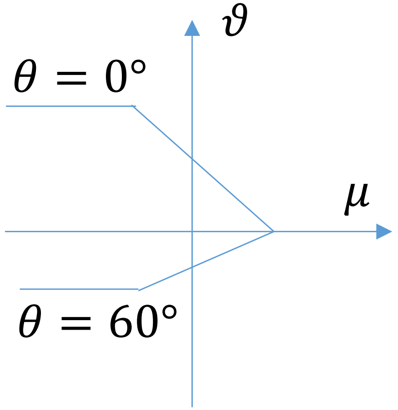
when comparing with the uniaxial stress state, we get so-called *substitute stress*: the stress in uniaxial stress state equivalent to the actual stress state:

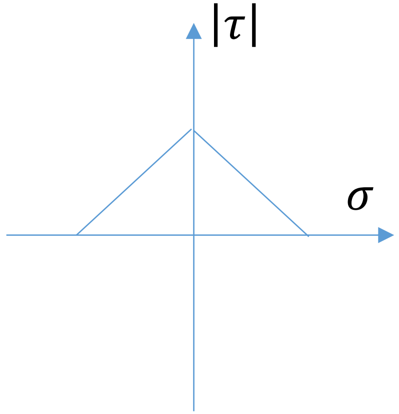
$$\sigma_G = \max(|\sigma_I|, |\sigma_{III}|)$$











(the criterion has only historical perspective)

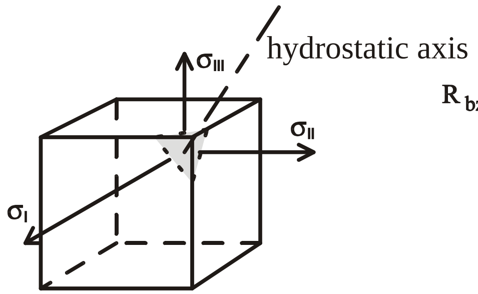
material bearing capacity for hydrostatic pressure is much greater than predicted

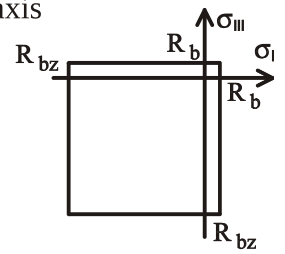
for shear case the predicted capacity is twice as actual one

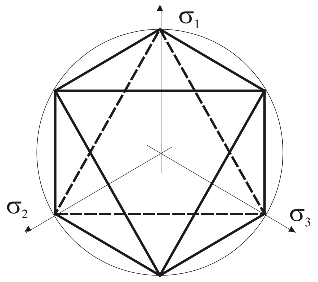
Galileo-Rankine-Clebsch criterion

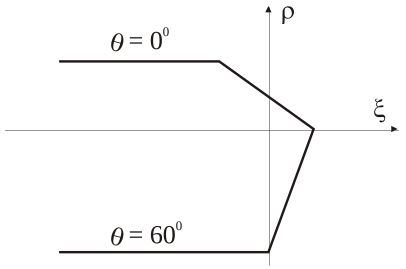
The G-R-C criterion takes account for different tensile and compression strengths (concrete and similar)

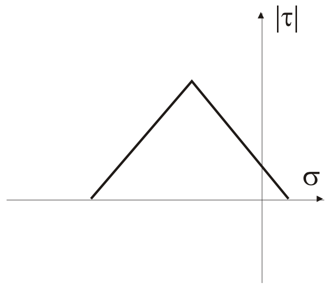
$$m_{GRC} = \max\left(\frac{\langle \sigma_I \rangle}{R_t}, \frac{|\sigma_{III}|}{R_c}\right), \qquad R_c \gg R_t$$









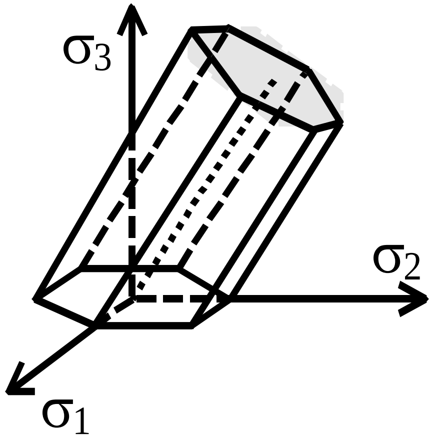


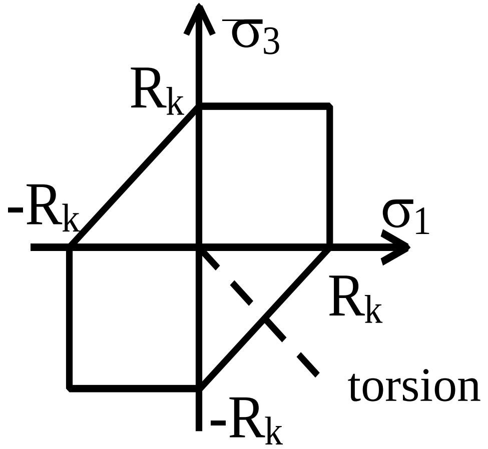
The criterion is used for ceramic materials.

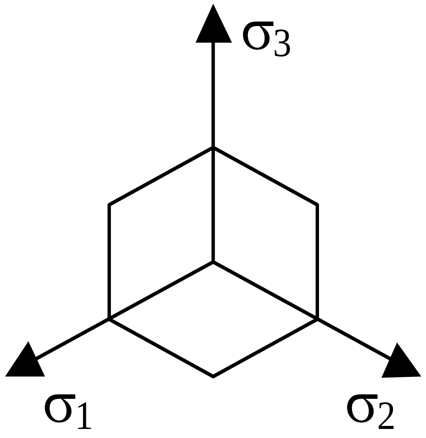
Coulomb-Tresca-Guest criterion

The exertion measure is the extreme value of shear stress.

$$\max\left(\left|\frac{\sigma_1-\sigma_2}{2}\right|,\left|\frac{\sigma_2-\sigma_3}{2}\right|,\left|\frac{\sigma_3-\sigma_1}{2}\right|\right)=\left|\frac{\sigma_I-\sigma_{III}}{2}\right|=\frac{\sigma_0}{2}\rightarrow\ \sigma_{CTG}=|\sigma_I-\sigma_{III}|$$





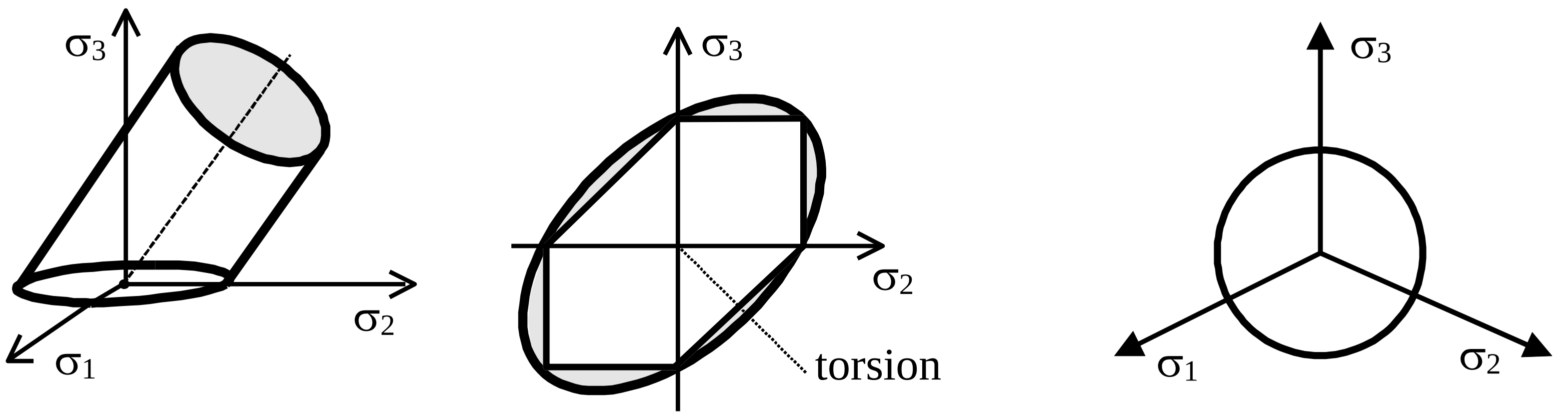


The criterion is popular for ductile materials.

Huber-Mises-Hencky criterion

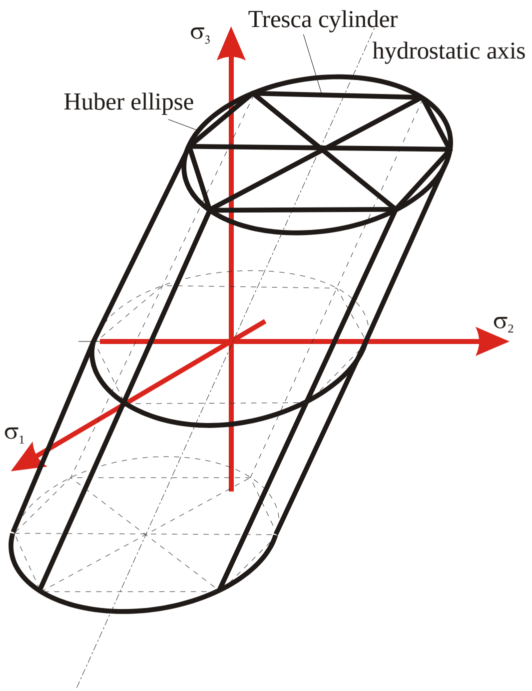
Maksymilian Tytus Huber – Polish academic (1904):
The exertion measure is the specific deviatoric energy (distortion energy density, octahedral shearing stress)

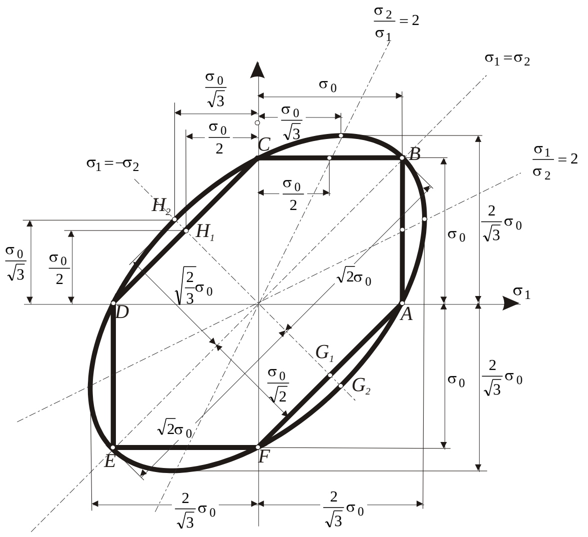
$$\frac{1}{\sqrt{6}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}=\frac{\sigma_0}{\sqrt{3}}\rightarrow\ \sigma_{HMH}=\frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}$$

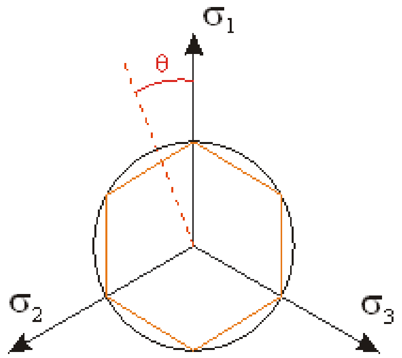


The best criterion for ductile materials (steel, aluminum, etc.), commonly used, and named von Mises criterion (1913, however priority of Huber has been proved and well-known)

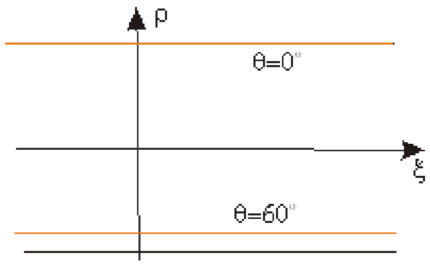
CTG and HMH criteria comparison



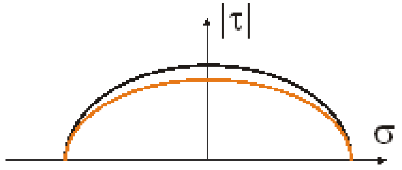




deviatoric plane (Meldahl)



meridian plane



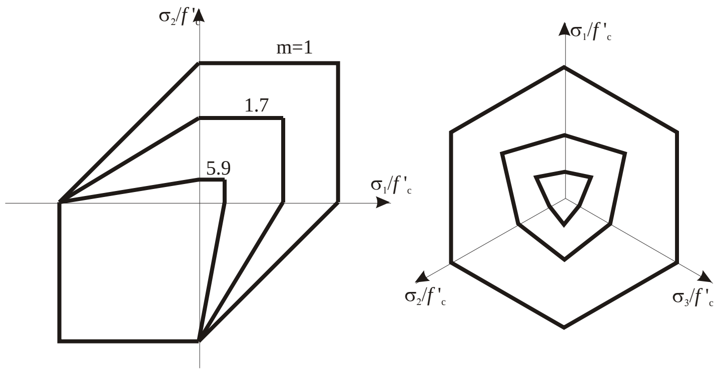
Mohr-Coulomb criterion

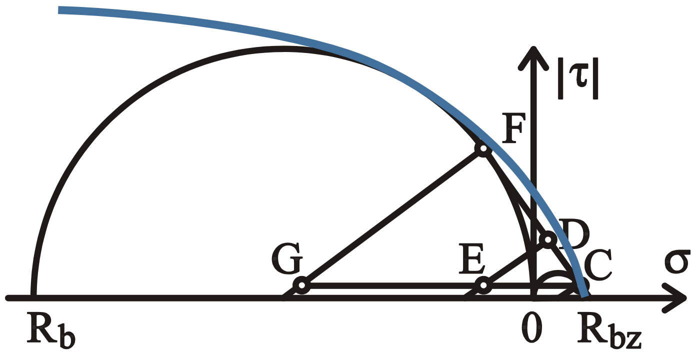
The criterion considers the limiting shear stress in a plane as a function of normal stress. The simplest form of the envelope of Mohr circles on the plane $\sigma - |\tau|$ is a straight line (Coulomb, 1772):

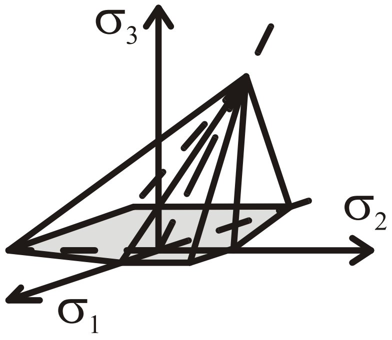
$$|\tau| = c - \sigma \tan\varphi$$

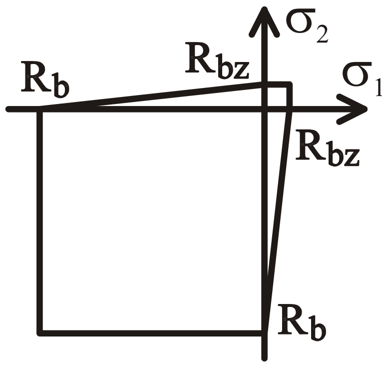
where:

c – cohesiveness
σ – pressure







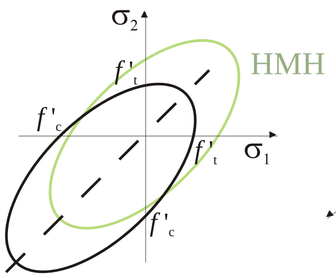


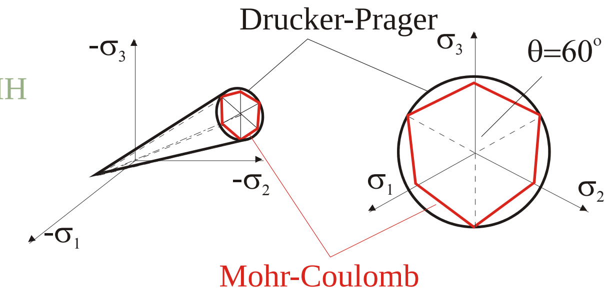
Commonly used in the soil mechanics

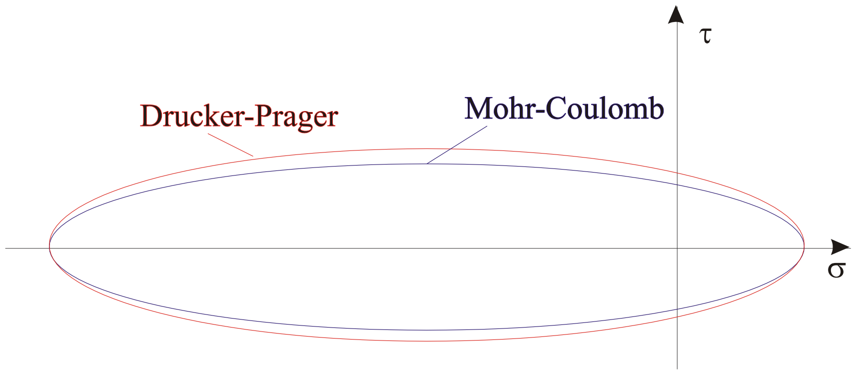
Drucker-Prager criterion

The criterion is a simple modification of HMH criterion by inclusion of an additional term accounting for hydrostatic stress:

$$m_{DP} = aI_1 + m_{HMH}$$







Practical formulae

Nine times out of ten, the complex stress state is limited to one normal stress σ, and one shear stress τ. In the simple case, the formulae of substitute stress are:

$$\sigma_G = \max\left(\frac{\sigma}{2} + \sqrt{\left(\tfrac{\sigma}{2}\right)^2 + \tau^2},\quad \frac{\sigma}{2} - \sqrt{\left(\tfrac{\sigma}{2}\right)^2 + \tau^2}\right),$$

$$\sigma_{\mathrm{CTG}} = \sqrt{\sigma^2 + 4\tau^2},$$

$$\sigma_{\mathrm{HMH}} = \sqrt{\sigma^2 + 3\tau^2},$$

$$\sigma_C = (1-k)\frac{\sigma}{2} + (1+k)\sqrt{\left(\tfrac{\sigma}{2}\right)^2 + \tau^2}.$$

The exertion hypotheses problems are cunning calculation problems for tests and exams. In one problem all particular SoM cases may be included. The effort calculation serves as pretext only.

Thank you for your attention!