Strength of Materials

14. Summary

Why repetition?

Repetitio est mater studiorum.
Repetition is the mother of knowledge.

I tell a student that the most important class you can take is technique. A great chef is first a great technician. ”If you are a jeweler, or a surgeon or a cook, you have to know the trade in your hand. You have to learn the process. You learn it through endless repetition until it belongs to you.”

If you want to be engineers by trade you have to master appropriate skills.

Knowledge of facts and methods and association of ideas.
The repetitions are necessary to get a broad view of the matter, to synthesize the methods and facts.

SoM – the basic cases

Tension – uniaxial and uniform stress state
Compression – uniaxial and uniform stress state as long as the stability is preserved
Bending – uniaxial but non-uniform stress state

- simple bending with constant bending moment and curvature of axis
- biaxial bending with two bending moments acting simultaneously
- eccentric tension (two bending moments and axial force)
- transverse bending with variable bending moment and non-zero shear force

Torsion – (unconfined) shear stress
Stability – compression of slender bars, check of equilibrium state

SoM – other material models

Elasticity – linearity and superposition

Plasticity – permanent strains, residual stress, loading, unloading and reloading, nonlinear relations, no superposition

Rheology – creep, relaxation, inelastic recovery, ageing, fading memory, solid and fluid type of material, time as a new independent variable: observation time, material age

Mathematics

Linear equations set for (linear) elasticity
Nonlinear relationships between the cross-sectional forces and the bar response (elongation, curvature)
Time depending relationships for material behavior as well as for load action (time of loading, duration of loading, material age at loading)

Linear algebra for most of cases (exception: stability)
Nonlinear algebra
Differential equations or Volterra integral equations in rheology

BVP

- in elasticity the set of 15 equations with 15 unknowns
- in plasticity 15+1 yielding equation (inequality)
- in rheology the spacetime

Models used

Hooke model – spring (one parameter)
de Saint-Venant model – friction connection (with or without work hardening)
Newton model – linear or nonlinear dependence on the stress level (piston in regular or irregular cylinder)

Structural models in rheology

First things first

Comprehension of basic facts

There is necessary to take in some formulas (by heart)

The knowledge of methods

- cross-sectional characteristics first
- cross-sectional forces first
- the stress matrix
- result checking (range and validity of the solution – the substitute stress hypothesis)

Tension

The tensile stress diagram
The first mention of schematization and creep
BVP solution:

stress matrix

$$T_\sigma = \begin{pmatrix} q & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

strain matrix

$$T_\varepsilon = \begin{pmatrix} \frac{q}{E} & 0 & 0 \\ 0 & -\upsilon\frac{q}{E} & 0 \\ 0 & 0 & -\upsilon\frac{q}{E} \end{pmatrix}$$

displacement vector

$$u = \frac{q}{E}x, v = -\upsilon\frac{q}{E}y, w = -\upsilon\frac{q}{E}z$$

The first mention of the de Saint-Venant principle
Compression of thick bars
The formulae ready to use

- stress, $\sigma_x = \frac{N}{A}$ (in [MPa])
- strain, $\varepsilon_x = \frac{N}{EA}$, EA – tension stiffness (in [N])
- elongation, $\Delta l = \frac{Nl}{EA}$

The self-support length
The degree of static and kinematic indeterminacy
The composed member and the idea of the prestressing
The rings and boilers – the solution for a frankfurter
The indeterminate structures – the importance of the temperature of humidity variations

Bending

The basic formulae:

$$\sigma_x = \frac{M_y}{I_y} z \qquad \frac{|M_y|}{W_y} \leq R \qquad \kappa = \frac{|M_y|}{E I_y}$$

The Bernoulli theorem of the cross-sections planes
(the existence of a symmetry plane)
The importance of the principal central inertia axes
The position of the most exerted fibers
The case of a sign-dependent material
The composite cross-section – different materials
The non-prismatic members
The first mention on the residual stress
The difference between the maximum strength and maximum stiffness cross-section (different proportions)

Biaxial bending

The neutral axis position – to facilitate the search of the most distant fibers
The lack of simple correlation between the bending moment and the neutral axis directions
The limit strength state

$\max |\sigma_x| \leq R$
or
$\max \sigma_x \leq R_t$ for tension
$\min \sigma_x \geq R_c$ for compression

The conditions of the fully plastic load – the first mention
The composed bending (with the axial force)
The cross-section core and its importance (the resultant force position)
The dual interpretation of the neutral axis equation

Transverse bending

Why the cross-sections shouldn't remain plane and we assume they might be plane
The shear force as a new ”playmaker”
Pure shear and shearing stress distribution
Mean shear stress formula and shear stress distribution

$$\tilde{\tau}_{zx} = \frac{Q_z(x) \cdot S_y^*(z)}{I_y \cdot b(z)} = \tilde{\tau}_{xz}$$

The principal stress trajectories (and the second mention about the prestressing)
The shear flow – composed cross-section beams
The shear stress in the thin-walled profiles
The shear center (with an homemade experiment)

Beam deflection

$$\frac{M_y(x)}{EJ_y} = -w''(x)$$

The three methods:

- simple analytic (kinematic and compatibility conditions)
- Macaulay (constant bending stiffness and no hinges)
- Mohr (for calculation the deflection at a point, the stiffness change and hinges permitted)
- virtual works principle – unit force theorem

The shear impact on the beam deflections

Torsion

The kinematic approach to the BVP – de Saint-Venant assumptions

$$\theta = \frac{M_x}{GI_x}$$

The solution to the circular shafts – no warping
Prandtl and hydrodynamical analogy
Solution to the rectangular cross-section
Thin-walled profiles

- developable (C-channels, Z-sections, angles)
- non-developable (I-beams, H-beams, T-beams) but open
- closed profiles
- mixt (open-closed)

Bredt's first and second formulae
Torsion of composite section

Energy and stability

Specific volumetric energy (volumetric energy density)
Specific distortion energy (distortion energy density)
Variational principles and theorems of Castigliano and Menabrea, Betti principle
Generalized forces and displacements
Potential energy for bar structures
Stable state loss – stability loss with buckling
The effective length and slenderness
The critical force

- Euler formula, in elastic range
- Tetmeyer-Jasiński and Johnson-Ostenfeld formulae for nonlinear range
- Rankine-Gordon formula

Effect of imperfections
Energy method
Stability of a system – compatibility conditions

Exertion criteria

Comparison of the composed three-axial stress state with the uniaxial uniform stress state – „magic” formulae

- CTG criterion
- HMH criterion
- Mohr criterion
- Coulomb criterion
- Drucker-Prager criterion

Different ways of visualization: in the Westergaard space, …

Plasticity

Stress distribution in the elastic range, elastic-plastic range, and for fully-plastic stage
The residual stress distribution, the residual curvature
Quasi-cycles
The shape coefficient
Limit analysis of structures:

- static approach (the lower bound)
- kinematic approach (the upper bound)

Plastic hinges and yield lines
Nonlinear moment-curvature interdependence
Interaction curves

Fracture mechanics and rheology

Stress concentration factor
Lennard-Jones model of fracture
Cyclic load – fatigue

- low cycle fatigu**e**
- high cycle fatigue

Cracking modes I, II, and III
Stress intensity factors
Fracture mechanics example

Time as a new independent variable

- creep
- relaxation
- ageing
- fading memory

Simple structural models:

- Maxwell
- Kelvin
- Standard
- Burgers
- Maxwell and Kelvin generalized

Hereditary theories

- invariant
- non-invariant

Boltzmann superposition theorems
Three stages of metal alloys creep

Unmentioned topics

Multiparameter elasticity
Nonlinear elasticity
Hyperelasticity (Mooney-Rivlin material for rubber and elastomers)
Beams on elastic foundation
Settlement of supports (or soil)
Curved bars
Confined torsion
Bending assisted compression
Work hardening in plasticity
Nonlinear creep (nonlinear dependence between strain and stress)
Dynamic loads & impacts (ship collision with a bridge pillar)
Contact stress
Nonlinear geometry of structures
Cables

Thank you for your attention!