MICROMECHANICAL MODEL OF HYPERELASTIC BEHAVIOUR OF CELLULAR MATERIALS

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Cellulars are an increasingly investigated class of materials solids as they reveal unique properties. Generally these materials due to their geometric structure of skeleton are characterized by high deformability and reversibility of deformation thus showing hyperelastic behaviour. Due to large pore volume fraction they can easily experience large deformations since such a deformation on the macroscopic level usually require smaller deformations of the individual walls constituting skeleton. The formulation based on micromechanical modeling [1] assumes that essential macroscopic features of mechanical behaviour on a macro scale can be inferred from the deformation response of a representative volume element. Open-cell materials with diverse regular skeleton structures as shown in Fig 1. are considered.

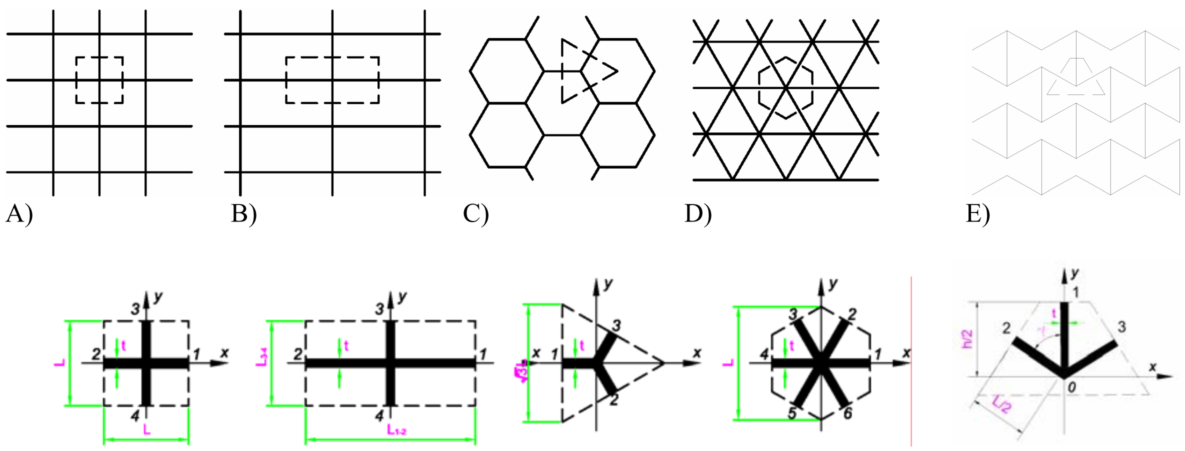

Fig. 1. Typical material structures and their representative unit cells.

Microstructure of these materials are modelled by idealized regular repeating pattern of unit cells where skeleton of a cell is modelled as elastic beam structure of unit depth with stiff joints. The idea of two-scale modelling leads to formulation of equivalent continuum, which elastic properties : elastic stiffness matrix, yield stresses and limit strains describing deformability in the elastic range depend on material properties of a solid phase of the cell and topological arrangement of it's structure.

Interest is focused on geometric nonlinearity which refers to large displacement and small strain case. The present paper extends recent effective linear anisotropic elasticity model [2,3] for cellular materials to geometric nonlinearity, which in the frame strain potential formulation [4,5] is described by constitutive relation between Green's Lagrangean strain tensor **E** and second Piola-Kirchoff stress tensor $\mathbf{\Pi}$, as follows:

(1) $$\mathbf{\Pi}(\mathbf{E}) = \left.\frac{\partial \mathbf{\Pi}}{\partial \mathbf{E}}\right|_{\mathbf{E}=0} : \mathbf{E} = \mathbf{S_0} : \mathbf{E}$$

where: $\mathbf{S_0}$ is initial elasticity tensor (initial tangent operator).

Examples involving numerical test on cellular materials under homogenoeous strain for uniaxial or biaxial loading in the tensile and compressive range, and under shearing are considered. The calculations are performed for cellular materials exhibiting structures specified in Table 1.

Structure type	L [mm]	h [mm]	t [mm]	γ	R_e [MPa]	E_s [GPa]
A)	20.00	-	1.00	-	200.0	20.0
C)	20.00	-	1.00	-	200.0	20.0
D)	20.00	-	1.00	-	200.0	20.0
E)	6.0	8.0	0.15	60^o	100.0	10.0

Table 1. Specification of cellular microstructure.

The effect of nonlinearity is shown on example of uniaxial test. The load is applied subsequently in three choosen directions ξ with respect to x axis (shown in Fig.1) given by β angle. Due to isotropic mechanical properties the materials of structure type C) and D) are not sensitive to this load direction. The relevant plots of normal nominal stress in dependence of material stretch are given in Fig. 2**.**

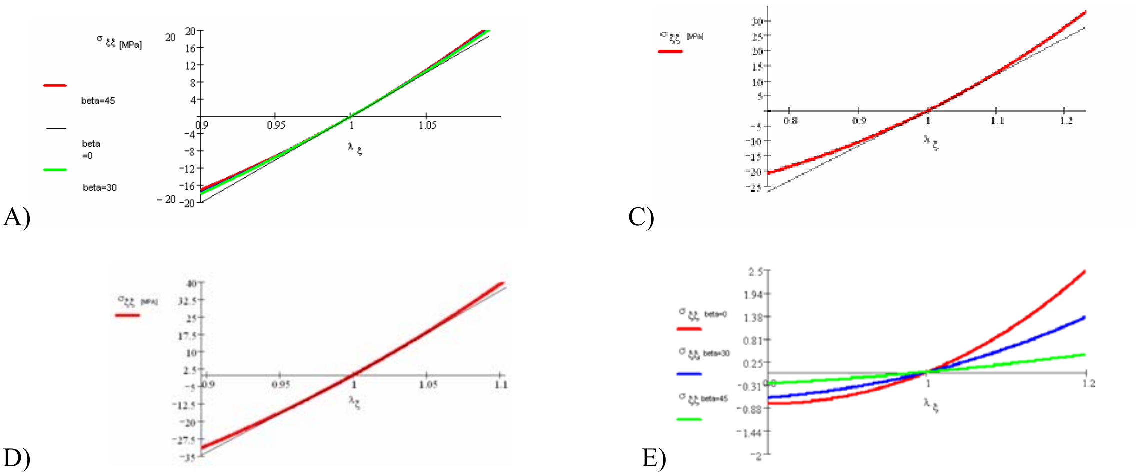

Fig2. Stress-strain curves in elastic range

The conclusion is that significant differences between infinitesimal strain behaviour and small strain regime can be observed. The influence of material structure and microstructural parameters on nonlinear effect is clearly visible and it gives the hints for material selection.

References

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