P R O J E K T

Z

W Y T R Z Y M A Ł O Ś C I M A T E R I A Ł Ó W

Projekt belki zginanej poprzecznie

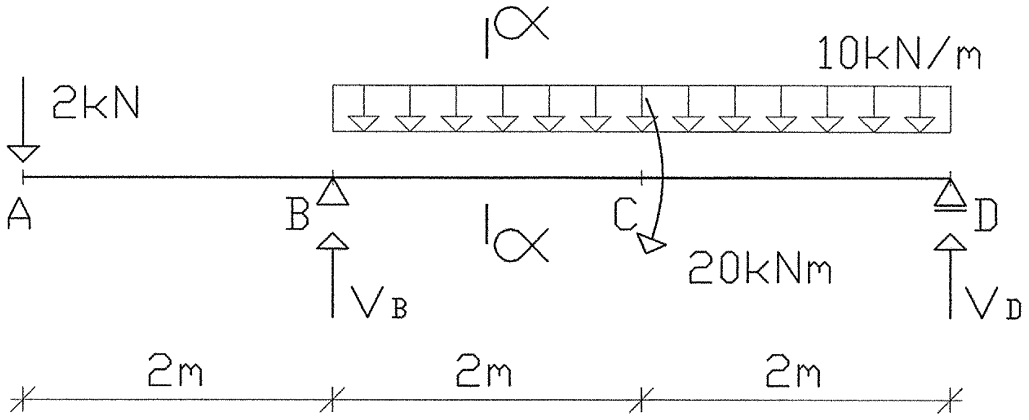
stan naprężeń i ugięcia

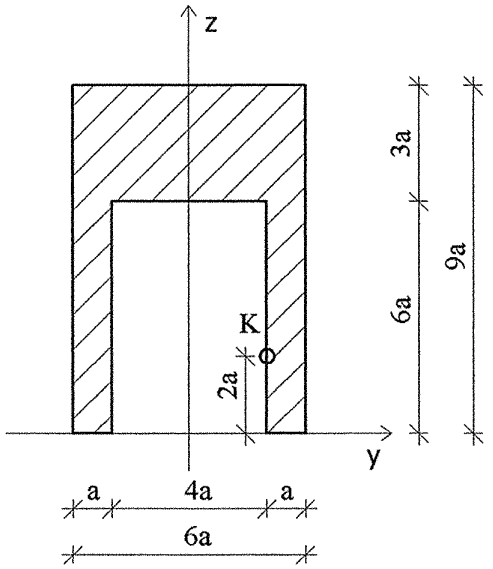
Zaprojektować wymiary przeroju poprzecznego zginanej belki ze względu na stan graniczny nośności i użytkowania.

Po zaprojektowaniu wyznaczyć rozkład naprężeń normalnych i stycznych w przekroju α-α oraz obliczyć naprężenia główne i ich kierunki w punkcie K przekroju.

Otrzymane wyniki sprawdzić programami komputerowymi STATYKA i PRZEKRÓJ, załączyć wydruki rezultatów obliczeń.

R = 175 MPa **$R_t = 0.6 \cdot R$** **$f_{dop} = l_{max} / 250$** **E = 205 GPa**





STATYKA

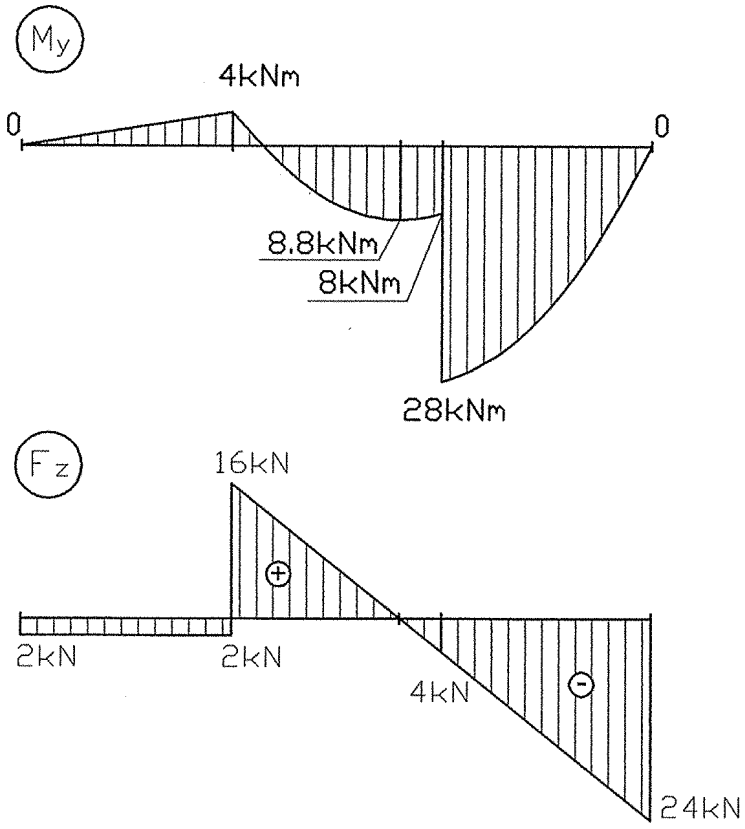
$\Sigma M(B) = 0$
$V_D \cdot 4 + 2 \cdot 2 - 20 - 10 \cdot 4 \cdot 2 = 0$
$V_D = (100 - 4) : 4$
$V_D = 24$

$\Sigma M(D) = 0$
$2 \cdot 6 + 10 \cdot 4 \cdot 2 - V_B \cdot 4 - 20 = 0$
$4 \cdot V_B = 72$
$V_B = 18$

Spr. Σ "Z" = 0
$-10 \cdot 4 - 2 + 24 + 18 = 0$

$M(A) = 0$
$M(B) = -2 \cdot 2 = -4$
$M(C)^L = -2 \cdot 4 + 18 \cdot 2 + 10 \cdot 2 \cdot 1 = -8 + 36 - 20 = 8$
$M(C)^P = 24 \cdot 2 - 10 \cdot 2 \cdot 1 = 48 - 20 = 28$
$M(D) = 0$

$F_z(A) = -2$
$F_z(B)^L = -2$
$F_z(B)^P = -2 + 18 = 16$
$F_z(C) = -24 + 20 = -4$
$F_z(D) = -24$



GEOMETRIA PRZEKROJU

$F = 2\cdot(a\cdot 6a) + 3a\cdot 6a = 12a^2 + 18a^2 = 30a^2$

$S_y = 2\cdot(a\cdot 6a\cdot 3a) + 3a\cdot 6a\cdot 7.5a = 36a^3 + 135a^3 = 171a^3$

$z_o = \frac{171a^3}{30a^2} = 5.7a$

MOMENT BEZWŁADNOŚCI

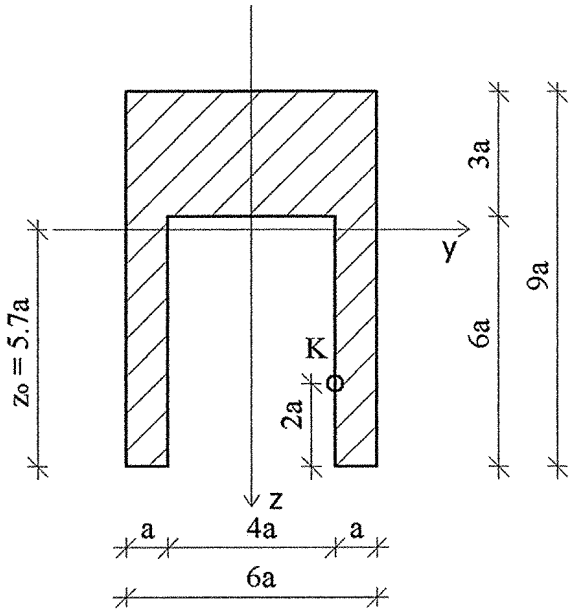
$J_{yo} = [\ \frac{6a\cdot(3a)^3}{12} + 6a\cdot 3a\cdot(1.8)^2\ ] + 2\cdot\ [\frac{a\cdot(6a)^3}{12} + a\cdot 6a\cdot(2.7)^2] =$

$= [\ 13.5a + 58.32\ ]\ a^4 + 2\cdot\ [\ 18 + 43.74\ ]\ a^4 = 71.82a^4 + 123.48a^4 = 195.3a^4$

WSKAŹNIK WYTRZYMAŁOŚCI

$|\ z_{max}\ | = 5.7a$

$W_y = J_{yo} / |\ z_{max}\ | = 195.33a^4 / 5.7a = 34.26a^3$



Warunek projektowania ze względu na naprężenie normalne:

$\frac{M\ max}{Wy} \le R \qquad => \qquad \frac{M\ max}{R} \le W_y$

$\frac{28kNm}{175MPa} \le 34.26a^3 \qquad 28\cdot 10^3\ Nm \le 175\cdot 10^6\cdot N/m^2\cdot 34.26\ a^3$

$0.16\cdot 10^{-3} :\ 34.26\ m^3 \le a^3$

$a^3 \ge 0.00467\cdot 10^{-3}\ m^3$

$a \ge 0.0167\ m$

$a \ge 1.67$ cm

Warunek projektowania ze względu na naprężenia styczne:

$$\tau_{max} = \frac{Fz\,max \cdot Sy(0)}{Jyo \cdot b(0)}$$

$S_y(0) = 2\cdot(a \cdot 0.3a \cdot 0.15a) + 3a \cdot 6a \cdot 1.8a = 0.09a^3 + 32.4a^3 = 32.49a^3$

$b(0) = 2a$
$F_{z\,max} = 24\ kN$

$$\tau_{max} \le R_t \quad => \quad \frac{28\,kN \cdot 32.49a^3}{195.3a4 \cdot 2a} \le 0.6\cdot R$$

$1.9963 \cdot 10^3\ m^2 \le 105 \cdot 10^6 \cdot a^2$

$a^2 \ge 0.019 \cdot 10^{-3}\ m^2$
$a^2 \ge 0.0019 \cdot 10^{-2}\ m^2$
$a \ge 0.0435 \cdot 10^{-1}\ m$
$a \ge 0.00435\ m$

$a \ge 0.435$ cm

Warunek projektowania ze względu na ugięcia:

$$M(x) = -2\cdot x\,\big|_{AB} + 18\cdot(x-2) - 10\cdot\tfrac{1}{2}\cdot(x-2)^2\,\big|_{BC} + 20\cdot(x-4)^0\,\big|_{CD}$$

$$EJ_y w''(x) = 2\cdot x\,\big|_{AB} - 18\cdot(x-2) + 5\cdot(x-2)^2\,\big|_{BC} - 20\cdot(x-4)^0\,\big|_{CD}$$

$$EJ_y w'(x) = C + x^2\,\big|_{AB} - 9\cdot(x-2)^2 + 5/3\cdot(x-2)^3\,\big|_{BC} - 20\cdot(x-4)^1\,\big|_{CD}$$

$$EJ_y w(x) = D + C\cdot x + 1/3\cdot x^3\,\big|_{AB} - 9/3\cdot(x-2)^3 + 5/12\cdot(x-2)^4\,\big|_{BC} - 20/2\cdot(x-4)^2\,\big|_{CD}$$

$$EJ_y w(x) = D + C\cdot x + 0.33\cdot x^3\,\big|_{AB} - 3\cdot(x-2)^3 + 0.4166\cdot(x-2)^4\,\big|_{BC} - 10\cdot(x-4)^2\,\big|_{CD}$$

Kinematyczne Warunki Brzegowe:

$w(2) = 0$
$0 = D + 2\cdot C + 0.33\cdot 8$
$0 = D + 2\cdot C + 2.66$

$w(6) = 0$
$0 = D + 6\cdot C + 0.33(3)\cdot 216 - 3\cdot 64 + 0.416(6)\cdot 256 - 10\cdot 4$
$0 = D + 6\cdot C + 72 - 192 + 106.66 - 40$
$0 = D + 6\cdot C - 53.34$

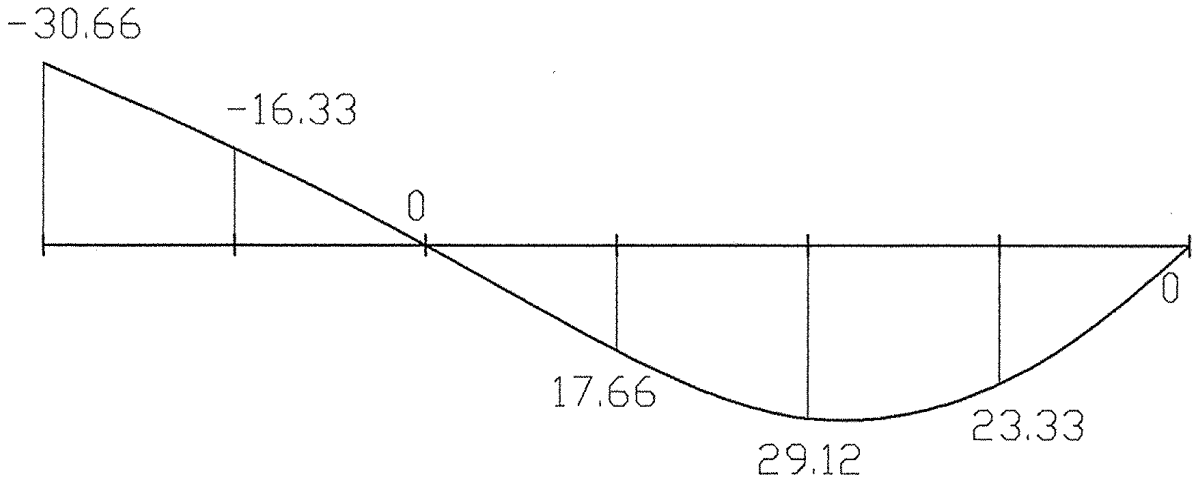
$0 = D + 2\cdot C + 2.66$
$0 = D + 6\cdot C - 53.34$

$0 = -4\cdot C + 56$ $\qquad$ $0 = D + 2\cdot 14 + 2.66$
$4\cdot C = 56$ $\qquad$ $-D = 30.66$
$C = 14$ $\qquad$ **$D = -30.66$**

$$EJ_y w'(x) = 14 + x^2 \Big|_{AB} - 9\cdot(x-2)^2 + 1.66\cdot(x-2)^3 \Big|_{BC} - 20\cdot(x-4)^1 \Big|_{CD}$$
$$EJ_y w(x) = -30.66 + 14\cdot x + 0.33\cdot x^3 \Big|_{AB} - 3\cdot(x-2)^3 + 0.4166\cdot(x-2)^4 \Big|_{BC} - 10\cdot(x-4)^2 \Big|_{CD}$$

pkt A	x = 0	$EJ_y w'(0) = 14$ $EJ_y w(0) = -30.66$
	x = 1	$EJ_y w'(1) = 14 + 1^2 = 15$ $EJ_y w(1) = -30.66 + 14\cdot 1 + 0.33\cdot 1^3 = -16.33$
pkt B	x = 2	$EJ_y w'(2) = 14 + 2^2 = 18$ $EJ_y w(2) = 0$
	x = 3	$EJ_y w'(3) = 14 + 3^2 - 9\cdot(3-2)^2 + 1.66\cdot(3-2)^3 = 15.66$ $EJ_y w(3) = -30.66 + 14\cdot 3 + 0.33\cdot 3^3 - 3\cdot(3-2)^3 + 0.4166\cdot(3-2)^4 = 17.66$
pkt C	x = 4	$EJ_y w'(4) = 14 + 4^2 - 9\cdot(4-2)^2 + 1.66\cdot(4-2)^3 = 7.28$ $EJ_y w(4) = -30.66 + 14\cdot 4 + 0.33\cdot 4^3 - 3\cdot(4-2)^3 + 0.4166\cdot(4-2)^4 = 29.12$
	x = 5	$EJ_y w'(5) = 14 + 5^2 - 9\cdot(5-2)^2 + 1.66\cdot(5-2)^3 - 20\cdot(5-4) = -17.18$ $EJ_y w(5) = -30.66 + 14\cdot 5 + 0.33\cdot 5^3 - 3\cdot(5-2)^3 + 0.4166\cdot(5-2)^4 - 10\cdot(5-4)^2 = 23.33$
pkt D	x = 6	$EJ_y w'(6) = 14 + 6^2 - 9\cdot(6-2)^2 + 1.66\cdot(6-2)^3 - 20\cdot(6-4) = -27.76$ $EJ_y w(6) = 0$

Wykres ugięcia belki



$w_{max} \leq w_{dop}$

$$w_{max} = \frac{30.66}{E\,Jy}\ [\frac{kNm^3}{GPa}] = \frac{30.66}{205 \cdot 195.3} \cdot 10^{-6} \cdot a^{-4} \cdot m^5 = 0.0007658 \cdot 10^{-4} \cdot a^{-4} \cdot m^5 =$$
$$= 7.658 \cdot 10^{-10} \cdot a^{-4} \cdot m^5$$

$$w_{dop} = \frac{l}{250}\,m = \frac{6}{250}\,m = 0.024\ m$$

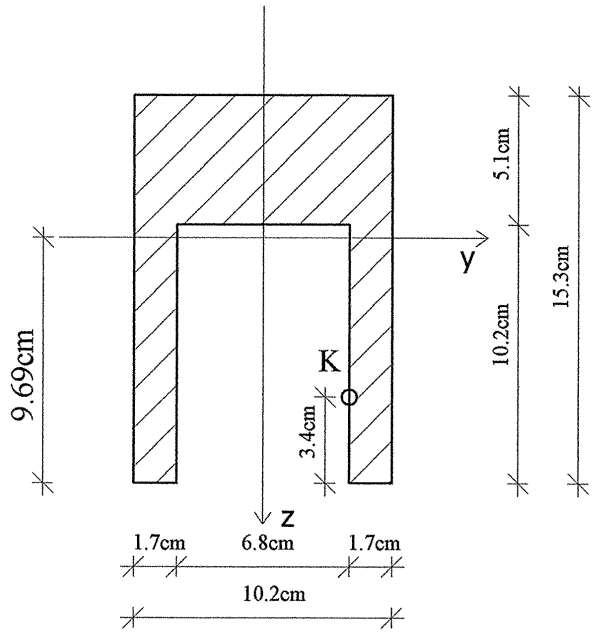
$7.658 \cdot 10^{-10} \cdot a^{-4} \cdot m^5 \leq 0.024\ m$
$7.658 \cdot 10^{-10} \cdot m^4 \leq 0.024 \cdot a^4$
$2.4 \cdot 10^{-2} \cdot a^4 \geq 7.658 \cdot 10^{-10} \cdot m^4$
$a^4 \geq 7.658 : 2.4 \cdot 10^{-8} \cdot m^4$
$a \geq 1.33 \cdot 10^{-2} \cdot m$
a ≥ 1.33 cm

PODSUMOWANIE:

a ≥ 1.67 cm ∧ **a ≥ 0.435 cm** ∧ **a ≥ 1.33 cm**

Przyjmujemy do obliczeń:

a = 1.7cm



ROZKŁAD NAPRĘŻEŃ NORMALNYCH W PRZEKROJU α-α

$J_{yo} = 195.3 \cdot a^4 = 195.3 \cdot (1.7)^4 \cdot 10^{-8}\ m^4 = 195.3 \cdot (1.7)^4 \cdot 10^{-8}\ m^4 =$
$= 1631.16 \cdot 10^{-8}\ m^4 = 0.1631 \cdot 10^{-4}\ m^4$

$M^{\alpha-\alpha} = 7\ kNm \qquad \sigma_x = \frac{My}{Jy} \cdot z,$

$$\frac{My}{Jy} = \frac{7}{0.1631} \cdot 10^7 \cdot N \cdot m^{-3} = -42.92 \cdot 10^7 \cdot N \cdot m^{-3} = -4.292 \cdot 10^8 \cdot N \cdot m^{-3}$$

$\sigma_x(z = 10.2) = -4.292 \cdot 10^8 \cdot N \cdot m^{-3} \cdot 0.102\ m = 0.4377 \cdot 10^8 \cdot N \cdot m^{-2} = -43.77\ MPa$

$\sigma_x(z = 0.51) = -4.292 \cdot 10^8 \cdot N \cdot m^{-3} \cdot (-0.0051)\ m = 0.02188 \cdot 10^8 \cdot N \cdot m^{-2} = 2.188\ MPa$

$\sigma_x(z = -5.61) = -4.292 \cdot 10^8 \cdot N \cdot m^{-3} \cdot (-0.0561)\ m = -0.2407 \cdot 10^8 \cdot N \cdot m^{-2} = 24.07\ MPa$

ROZKŁAD NAPRĘŻEŃ STYCZNYCH PRZEKROJU α-α

$J_{yo} = 0.1631 \cdot 10^{-4}\ m^4 \qquad F_z^{\alpha-\alpha} = 6\ kN$

$$\tau_{max}(z) = \frac{Fz \cdot Sy(z)}{Jyo \cdot b(z)}$$

$z = -0.0561\ m \qquad b = 0.102\ m$
$S_y(-5.61cm) = 0$
$\tau_{max} = 0$

$z = -0.0051\ m \qquad b = 0.102\ m$
$S_y(-0.51cm) = 10.2 \cdot 5.1 \cdot 2.55 \cdot 10^{-6}\ m^3 = 132.651 \cdot 10^{-6}\ m^3$
$$\tau_{max} = \frac{6 \cdot 132.651}{0.1631 \cdot 0.102} \cdot 10\ Pa = 4.78 \cdot 10^4 \cdot 10\ Pa = 0.478\ MPa$$

$z = -0.0051\ m \qquad b = 0.034\ m$
$S_y(-0.51cm) = 132.651 \cdot 10^{-6}\ m^3$
$$\tau_{max} = \frac{6 \cdot 132.651}{0.1631 \cdot 0.034} \cdot 10\ Pa = 1.43 \cdot 10^5 \cdot 10\ Pa = 1.43\ MPa$$

$z = 0\ m \qquad b = 0.034\ m$

$$S_y(0) = 132.651\cdot10^{-6} + 2\cdot(1.7\cdot0.3\cdot1.7\cdot0.255)\ m^3 =$$
$$= 132.651\cdot10^{-6} + 0.4421\cdot10^{-6}\ m^3 = 133.093\cdot10^{-6}\ m^3$$
$$\tau_{max} = \frac{6\cdot133.093}{0.1631\cdot0.034}\cdot 10\ Pa = 1.44\cdot 10^5\cdot 10\ Pa = 1.44\ Mpa$$

z = 0.0969 m b = 0.034 m

S_y (9.69cm) = 0

$\tau_{max} = 0$